GEORG-AUGUST-UNIVERSITÄT
GÖTTINGEN

Price Discovery Measures in Agricultural Economics

Dissertation

zur Erlangung des Doktorgrades

der Fakultät für Agrarwissenschaften

der Georg-August-Universität Göttingen

vorgelegt von

Evgenia Pavlova

geboren in Moskau, Russland

Göttingen, Februar 2018

D 7

1. Referent: Prof. Dr. Stephan von Cramon-Taubadel

2. Korreferent: Prof. Dr. Thomas Kneib

Tag der mündlichen Prüfung: 22. Februar 2018

Acknowledgements

I want to say thank you to everyone who helped me on my way to my Ph.D., because I could have not accomplished it by myself.
To my supervisor, Stephan, for believing in me, giving me the freedom to research, and for always being supportive and encouraging. I would like to thank you especially for finding time and being there during my first “foreign” presentation in Boston in 2016.
To the head of our RTG, Thomas, I thank you for all your help with my thesis defense and the time leading up to this, and for always being available to talk and to listen.
To Barbara Strauß und Dede Dörte for all your support all these years and for creating the incredible atmosphere in our RTG.
To all my colleagues from Courant Center and DARE Department, I am lucky to have been a part of such an amazing and diverse team.
To my parents, step-parents and grandparents, thank you for believing in me no matter what and for being there for me despite the distance.
To all of you and to everyone else who was there with me on the way, no matter how long or how far, I thank you sincerely.

Contents

Introduction

The notion of price discovery has been of interest for many researches in the last decades, mostly in application to the financial markets (see eg. *Journal of Financial Markets, 2002 (5)* where the whole issue is devoted to the subject of price discovery). Why is it so?

For any market participant it is important to understand where price information is being produced. Suppose we have a set of informationally-linked markets trading some homogeneous commodity (or asset) at prices that are compatible with the Law of One Price (LOP). If upon arrival of the new information each of the markets sets a different new price, subsequent intermarket arbitrage will prevent the prices from drifting too far apart from each other and eventually force them to return to the LOP values. Some market (or markets) will play a leading role in this process, i.e. vary their price only a little, and thus dominate the price discovery, whereas remaining markets will be mostly adjusting to the new price level. Econometrically speaking, in this case prices are $I(1)$ cointegrated variables, sharing one (or, more rarely, several) common stochastic trends or factors. This common stochastic factor is referred to as the unobservable efficient price, and price discovery can be viewed as the process of uncovering this fundamental value.

Let P_t^* denote the unobservable permanent price that reflects the *fundamental value* of a commodity (an asset). The observable market price P_t is distinct from it and can be decomposed into two components

$$P_t = P_t^* + \varepsilon_t, \tag{0.0}$$

where ε_t stands for various transitory effects, market noise etc.

More detailed, price discovery is the process by which security or commodity markets attempt to identify permanent changes in the equilibrium transaction prices. However, there is no unambiguous definition of price discovery among researches. For example, the above definition is consistent with Harris' view of price discovery as "the process by which security markets attempt to identify permanent changes in equilibrium transaction prices" ([Harris et al., 2002b, p.2]), understanding it in broad terms as prices reacting to new information.

At the same time, Hasbrouck in [Hasbrouck, 1995] defines price discovery as "who moves first in the process of price adjustment", focusing attention on the *speed* component of the process. Some other studies ([Cao et al., 2009]) interpret price discovery by how informative prices are in depicting the true permanent value. In a special issue of *Journal of Financial Markets*, mentioned above, price discovery was defined as "efficient and timely incorporation of the information ... into market prices", highlighting both *speed* and *efficiency* characteristics of the process. There are other studies that accept this definition, eg. [Yan and Zivot, 2010] and [Putnins, 2013].

Price discovery was historically considered to be one of the central functions of secondary markets and thus predominantly applied to the analysis of the highly homogeneous financial assets. Therefore, existing literature on the price discovery has concentrated mostly on financial markets.

One of the first price discovery models is found in [Garbade and Silber, 1983] and considers the prices on spot and futures markets. Financial futures markets do not only allow risk transferring or hedging or provide an opportunity for speculation, but also facilitate the process of price discovery. To understand why, assume that the spot and futures prices both measure the permanent price P_t^* with some error. Then the price discovery quantifies the contribution of spot and futures prices to the revelation of P_t^* and allows understanding which markets dominates, and which is a leader's follower. The model is used to analyze price discovery in various markets, eg. of the financial assets for soft commodities like wheat, corn, oats, orange juice and a number of hard commodities. The authors find that futures play crucial role in price discovery on almost any market, since more than 75% of their pricing occurs in the futures market.

Up until now substantial amount of studies, both applied and theoretical, consider price discovery in relation to spot and futures markets, including those for agricultural commodities, cf. eg. [Kumar, 2004], [Hernandes and Torero, 2010].

However, since the present study focuses on the *spatial* price discovery across markets (same or similar commodity traded across geographical regions, countries, regions of the same country etc.) rather than the duality between spot and futures markets, we will concentrate on the following price discovery measures that are applicable to our research.

1. Permanent-Transitory decomposition in [Gonzalo and Granger, 1995], hereafter referred to as PT, and
2. Information Shares in [Hasbrouck, 1995], referred to as IS.

PT measure is based on the Vector Error Correction Model (VECM) of the prices time series and identifies long-run effect as a component that is not Granger caused by some temporary changes. From this, a special linear combination of the variables is build, and the coefficients of this combination represent the price discovery measure. Unlike this, IS relies on the Vector Moving Average (VMA) representation of the prices system, and defines price discovery as the percentage of total variance, explained by each variable.

There has been a substantial amount of research on the subject of similarities and differences between these two main approaches. [de Jong, 2002], [Baillie et al., 2002] and [Lehmann, 2002] compare them and show that the approaches are closely related (at least for financial assets). [Lehmann, 2002] suggests that it is desirable to report both metrics when analyzing a particular market in order to get a full picture of price discovery process. [Baillie et al., 2002] come to a conclusion that it is impossible to pick a superior measure, however, IS has more general economic appeal and interpretation.

As mentioned above, the vast majority of existing studies on the subject analyzes highly homogeneous financial assets. The aim of this thesis is therefore somewhat explorative: to investigate whether the price discovery approach can provide useful insights into pricing on agricultural markets.

Agricultural commodities differ from financial instruments in a number of ways, most importantly in less frequent observations available for the analysis: one trading of day stock exchange provides 21,600 observations, whereas much agricultural price data is only weekly or monthly. Another difference is that the same agricultural commodities traded at different venues are less homogeneous than identical financial assets. For latter it is usually assumed that the cointegrating vector is $(1, -1)$, which is not necessarily the case in agricultural commodity settings. We account for these special characteristic and show how they influence the price discovery measures.

Our empirical analysis focuses on the pork meat market in European Union (EU) in the period spanning almost three decades from 1987 to 2015. This choice is well justified: pork is the most consumed and produced meat in many countries of the EU[1], and pork meat industry plays an important role in the EU economics[2]. Many prominent researches were devoted to the analysis of price processes in the pork markets worldwide, cf. eg. [Holst and von Cramon-Taubadel, 2013], [Goodwin and Harper, 2000], [Bakucs and Ferto, 2005], however focusing mainly on the horizontal (spatial) or vertical price transmission. Price discovery analysis of the agricultural markets is still in its early stages, and to our knowledge there are currently no studies analyzing price discovery of the commodity pork meat market.

This thesis is organized as follows: in Chapter 1 we introduce different price discovery metrics in greater detail, outline differences between agricultural commodities and financial assets as well as their impact on the price discovery metrics, and present the results of the empirical analysis. Chapter 2 expands these findings to the multivariate case, also including empirical application. In Chapter 3 we explore alternative methodology to the decomposition of the time series variance, State Space (SS) model. Technical calculations are carried out in Appendices A - C.

[1]Cf. eg. recent OECD Report https://data.oecd.org/agroutput/meat-consumption.htm
[2]Eurostat: Meat Production Statistics

Chapter 1

Price Discovery in the Bivariate Case

After an overview of the literature on the subject of price discovery we now turn to the practical aspects of the methodology. We start by analyzing the bivariate case of two countries trading the same agricultural commodity and discuss the process of calculating the PT and IS price discovery measures, including outlying potential drawbacks of the approaches and possible modifications. We also state a number of specific features of the agricultural commodities that distinguish them from the financial assets. The Chapter closes with the empirical analysis of the pork meat prices in four European countries in 1987-2015, performed pairwise for each combination of the countries.

1.1 Classical Price Discovery measures

1.1.1 PT measure

Consider two $I(1)$ time series y_{1t} and $y_{2t} := y_t \in \mathbb{R}_{2\times T}$ that are cointegrated with vector $\beta = (\beta_1, \beta_2)$, meaning that the linear combination $z_t = \beta_1 y_{1t} + \beta_2 y_{2t}$ is stationary or $I(0)$ process.

[Stock and Watson, 1988] show that if the series are cointegrated, there must be a common factor representation of the form

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} -\beta_2 \\ \beta_1 \end{pmatrix} f_t + \begin{pmatrix} \tilde{y}_{1,t} \\ \tilde{y}_{2,t} \end{pmatrix}, \tag{1.1}$$

where f_t (common factor) depicts the long-run dynamics of the system. The above equation allows us to decompose the time series into a permanent component f_t that represents the long-run equilibrium, and a cyclical (or transitory) component $(\tilde{y}_1, \tilde{y}_2)'$[1].

Estimation of f_t is based on two conditions:

1. f_t is a linear combination of $(y_{1,t}, y_{2,t})$,
2. the transitory component $(\tilde{y}_{1,t}, \tilde{y}_{2,t})'$ has no permanent effect on the variables $(y_{1,t}, y_{2,t})'$, meaning that f_t solely represents the long-run behavior of the system.

The starting point for computation of f_t is the Vector Error Correction Model (VECM) of the form

$$\Delta y_t = \underbrace{\alpha\beta' y_{t-1}}_{long-run effect} + \underbrace{\sum_{j=1}^{k} A_j \Delta y_{t-j}}_{short-run effects} + e_t, \quad t = 1, \dots T \tag{1.2}$$

where $\beta \in \mathbb{R}_{2\times 1}$ is a cointegrating vector, $\alpha \in \mathbb{R}_{2\times 1}$ is an error-correction vector and $e_t = (e_{1t}, e_{2t})' \in \mathbb{R}_{2\times T}$ are innovations with zero mean and covariance matrix Ω

$$\Omega = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}. \tag{1.3}$$

[1] Alongside with the price discovery analysis, estimating this common factor can also help to analyze cointegration in large systems, where the model is very complex and one is only interested in an information provided by a smaller set of long-run factors, cf. [Gonzalo and Granger, 1995]

It can be shown (cf. [Gonzalo and Granger, 1995, Proposition 2]) that the only linear combination of (y_{1t}, y_{2t}) satisfying the above conditions is

$$f_t = \gamma y_t, \tag{1.4}$$

where $\gamma = (\gamma_1, \gamma_2)$ is a vector orthogonal to error-correction vector $\alpha = (\alpha_1, \alpha_2)$.

Since γ (as a linear combination) represents the weights of both prices in the common factor, it can also be viewed as a price discovery metric, meaning that the price that moves closer to the common factor must be dominating the process of price discovery. The price that shows greater adjustment to the common factor is hence the one that is following the leader. The PT measure of price discovery is based on this considerations and is therefore defined by γ.

Estimation of γ as an orthiogonal vector in the bivariate case is trivial. In order to interpret it as a price discovery metric and to ensure that the measures for two countries sum up to 1 (or 100%), we impose additional normalization condition of $\gamma_1 + \gamma_2 = 1$. We then get that

$$PT_1 = \gamma_1 = \frac{\alpha_2}{\alpha_2 - \alpha_1}, \quad PT_2 = \gamma_2 = \frac{\alpha_1}{\alpha_1 - \alpha_2}. \tag{1.5}$$

The fact that the common factor weights γ is orthogonal to α is intuitive, cf. for example [Gonzalo and Granger, 1995] for an elaborate example concerning the macroeconomical model of GDP and consumption. According to the model, income shows adjustment to the error-correction term, but consumption doesn't, i.e. $\alpha = (0, 1)'$ and so $\alpha_\perp := \gamma = (1, 0)'$. The whole weight in the common factor goes to consumption, and more "dependent" income that shows 100% adjustment does not play any role in the price discovery process.

According to (1.5), PT measure is non-negative (and hence interpretable), if

$$\begin{matrix} \alpha_2 \geq 0 & \text{or} & \alpha_2 \leq 0 \\ \alpha_2 > \alpha_1 & & \alpha_2 < \alpha_1, \end{matrix}$$

which as we will see in Section 1.3, is not always the case. This is a limitation of the PT approach to the price discovery.

1.1.2 IS measure

Let us now consider the second price discovery measure. Unlike the above, Hasbrouck uses the Vector Moving Average (VMA) representation of the equation (1.2)

$$\Delta y_t = \Psi(L) e_t, \tag{1.6}$$

where $\Psi(L)$ is a matrix polynomial in the lag operator L. The metric is derived with the integrated form of VMA, given as

$$y_t = \Psi(1) \sum_{s=1}^{t} e_s + \Psi^*(L) e_t. \tag{1.7}$$

Here, the matrix $\Psi(1)e_t \in \mathbb{R}_{2\times 2}$ connects the variables y_t and the shocks, or innovations, e_t and thus represents the permanent impact of these market innovations on both prices. If eg.

$$Psi(1) = \begin{pmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{pmatrix},$$

then $\psi_{11} e_{1t} + \psi_{12} e_{2t}$ is the long-run impact of innovations on the first price.

The IS price discovery measure is defined as a share of the total variance of innovations explained by each of the time series shocks. In the above example, we would want to decompose $var(\psi_{11}e_{1t} + \psi_{12}e_{2t})$ to account for the variance percentage explained by e_{1t} and e_{2t}, respectively. These variances are found in the diagonal elements of the matrix $\Psi(1)\Omega\Psi(1)'$ with Ω in (1.3).

Before we turn to the formal derivation of the IS metric, note that Hasbrouck's model was initially developed for the analysis of the financial markets where the following assumptions are usually made:

1. cointegrating vector $\beta = (1, -1)$, and
2. the effect of innovation is the same for both prices, meaning that the rows of the matrix $\Psi(1)$ are identical, i.e.

$$\Psi(1) = \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_1 & \psi_2 \end{pmatrix} = \begin{pmatrix} \psi \\ \psi \end{pmatrix}. \tag{1.8}$$

Furthermore, both models (1.2) and (1.7) are connected through the following relation between VECM and VMA (cf. [Johansen, 1991, Theorem 4.1]):

$$\Psi(1) = \beta_\perp \Pi \alpha_\perp^{'}, \tag{1.9}$$

where

$$\Pi = \left(\alpha_\perp^{'} \left(I - \sum_{j=1}^{k} A_j\right) \beta_\perp\right)^{-1}. \tag{1.10}$$

It is easy to see that in the bivariate case matrix Π is a scalar (if there is one common factor in the system). Furthermore, from the assumption of $\beta = (1, -1)$ it follows that $\beta_\perp = (1, 1)$. Then from (1.8) and (1.9)

$$\Psi(1) = \Pi \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_1 & \gamma_2 \end{pmatrix}, \tag{1.11}$$

and

$$\frac{\psi_1}{\psi_2} = \frac{\gamma_1}{\gamma_2}, \tag{1.12}$$

cf. [Baillie et al., 2002]. Therefore, in this case the estimation of VMA is even not required, since the matrix $\Psi(1)$ can be recovered from the VECM representation (1.2).

Since the matrix $\Psi(1)$ has identical rows, the total variance of innovations $var(\psi_{11}e_{1t} + \psi_{12}e_{2t})$ becomes

$$var(\psi e_t) = var\left(\psi_1 e_{1t} + \psi_2 e_{2t}\right) = \psi_1^2\sigma_1^2 + 2\psi_1\psi_2\rho\sigma_1\sigma_2 + \psi_2^2\sigma_2^2.$$

If the innovations across markets are uncorrelated ($\rho_{e1t,e_{2t}} = 0$ and matrix Ω is diagonal), then

$$var(\psi e_t) = \psi_1^2\sigma_1^2 + \psi_2^2\sigma_2^2, \tag{1.13}$$

and the information share of the market $i, i = 1, 2$ becomes

$$IS_i = \frac{\left(\psi_i^2\sigma_i^2\right)^2}{\psi_1^2\sigma_1^2 + \psi_2^2\sigma_2^2} = \frac{\left(\gamma_i^2\sigma_i^2\right)^2}{\gamma_1^2\sigma_1^2 + \gamma_2^2\sigma_2^2}, \tag{1.14}$$

calculated by using solely the VECM representation in (1.2).

However, the assumption of the uncorrelated innovations is not very realistic, i.e. in general $\rho_{e1t,e_{2t}} \neq 0$ and (1.13) cannot be applied. To eliminate the contemporaneous correlations, Hasbrouck uses the Cholesky decomposition of matrix $\Omega = MM'$, where M is a lower triangular matrix

$$M = \begin{pmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sigma_2\sqrt{1-\rho^2} \end{pmatrix}. \tag{1.15}$$

In this case, the metric becomes

$$IS_i = \frac{([\psi M]_i)^2}{\psi \Omega \psi'}, \tag{1.16}$$

or more specifically, for a bivariate case,

$$IS_1 = \frac{(\gamma_1 m_{11} + \gamma_2 m_{21})^2}{(\gamma_1 m_{11} + \gamma_2 m_{21})^2 + (\gamma_2 m_{22})^2}, \quad IS_2 = \frac{(\gamma_2 m_{22})^2}{(\gamma_1 m_{11} + \gamma_2 m_{21})^2 + (\gamma_2 m_{22})^2}. \tag{1.17}$$

It should be noted that the attribution of information in Cholesky decomposition depends on the ordering of the variables preliminary to the decomposition (i.e. whether y_1 or y_2 is considered to be "first" or "second" market price): each ordering would render different M and hence different IS values. It is therefore common to compute the IS measure for both orderings and then take a mean value as a final estimate, cf. eg. [Hasbrouck, 1995], [Hasbrouck, 2002], [Putnins, 2013]. In the analysis of financial markets this approach proved to be justified, due to high data frequency and hence narrow bands of the IS metric under alternative orderings. In the analysis of agricultural markets and commodities high frequency data is rarely available, which makes this approach questionable. We explore possible alternatives for the calculation of the IS metric below.

1.1.3 Structural Vector Autoregressive model

In Section 1.1.2 we saw that the main reason for the discrepancies in the IS measures under different orderings of the variables is the fact that the innovations in (1.7) are correlated. Model (1.7) is referred to as a *reduced form* VMA, that is equivalent to the following *reduced form* Vector Autoregressive (VAR) model

$$y_t = a_0 + A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + e_t, \tag{1.18}$$

where $a_0 \in \mathbb{R}_{2\times 1}$ is a vector of constants and $A_i \in \mathbb{R}_{2\times 2}$, $i = 1, \dots p$ are the time invariant coefficient matrices. The shocks, or innovations e_t are serially uncorrelated, but can be correlated with each other. Let $\Omega := \mathbb{E}(e_t e_t')$ denote the covariance matrix of innovations as in (1.3).

There is however an alternative representation of (1.18) that takes care of the correlation issue. It is called the Structural Vector Autoregressive (SVAR) model and is obtained from (1.18) by means of multiplying both parts of the equation with a certain matrix $B \in \mathbb{R}_{2\times 2}$,

$$
\begin{aligned}
By_t &= Ba_0 + BA_1 y_{t-1} + BA_2 y_{t-2} + \dots + BA_p y_{t-p} + Be_t, \\
By_t &= b_0 + B_1 y_{t-1} + B_2 y_{t-2} + \dots + B_p y_{t-p} + u_t, \quad t = 1, \dots, T
\end{aligned} \tag{1.19}
$$

with the shocks u_t are now orthogonal and uncorrelated. These shocks are referred to as *structural* innovations, as opposed to *reduced-form* innovations e_t from (1.18). The covariance matrix of u_t is normalized such that

$$
\mathbb{E}(u_t u_t') := \Sigma = I_2. \tag{1.20}
$$

The connection between the structural and the reduced-form VAR is given by

$$
u_t = Be_t \qquad \text{or, alternatively} \qquad e_t = B^{-1} u_t. \tag{1.21}
$$

Recall the Cholesky decomposition of the covariance matrix $\Omega = MM'$ with M lower triangular matrix in (1.15). From this and the fact that u_t have unit variances one could derive $e_t = Mu_t$ and hence $B^{-1} := M$. This is, however, only one possible admissible solution. Let Q a square orthogonal matrix from a class of orthogonal matrices $\mathcal{Q}$ with $QQ' = Q'Q = I_2$, then $\Omega = MQQ'M' = MM'$ and therefore,

$$B^{-1} = MQ. \tag{1.22}$$

The central question is of course regarding the identification of matrix B (and then, Q). For the covariance matrix Ω of e_t we have

$$\Omega = (e_t e_t^{'}) = \mathbb{E}(B^{-1} u_t u_t^{'} B^{-1'}) = B^{-1}\mathbb{E}(u_t u_t^{'})B^{-1'} = B^{-1}B^{-1'},$$

and due to the symmetry of Ω, we have to fix 1 parameter in matrix B, since there are 4 parameters altogether and 3 restrictions implied by orthonormality.

There are several major types of identifying restrictions. One idea is to use economic theory: structural innovations u_t are required to have economical meaning and interpretation, from which one can infer the structure of the matrix B. Most common here are exclusion (or zero) restrictions and sign restrictions.

As the name suggests, zero restrictions imply that some elements of the B^{-1} matrix are 0, i.e. some shock(s) have no impact on some variable(s). Eg. [Kilian, 2009] analyses global crude oil market and assumes that the demand shocks do not affect the production within the same time period. These restrictions can be short-run, and focus on the contemporaneous effects, as well as long-run, considering the coefficients in the long-run effect matrices B_1, B_2, etc. instead.

Sign restrictions are based on the identification involving the sign of some shocks on some variables. This approach is more flexible than the zero restriction in a sense that it would produce the whole set of admissible matrices B^{-1}. The approach goes back to [Uhlig, 2005] who notes that sign restrictions allow the researcher to "throw out all impulse responses inconsistent with some given set of theories, some of which are at odds with the conventional wisdom". It is also possible to, say, combine the sign and exclusion restrictions, cf. eg. [Mountford and Uhlig, 2009].

Finally, there is another class of identifying restrictions in the SVAR models that are not economically motivated, but make use of the statistical properties of the data instead, e.g. heteroscedasticity restriction. Here one assumes that the covariance structure of the model has changed at least once during the observation period. The approach requires knowledge of the time of the break, though there are some statistical procedures to help the identification, cf. eg. [Lanne et al., 2010]. Another group of models takes advantage of non-normality of the data and constructs the structural shocks u_t that are stochastically independent rather than uncorrelated, eg. [Herwartz and Plodt, 2016].

After the matrix B^{-1} is identified, we could infer the Structural Vector Moving Average (SVMA) representation from (1.7) using $e_t = B^{-1}u_t$. Then $\Psi(1)e_t$ becomes $\Psi(1)B^{-1}u_t := \Theta u_t$. Covariance matrix of the structural shocks u_t is $\Sigma := I$, and hence the variance decomposition is done by means of calculating $\Theta\Theta'$. On the main diagonal of this matrix we find the variances of innovations, and the portions explained by the processes on the market $i, i = 1, 2$ can be found with the help of the i-th row of the matrix Θ. Therefore, the IS measure becomes

$$IS_{ij} = \frac{\left([\Theta]_{ij}\right)^2}{[\Theta\Theta']_{ii}}, \tag{1.23}$$

with IS_{ij} being the percentage of the total variation on the market i explained by the market j[2].

Due to its flexibility, in the present work we apply sign restrictions to identify matrix B^{-1}. Zero restrictions may be too exclusive and lack economic reasoning to say that shocks in one market have no impact on the shocks in the other market. Moreover, in a bivariate case they provide the same result as ordering the variables in Cholesky decomposition in VAR. Instead, we assume that each market reacts

[2] Cf. Section 1.2.3 for further details

positively on its own shocks, i.e. the structure of the matrix B would be

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} = \begin{pmatrix} + & * \\ * & + \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \tag{1.24}$$

We have no apriori knowledge about the signs of other elements, i.e. how market reacts on the shock coming from another market.

As we will see in Section 1.3, this will produce a very broad spectrum of matrices B and hence, the IS measures that can differ a lot. Indeed, under (1.24) it is both permissible to have the first market as a leader in price discovery as well as the follower. The problem identified in the VAR analysis in Section 1.1.2 is therefore not solved. It might be beneficial for further research to consider other identifying restrictions, especially those based on non-normality, and calculate IS measures based on these assumptions. The sign restrictions applied in the present work should be viewed as a reasonable assumption to investigate the applicability and performance of the SVAR methodology. Currently there are not many studies applying SVAR to the price discovery analysis in the agriculture. From the works known to us, the similar approach with sign restrictions was implemented in [Pozo et al., 2016] for the multivariate analysis of live cattle's market.

Bearing the above in mind, the algorithm to calculate the IS price discovery measure is as follows:

1. Estimate A_1 and Ω from VAR (1.18)

2. Calculate lower triangular matrix M from the Cholesky decomposition of Ω

3. Draw orthogonal matrix Q from $\mathcal{Q}$ and calculate MQ

4. If MQ is admissible (i.e. in agreement with the sign restrictions), then $B^{-1} := MQ$

5. Calculate $\Theta := A_1 \cdot B^{-1}$ (since in VMA and VAR representations $\Psi(1) = A_1$)

6. Variance decomposition as in (1.23)

To construct $\mathcal{Q}$ and draw orthogonal $Q \in \mathcal{Q}$ Givens rotation matrices are used. Those have the form

$$Q(\phi) = \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ sin(\phi) & cos(\phi) \end{pmatrix}, \quad \phi \in [0, 2\pi]. \tag{1.25}$$

The value of ϕ is sampled randomly from $[0, 2\pi]$ to produce matrix Q, and this process is repeated multiple times. Alternatively, a grid for ϕ can be defined between these values, and for each point the matrix is calculated and then used in the algorithm above.

1.1.4 ILS measure

As noted above, different price discovery measures are based on the different definitions of the price discovery process itself. Some studies (cf. [Yan and Zivot, 2010]) argue, that in the number of cases PT and IS measures only capture either the *speed* or *accuracy* of the price discovery, whereas the process is simultaneously defined by both of these characteristics. According to these studies, two measures provide an adequate estimator for price discovery only if the time series under consideration display similar noise level.

To account for this potential drawback, another metric was proposed by [Putnins, 2013]: Information Leadership Share (ILS), combining both PT and IS

$$ILS_1 = \frac{IL_1}{IL_1 + IL_2}, \quad ILS_2 = \frac{IL_2}{IL_1 + IL_2}, \tag{1.26}$$

where

$$IL_1 = \left|\frac{IS_1 PT_2}{IS_2 PT_1}\right|, \quad IL_2 = \left|\frac{IS_2 PT_1}{IS_1 PT_2}\right|. \tag{1.27}$$

In his paper, Putnins provides extensive simulations to demonstrate that in many cases ILS outperforms the classical price discovery measures. However, there is currently no version of the measure for the multivariate case, so it can only be used for the analysis of two time series. Another point is that despite its good performance on the simulated data sets, when applying ILS measure to the real life examples it often provides results that are difficult to interpret (extreme values close to 99% and 1%, even if PT and IS measures for the first time series were eg. 0.8 and 0.5). In Appendix A we analytically investigate possible reasons for this and provide some simulations to visualize our findings. The idea to combine the PT and IS measures is interesting and promising, by accounting for both the speed and the efficiency of the price adjustment one would more precisely extract information from the underlying time series. However, based on the above potential limitations we are not considering the ILS metric in the present analysis.

1.2 Special characteristics of agricultural commodities

Price discovery models are commonly applied to the analysis of financial markets. Our work focuses on the prices of agricultural commodities, that are different in a number of ways. Below we outline some of the distinctions that are relevant for the calculation of the price discovery measures.

1. Stationarity of agricultural prices,

2. Non-homogeneity of agricultural products,

3. Different structure of cointegration relation (long-run equilibrium).

1.2.1 Stationarity of agricultural prices

:

Price discovery measures are derived from the VECM that is linked to the concept of cointegration. Moreover, these two notions are often considered isomorphic. The reason for this is the Granger representation theorem (cf. [Engle and Granger, 1987]), stating that two (or more) integrated $I(1)$ time series that are co-integrated, have an error-correction representation, and two (or more) time series that are error-correcting are cointegrated. However, this isomorphism only folds for the processes that is integrated. The question whether ECM can have applications for not integrated time series remains open.

As mentioned above, cointegration means that linear combination of two $I(1)$ time series, $y_{1,t}$ and $y_{2,t}, t = 1, \ldots, T$ is stationary or $I(0)$.

If this is not true and time series in question are $I(0)$ to begin with, we cannot find a unique cointegration relation, since every linear combination of two $I(0)$ variables will also be $I(0)$. If we believe that cointegration and VECM imply each other, we cannot apply error-correction method to the data in this case.

However, there is another point of view, argued by eg. [Williams, 1993] and [Beck, 1993], for example, to analyze the political data such as Supreme Court approval rates, that is stationary. They maintain that error-correction models were historically developed earlier than the theory of cointegration and are thus are flexible enough to model stationary data. The advantage of VECM is that it allows to decompose time series into long- and short-term effects. One might want to make use of this advantage and model long-term effects even for the processes that are not integrated.

To test whether the data is stationary, the following unit root tests are commonly used:

1. Augmented Dickey-Fuller (ADF) test,

2. Phillips-Perron (PP) test,

3. Elliot-Rothenberg-Stock (ERS) test,

as well as the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) stationarity test. Unit root tests and stationarity tests differ in their null and alternative hypotheses. In the unit root tests, H_1, an alternative, is that time series is stationary, whereas in stationariy tests $I(0)$ is a null hypothesis H_0.

It is not uncommon that different unit root tests provide different results. ADF and PP tests are often criticized for having a low power if the process is stationary but with a root close to the non-stationary boundary (cf. [DeJong et al., 1992]). This drawback, however, should be corrected in the ERS and KPSS tests. This ambiguity has been noted in numerous other studies (e.g. [Hjalmarsson and Oesterholm, 2007]). There is little theoretical reason to expect a strict unit root in economic time series, and the time series with long enough time span may behave as a near-integrated process.

In the present study we analyze European pork market prices, available to as weekly observations. We applied the above tests to four countries (Germany, Netherlands, Belgium, France, uses in further analysis). The testing results are given in Appendix B, for the total data set of $T = 1439$ observations as well as three subsamples of 480 (479) observations each. For the full periods the tests show contradictory results, and therefore we do not have enough evidence to assume that time series are integrated. In these smaller subsets unit root tests uniformly indicate that price time series are $I(1)$. In the following we proceed with the separate analysis of these three subsample periods.

1.2.2 Non-homogeneity of agricultural products

Agricultural products are indeed less homogeneous than the financial assets that are commonly used in the price discovery analysis. However, modern food markets operate with largely "standardized and homogeneous products"[3]. There are studies investigating market integration that come to a conclusion that European national pork markets are closely integrated and hence, pork can be viewed as a homogeneous product, see eg. [Sanjuan and Gil, 2001].

1.2.3 Different structure of cointegration relation

One of the assumptions of the IS measure is that the cointegrating vector $\beta = (1, -1)$, meaning that in the long run, the equilibrium prices on two markets become equal. However, this does not necessarily hold for less homogeneous agricultural commodities.

Consider $\beta = (\beta_1, \beta_2)$ with $\frac{\beta_1}{\beta_2} \neq -1$. Further, let $\beta_\perp = (\beta_{1\perp}, \beta_{2\perp})$ be a vector orthogonal to β.

Now consider

$$\Psi(1) = \begin{pmatrix} \psi_{11} & \psi_{12} \\ \psi 21 & \psi_{22} \end{pmatrix} = \Pi \begin{pmatrix} \beta_{1\perp}\gamma_1 & \beta_{1\perp}\gamma_2 \\ \beta_{2\perp}\gamma_1 & \beta_{2\perp}\gamma_2 \end{pmatrix}. \tag{1.28}$$

In order for the rows of this matrix to be identical (as in the financial markets case) it must hold that

$$\begin{cases} \beta_{1\perp}\gamma_1 = \beta_{2\perp}\gamma_1 \\ \beta_{1\perp}\gamma_2 = \beta_{2\perp}\gamma_2 \end{cases} \Leftrightarrow \quad \beta_{1\perp} = \beta_{2\perp}.$$

[3] [Grebitus et al., 2011]

Therefore, $\beta_{\perp} \propto (1,1)$ meaning that for the cointegrating vector $\beta = (\beta_1, \beta_2)$

$$\beta_1 + \beta_2 = 0,$$

and $\beta \propto (1,-1)$, which contradicts the assumption above. Hence, matrix $\Psi(1)$does not have identical rows and the relation (1.12) does not hold. Therefore, VECM (1.2) does not contain all the information required for the computation of the IS measure, s.t. $\Psi(1)$ must be estimated from (1.7).

1.3 Empirical results

Below we demonstrate application results of calculating the PT and IS price discovery measures for weakly pork meat prices in four European countries: Germany, Netherlands, France and Belgium for the period from 1987 untill 2015.Figure 1.1 depicts the data. The data set was divided into three subsets containing 480 (479) observations, as mentioned in Section 1.2.1:

1. Period 1: March 1987 - July 1996,
2. Period 2: July 1996 - January 2006,
3. Period 3: January 2006 - March 2015.

The reason for this partition is that the whole observation period covers around 30 years, during which the character of the underlying relations and interpedendecies in the system might have changed. We therefore concentrated on shorter periods close to a decade. The dividing time points also correspond to the inclusion of the new EU members (Austria, Finland and Sweden in 1995 and Romania and Bulgaria in 2007) that could have an impact on all price processes in the system.

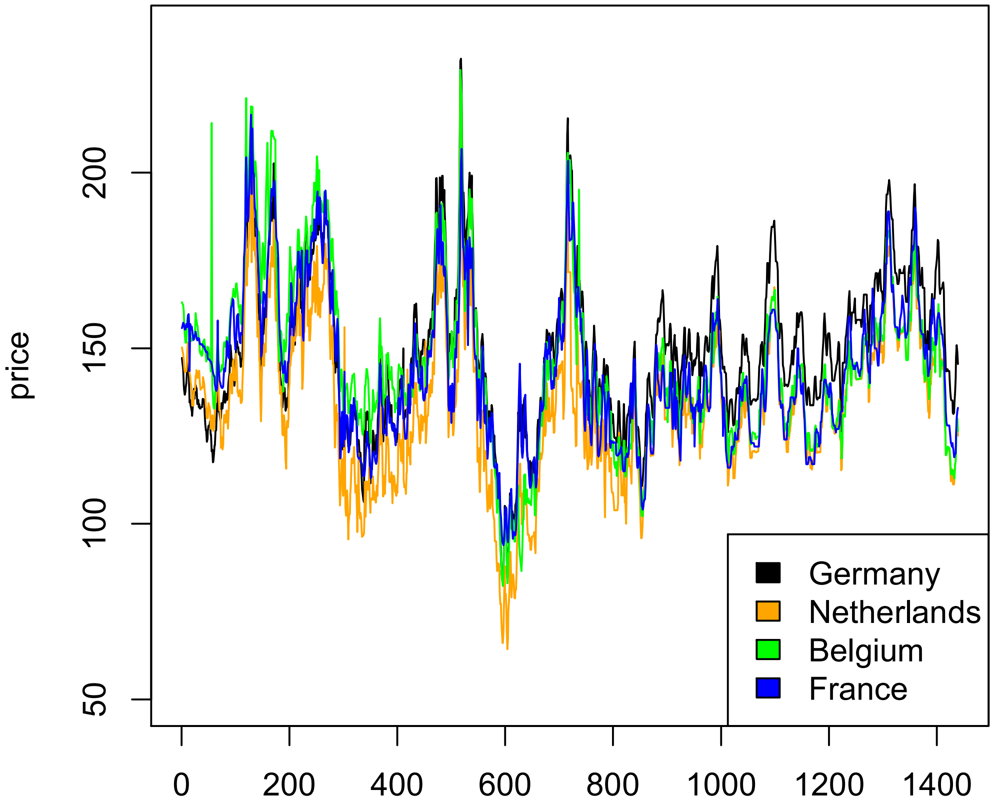


Figure 1.1: Pork meat prices (weekly) for four European countries in 1987-2015

PT price discovery measures calculated for all three periods are given in Table 1.1 to Table 1.3, representing the pairwise relations between countries. Apart from the Germany-Netherlands pair in the first period, all PT values are interpretable. For the Germany-Netherlands case it can also be assumed that Germany is 100% dominating Netherlands in terms of price discovery, as it is done in many empirical studies if the PT values happen to lie outside the $[0, 1]$ bound (cf. [Putnins, 2013]).

The causality relations, according to the PT metric, differ in all three periods. For example, in the first period Germany dominates all other countries, and Belgium always is the follower. However, in the second period this relation changes and Belgium becomes a dominant market to Germany. The third period demonstrates a shift in the dominating role from Belgium to France.

As noted in Section 1.1.4, lower and upper bounds of the IS measure based on VAR and Cholesky decomposition can be very different, depending on the ordering of the variables. To illustrate this, we refer to Table 1.4 the IS measures for the first observation period for Germany-Netherlands, Germany-France and Germany-Belgium are presented. The discrepancies are very dramatic in the majority of cases, interchanging the "leader" and "follower" positions. It is clear that by simply taking the mean value of both calculations would result in biased estimations.

Finally, we calculated the IS measure from the SVAR with sign identifying restrictions for three country pairs Germany-Netherlands, Germany-France and Germany-Belgium and three observational periods. As noted above, this approach may still lack precision in calculating the share of the price discovery, but may also bring new insights in the price discovery process. Therefore, we use only three data pairs to gather the initial insights.

According to the logic of Section 1.1.3 we defined a 10^4 points grid for $\phi \in [0, 2\pi]$ to produce matrices Q, and used only those that were in line with the structural sign requirements for B (1.24). We then calculated the IS measures, where IS_{11} depicts the influence on German market shocks on the price in Germany, IS_{12} - the influence of the Netherlands shocks on Germany, IS_{21} visa versa, and IS_{22} the effect of the Netherlands shocks on its own price.

Table 1.5 shows descriptive statistics of the calculated IS measures for the third observational period 2006-2015 for Germany-Netherlands pair, from which it is clear that the range of values is still very broad and it is currently unclear how to estimate the share of the price discovery in this case.

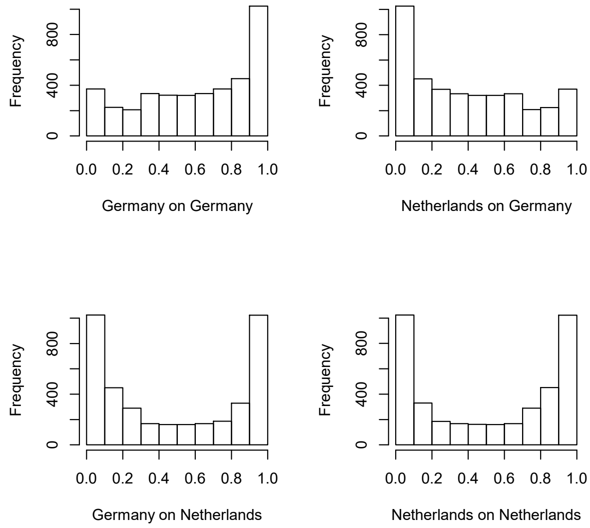
From the 10^4 points ϕ there were 3868 admissible B matrices. Figures 1.2-1.2 visualize the distribution of the IS metrics for the three country pairs in all three periods. One can see again, that the IS values differ from minimum of 0 to the maximum of 1 in all cases. However, judging by the frequencies we can infer, especially in the third period, that German pork meat price is mostly dominated by its own innovations, and the processes on the markets in Netherlands, Belgium and France do not play a crucial role there. We cannot really say anything definitive about the price discovery in Netherlands, Belgium shows slight tendency towards dominating its own price discovery as well as France. To sum up, we were able to recognize some patterns in the data, but the statemets do not allow for precise price discovery assessment. Partially, this may be due to the bias when analyzing the complex multivariate system by bivariate comparisons. On the other hand, further methodological improvements may be beneficiary in rending results that are more interpretable.

	Netherlands	Belgium	France
Germany	1.07 Germany -0.07 Netherlands	0.89 Germany 0.11 Belgium	0.91 Germany 0.09 France
Netherlands		0.54 Netherlands 0.46 Belgium	0.53 Netherlands 0.47 France
Belgium			0.43 Belgium 0.57 France

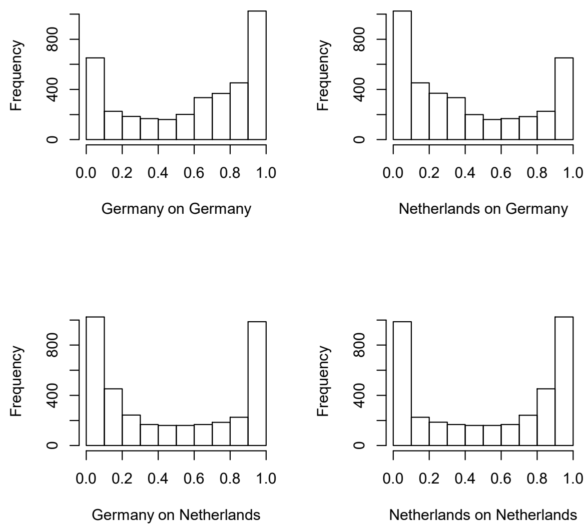
Table 1.1: PT measure for period 1, March 1987-August 1996

	Netherlands	Belgium	France
Germany	0.51 Germany 0.49 Netherlands	0.48 Germany 0.52 Belgium	0.80 Germany 0.20 France
Netherlands		0.23 Netherlands 0.77 Belgium	0.62 Netherlands 0.38 France
Belgium			0.79 Belgium 0.21 France

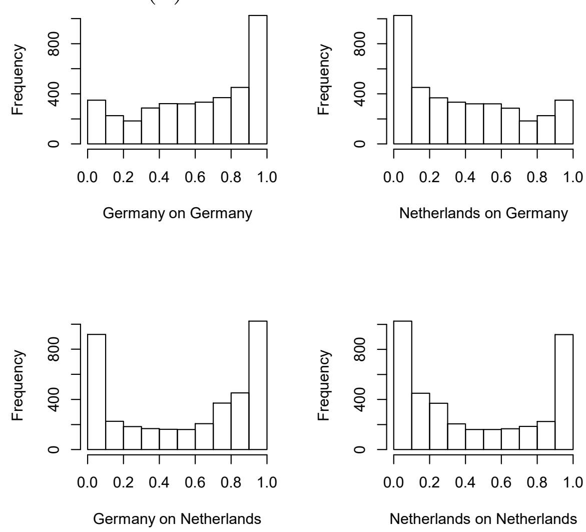
Table 1.2: PT measure for period 2, August 1996-January 2006



(a) Period 1: 1987-1996

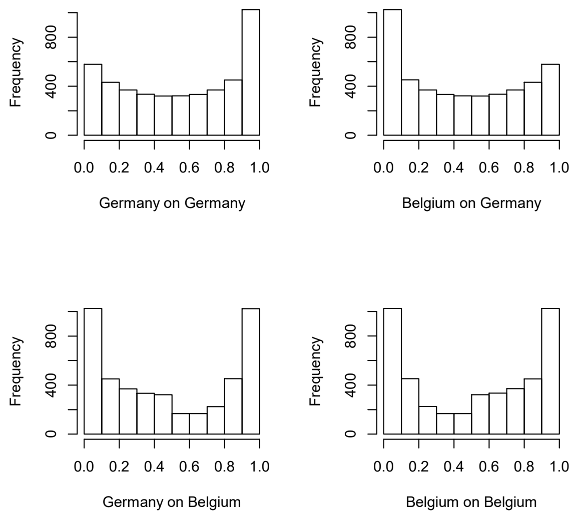


(b) Period 2: 1996-2006

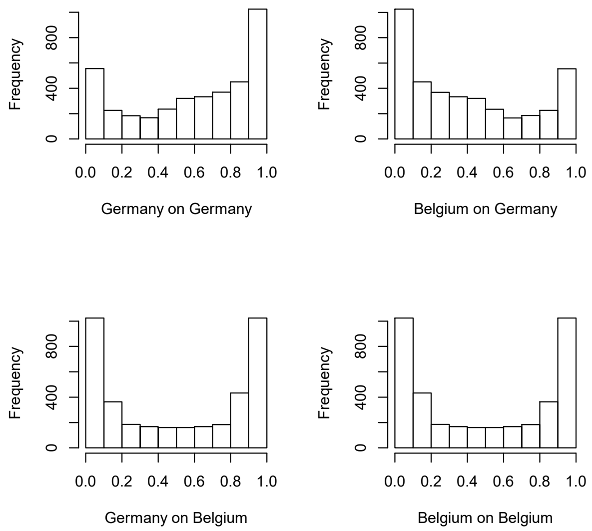


(c) Period 3: 2006-2015

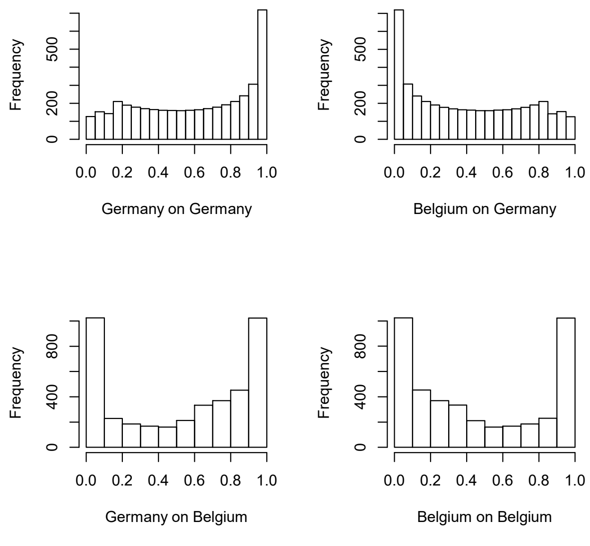
Figure 1.2: Histograms of the IS measure with sign restriction SVAR for Germany and Netherlands, three time periods



(a) Period 1: 1987-1996

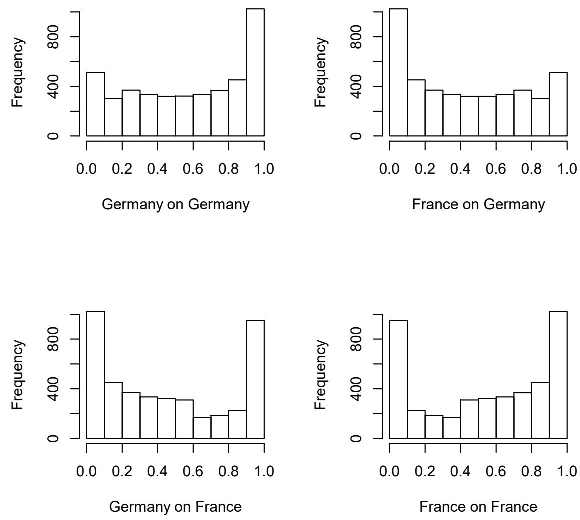


(b) Period 2: 1996-2006

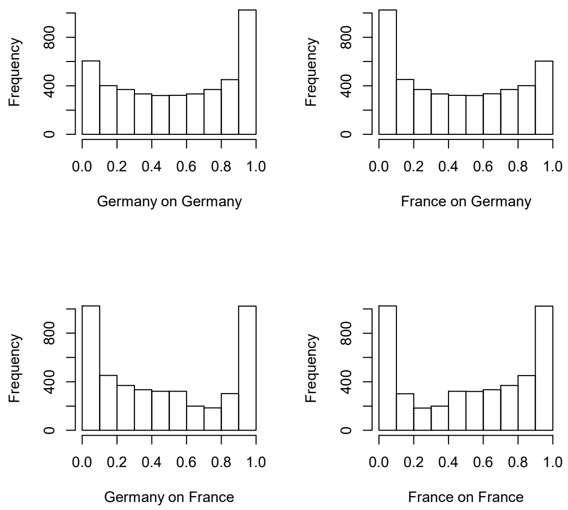


(c) Period 3: 2006-2015

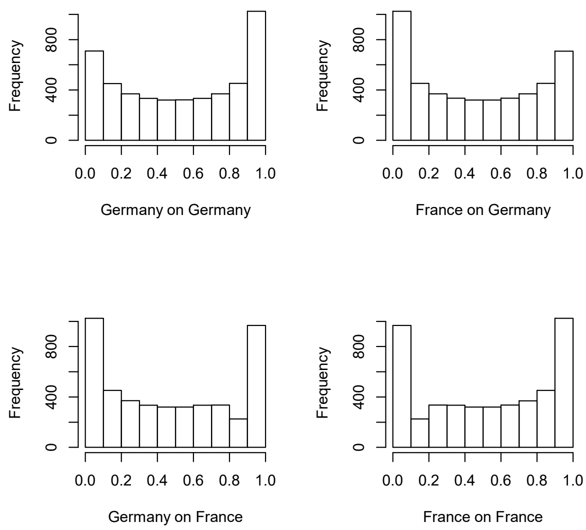
Figure 1.3: Histograms of the IS measure with sign restriction SVAR for Germany and Belgium, three time periods



(a) Period 1: 1987-1996



(b) Period 2: 1996-2006



(c) Period 3: 2006-2015

Figure 1.4: Histograms of the IS measure with sign restriction SVAR for Germany and France, three time periods

	Netherlands	Belgium	France
Germany	0.59 Germany 0.41 Netherlands	0.29 Germany 0.71 Belgium	0.50 Germany 0.50 France
Netherlands		0.19 Netherlands 0.81 Belgium	0.38 Netherlands 0.62 France
Belgium			0.30 Belgium 0.70 France

Table 1.3: PT measure for period 3, January 2006 - March 2015

Germany - Netherlands	0.74 0.29	0.26 0.72
Germany - France	0.40 0.16	0.60 0.84
Germany - Belgium	0.49 0.11	0.51 0.89

Table 1.4: IL measure, upper and lower bounds, for period 1, March 1987-August 1996

	IS_{11}	IS_{12}	IS_{21}	IS_{22}
Min	0.010	0.000	0.000	0.000
1st quartile	0.376	0.089	0.119	0.089
Median	0.674	0.326	0.671	0.329
Mean	0.618	0.383	0.546	0.454
3rd quartile	0.911	0.624	0.911	0.881
Max	1.000	0.999	1.000	1.000

Table 1.5: Descriptive statistic of the IS measure with sign restriction SVAR for Germany and Netherlands, 2006-2015

Chapter 2

Price Discovery in the Multivariate Case

In previous chapter we analyzed bivariate price series. Though bivariate analysis is an important steping stone to understanding price discovery, it fails to consider the price system as a whole, with its complex interdependencies. When important variables are omitted, the model becomes misspecificated, which may lead to inconsistend and/or biased estimators.

In this chapter we expand our analysis to the multivariate case of m countries with an aim to capture direct and indirect connections that may influence price discovery. We study the restrictions that arise in this case, and amendments that need to be done towards the price discovery measures.

Section 2.1 deals with the PT measure. It provides an updated formula for the calculation of the metric for $m \geq 2$ and covers the reasons why it may be rarely applicable in praxis. Section 2.2 is devoted to the IS measure. We apply SVAR approach to transform and orthogonalize the shocks in the model, and hence - to compute distinct IS measures for each country. Section **??** provides the results of the IS calculations for the pork meat prices data in European countries.

2.1 PT measure in the multivariate analysis

As shown in Chapter 1, in a bivariate case we can derive a closed form expression for the PT measure 1.5 using the fact that it is orthogonal to the error-correction vector α, and that the PT measures for both countries under consideration must add up to 1 to ensure their interpretability.

However, if $m > 2$ we cannot use the same approach to calculate the PT metric, but the facts proven by [Gonzalo and Granger, 1995] regarding the permanent-transitory decomposition of the system of time series still stand:

1. In permanent-transitory decomposition of the time series, the transitory component (i.e. temporary misalignments) does not Granger cause the permanent component (i.e. the equilibrium state),

2. This permanent component is a linear combination of the time series in the system, and

3. The required linear combination (and hence the PT measure) is given as $\alpha_\perp$, an orthogonal complement to the speed of adjustment vector (matrix) α.

The task is therefore to compute $\alpha_\perp$ in a general case of $m \in \mathbb{N}$. Following [Johansen, 1988] and [Johansen, 1991] one can choose a set of vectors $\alpha_\perp$ so that the matrix $[\alpha\alpha_\perp]$ is of full rank and $\alpha'\alpha_\perp = 0$. Therefore, one can define and compute the complement

$$\boxed{\begin{array}{l}\alpha_\perp \text{ as eigenvectors, belonging to the unit egenvalues } (\lambda = 1) \\ \text{of the matrix } I - \alpha(\alpha'\alpha)^{-1}\alpha' \end{array} \qquad (2.1)}$$

(for details, cf. [Trefethen and Bau, 1997, Chapter V]). This approach allows us to expand the PT methodology to multidimensional problems. Of course, (2.1) also holds in a bivariate case $m = 2$, as we show in Appendix C.

The procedure to obtain PT measures for $m \geq 3$ involves following steps:

1. Estimating the VECM

2. Extracting the error-correction vector (matrix) α, and then

3. Finding its orthogonal complement $\alpha_{\perp}$.

It holds, that the system of m time series can have up to $m - 1$ cointegrating vectors, therefore $rank(\alpha) \leq m - 1$. In agricultural markets with m time series, it is, however, common to have exactly $m - 1$ cointegrating relations (cf. eg. [Sekhar, 2012], [McNew and Fackler, 1997]), and so

$$dim(\alpha_{\perp}) = 1$$

and the orthogonal complement is therefore a vector. The elements of this vector, $(\alpha_{\perp,1}, \ldots, \alpha_{\perp,n})$ are the individual PT measures and show the fraction of price discovery that is due to each time series.

Let us now examine the interpretability of the PT measure $\alpha_{\perp}$. In order to view it as price discovery, two conditions must be met:

$$\alpha_{\perp,i} \in [0,1] \quad \forall i = 1, \ldots, n, \tag{2.2}$$

and

$$\sum_{i=1}^{n} \alpha_{\perp,i} = 1. \tag{2.3}$$

(2.1) contains no restictions on the elements of the vector $\alpha_{\perp}$. Also, already in a bivariate case we saw that the restrictions analogous to (2.2) - (2.3) were not always met (cf. Section 1.1.1). Hence, it is only logical to assume that the same might happen in the multivariate case.

As before, the main requirement for the elemenst of $\alpha_\perp$ is to be positive: if they partially violate (2.2) and are greater than 1, we can always apply L^1-norm (absolute value norm) to transform the values to be in $[0,1]$ and ensure, that the requirement (2.3) is still met:

$$\alpha_\perp^{new} = \left(\frac{\alpha_{\perp,1}}{||\alpha_\perp||_1}, \frac{\alpha_{\perp,2}}{||\alpha_\perp||_1}, \ldots, \frac{\alpha_{\perp,n}}{||\alpha_\perp||_1} \right),$$

with $||\alpha_\perp||_1 = |\alpha_{\perp,1}| + |\alpha_{\perp,2}| + \ldots + |\alpha_{\perp,n}|$. Then

$$\sum_{i=1}^{n} \alpha_{\perp,i}^{new} = \frac{\alpha_{\perp,1}}{||\alpha_\perp||_1} + \frac{\alpha_{\perp,2}}{||\alpha_\perp||_1} + \ldots + \frac{\alpha_{\perp,n}}{||\alpha_\perp||_1} = 1.$$

However, for negative values this approach would not render the desired results and the measure would, as such, be not interpretable. It imposes certain restrictions on the applicability of the PT measure, which, again, we have already seen in the bivariate analysis in Chapter 1.

Due to the computational complexity, it is not rational to provide a closed form analytical formula for the PT measure even with $m = 3$ time series and $m - 1 = 2$ cointegrating vectors (a common "set up" in the agricultural price analysis, as explained above). To visualize and further explore the potential issue of non-interpretability we turned to the simulated data.

We simulated $N = 1000$ samples of the trivariate cointegrated time series $y = (y_1, y_2, y_3)$ as VECM (Vector Error Correction Model). We used $T = 1000$ observations with $m - 1 = 2$ cointegrating relations in each sample to construct a fully integrated system and involve the above argumentation regarding the degree of cointegration in the agricultural markets. The simulated VECM included normalized to the first element cointegrating matrices $\beta \in \mathbb{R}_{3\times 2}$ of the form:

$$\beta = \begin{pmatrix} 1 & 1 \\ \beta_{21} & \beta_{22} \\ \beta_{31} & \beta_{32} \end{pmatrix}.$$

Both $\alpha \in \mathbb{R}_{3\times 2}$ and $\beta \in \mathbb{R}_{3\times 2}$ matrices were of rank 2 by construction, and the required $\alpha_{\perp} \in \mathbb{R}_{3\times 1}$ is, as such, a vector. Figure 2.1 depicts one of the simulated trivariate samples, i.e. three cointegrated time series that were later used in the calculation of the PT measure.

Figure 2.2 shows the computed PT measure (i.e. vector $\alpha_{\perp}$) for the first 300 samples of our simulation (to ensure better readability of the plot we did not plot the values for all 1000 samples). We see that in roughly half of the cases the values (at least one of three) are below zero, and hence cannot be used as price discovery measures. Out of the whole simulation run with $N = 1000$ we get the quantities in Table 2.1 for nonnegative outcomes: PT measure for y_1 meets this requirement in 534 cases out of 1000, y_2 in 478 cases and y_3 in 994 cases. However, to be able to interprete $\alpha_{\perp}$ as PT price discovery measure, we need all elements of the vector to be nonnegative, which was only achieved in 241 cases (i.e. 24%).

To sum up, the PT measure has an advantage of being easy to compute with using the procedure (2.1). However, if even one of the n elemets of the resulting vector $\alpha_{\perp} \in \mathbb{R}_{n\times 1}$ turns out to be negative, we cannot view these values as price discovery measure. Unfortunately, this seems to be the case quite often, as our simulation shows, which indicates at least limited applicability of the PT metric in the multivariate case.

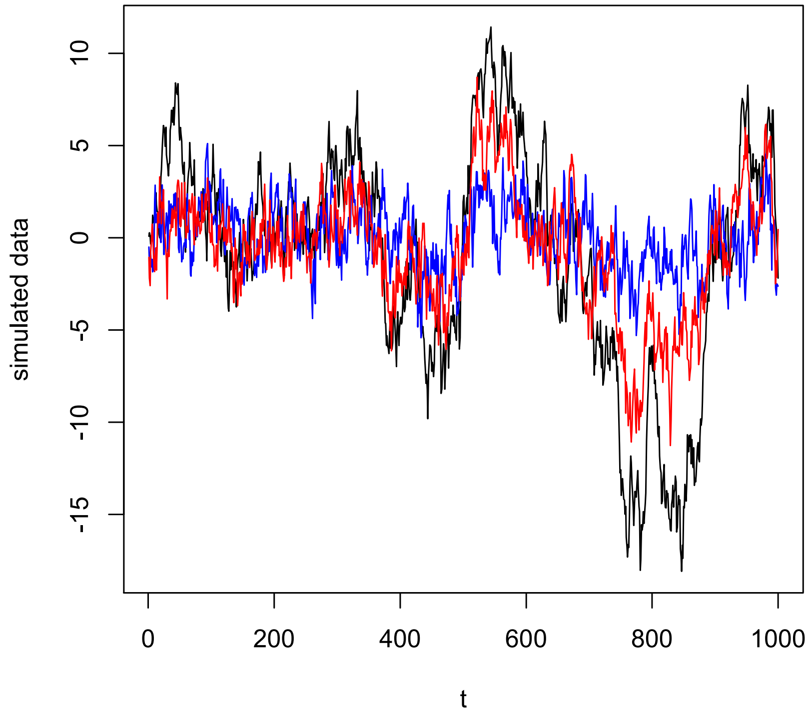


Figure 2.1: Simulated trivariate cointegrating time series (one sample), $T = 1000$: y_1 (black), y_2 (blue), y_3 (red).

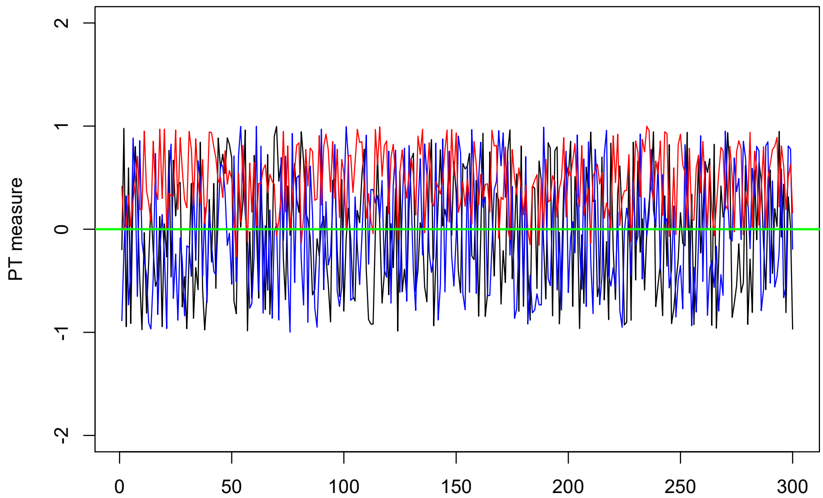


Figure 2.2: PT measured for three countries (black, blue and red) and the threshold value of zero (green).

	number of non-negative cases from the sample $N = 1000$
$\alpha_{\perp,1}$	534
$\alpha_{\perp,2}$	478
$\alpha_{\perp,3}$	994
$\alpha_{\perp} = (\alpha_{\perp,1}, \alpha_{\perp,2}, \alpha_{\perp,3})$	241

Table 2.1: Quantities of nonnegative PT measure values, sample $N = 1000$.

2.2 IS measure in the multivariate analysis

Let us now examine the IS measure in the multivariate case with m time series involved.

We follow the same logic as in the bivariate analysis in Chapter 1 and apply the Structural Vector Autoregressive (SVAR) model with sign restrictions to the data. In this case, the structural innovations are by construction uncorrelated with each other.

The details of the method are outlined in Section 1.1.3. Nonetheless, we briefly sketch the most important steps and equations for the multivariate case below.

We start with the Vector Autoregressive model of order p, VAR(p)

$$y_t = a_0 + A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + e_t, \tag{2.4}$$

where $y_t \in \mathbb{R}_{m\times 1}$ is a vector of observations at time t, $a_0 \in \mathbb{R}_{m\times 1}$ is a constant term (vector of intercepts), $A_i \in \mathbb{R}_{m\times m}, i = 1, \ldots, p$ are time-invariant coefficient matrices, and $e_t \in \mathbb{R}_{m\times 1}$ vector of normally distributed error terms with zero mean and covariance matrix Ω. The shocks e_t are serially uncorrelated, but in a general case could be contemporaneously correlated with each other, thus causing ambiguousity in calculation of the IS price discovery measure.

To obtain the Structural Vector Autoregressive (SVAR) representation of (2.4), we multiply both sides of the equation with a certain matrix B

$$By_t = B \cdot a_0 + B \cdot A_1 y_{t-1} + B \cdot A_2 y_{t-2} + \ldots + B \cdot A_p y_{t-p} + B \cdot e_t, \tag{2.5}$$

$$By_t = b_0 + B_1 y_{t-1} + B_2 y_{t-2} + \ldots + B_p y_{t-p} + u_t, \tag{2.6}$$

such that the covariance matrix Σ of the structural shocks $u_t \in \mathbb{R}_{m\times 1}$ is now an identity matrix

$$\Sigma = \mathbb{E}(u_t u_t^{'}) = I_{m\times m} = I_m.$$

As in the bivariate case, the connection between the shocks e_t from the reduced form VAR and e_t from the structural VAR is given by

$$e_t = B^{-1} u_t \qquad \text{or, equivalently} \qquad u_t = Be_t.$$

Let again M denote the lower triangular matrix from the Cholesky decomposition of $\Omega = MM'$. We can also write $\Omega = MQQ'M'$, where $Q \in \mathbb{R}_{m\times m}$ is the square orthogonal matrix such that $Q'Q = QQ' = I_m$. Therefore, $B^{-1} = MQ$ as in (1.22).

The task is therefore again to find admissible matrix (or matrices) B^{-1}. As in the bivariate case, we use the sign restrictions for identification. The identifying matrix has the form

$$\begin{pmatrix} + & * & * & \ldots & * \\ * & + & * & \ldots & * \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ * & * & * & \ldots & + \end{pmatrix}, \tag{2.7}$$

i.e. with the positive elements on the main diagonal. We follow here Section 1.1.3 and assume that each time series reacts positively on its own shock and that we have no knowledge of the shock effects on the other variables. This is the case of the partially identified model, since not all the schocks signs are accounted for. Again, it is clear that the set of admissible matrices is very broad, including examples both the case where one time series dominates all the others, and the case when this exact time series is dominated by the others, too.

Since the matrix M is given by the VAR model from the Cholesky decomposition of Ω, for B^{-1} we need to find admissible matrices Q, i.e. from the set of all orthogonal matrices $\mathcal{Q} = \{Q \in \mathbb{R}_{m\times m} : QQ' = Q'Q = I_m\}$ we need to pick only those that would satisfy the sign conditions (2.7).

In the bivariate case we used Givens rotation matrices to generate the set $\mathcal{Q}$. This approach can be applied to the trivariate model as well, by computing the product

$$Q(\phi_1, \phi_2, \phi_3) = Q_1(\phi_1) \times Q_2(\phi_2) \times Q_3(\phi_3)$$

of the following Givens rotation matrices

$$Q_1 = \begin{pmatrix} \cos(\phi_1) & -\sin(\phi_1) & 0 \\ \sin(\phi_1) & \cos(\phi_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} \cos(\phi_2) & 0 & -\sin(\phi_2) \\ 0 & 1 & 0 \\ \sin(\phi_2) & 0 & \cos(\phi_2) \end{pmatrix}$$

$$Q_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_3) & -\sin(\phi_3) \\ 0 & \sin(\phi_3) & \cos(\phi_3), \end{pmatrix}$$

where $\phi_1, \phi_2, \phi_3 \in [0, 2\pi]$. One can again define a finite-dimensional grid between 0 and 2π for each $\phi_i, i = 1, 2, 3$ and compute all possible matrices $Q(\phi_1, \phi_2, \phi_3)$.

As $\sin^2(\phi) + \cos^2(\phi) = 1$, it can be shown that the resulting matrix Q is indeed orthogonal. One would then retain only those of them that are in line with the desired sign restrictions.

For $m = 3$ in Givens rotation matrices there are already three parameters that need to be simulated, making calculation of candidate matrices Q considerably more involved than in a bivariate case. Generalization to $m > 3$ is theoretically possible, but is even more difficult to compute and is therefore rarely used in practice, cf. [Kilian and Lutkepohl, 2017].

Alternative approach used in multivariate case is Householder transformation that involves QR decomposition, proposed by [Rubio-Ramirez et al., 2010]. Any real square matrix W can be written as $W = QR$ with Q orthogonal matrix s.t. $QQ' = I$ ans R as upper triangular matrix. The algorithm involved drawing matrix $W \in mathbbR_{m\times m}$ as columns from the multivariate normal distribution $\mathcal{N}(0, I_m)$ and applying the decomposition to each W, therefore extracting matrix Q. If the matrix R is normalized to have positive diagonal elements, it can be shown that this method is equivalent to randomly selecting Q from the set of orthonormal matrices $\mathcal{Q}$.

With the QR decomposition, IS price discovery measure in the multivariate case can be calculated as follows:

1. Estimate A_1 and Ω from VAR (2.4)
2. Calculate lower triangular matrix M from the Cholesky decomposition of Ω
3. Create $m \times m$ matrix W by drawing each column from $(0, I_m)$
4. Perform QR decomposition of W.
5. If matrix R has non-positive diagonal element r_{ii}, reverse signs of all elements in the column i of the matrix Q

6. Calculate MQ

7. If MQ is admissible (i.e. in agreement with the sign restrictions), then $B^{-1} := MQ$

8. Calculate $\Theta := A_1 \cdot B^{-1}$

9. Calculate Variance decomposition

An advantage of the QR approach is that it can be applied to large systems with m time series to calculate IS price discovery measure under the sign restrictions. The result will be an $m \times m$ matrix with the elements

$$IS_{ij} = \frac{\left([\Theta]_{ij}\right)^2}{[\Theta\Theta']_{ii}}, \tag{2.8}$$

that show the impact of the shock $i, i = 1, \ldots, m$ on every time series $j, j = 1, \ldots, m$. With increasing m, the dimension of such matrix will also increase (quadratically), and may become difficult to analyse in terms of making an unambiguous statement about who is leading the price discovery process. For smaller system it can, however, provide useful insights into the market forces in the system. In the next section we provide an example

2.3 Data analysis

In this section we apply the discussed IS methodology to calculate price discovery measure on the European pork market. We focus on the same four countries as in Section 1.3: Germany, Netherlands, Belgium and France, see Figure 1.1. However, now our analysis is multivariate and performed for all these countries simultaneously.

In Section 1.3 we only considere three country pairs: Germany-Netherlands, Germany-Belgium and Germany-France. The values of IS measures under different draws of the B^{-1} matrix were very different, hence making the interpretation of the results difficult. The same problem is likely to arise in the multivariate case too, but it is still interesting to see if the bivariate and multivariate analysis provide similar findings.

We again divide the total set of observations into three subsets as in Section 1.3.

IS metric (2.8) was computed using the QR decomposition to find a set of admissible matrices $B^{-1} = MQ$. In accordance with the algorithm above, we simulated 10^5 matrices W, where the matrix columns are random multivariate normal vectors from $\mathcal{N}(0, I_4)$. Then, matrices R and Q from the QR decomposition were calculated. If any diagonal element of R was non.positive, we reversed the signs of the elements in the corresponding column of Q (by multiplying those with -1). Finally, the matrix MQ was calculated and checked against the sign restrictions 2.7. From the total amount of 10^5 draws, only around 5000 were in agreement with the sign restrictions.

As expected, the IS measures are still very different under different draws of the matrix B^{-1}. In a system of 4 countries we have $4^2 = 16$ IS measures. To illustrate this, we refer to Table 2.2 that shows the descriptive statistics for the IS price discovery measures for four countries in the third observational period from 2006 till 2015. Figure 2.3 depicts the frequencies of the different IS values for the same period for these four countries. The histograms clearly indicate that the most common outcome in every case was the minimal value of the IS measure, meaning that the sign restriction still did not provide enough identification for the model.

With increasing m the interpretation of the price discovery in the system becomes even more challenging. From Table 2.2 it is unclear what descriptive statistic measure would suit this purpose, if any.

All of these reasons are an indication that it might be beneficiary to consider different identifying restrictions, eg. the non-normality and non-homoscedasticity ones. Those restrictions are not based on the economical properties of the data (that may prove to be challenging to render in the case of spatial markets for agricultural commodities) but rather on the data-driven ones.

		Germany	Netherlands	Belgium	France
Germany	Min	0.00	0.00	0.00	0.00
	Mean	0.26	0.25	0.25	0.24
	Median	0.18	0.17	0.16	0.15
	Max	0.99	0.99	0.99	0.99
Netherlands	Min	0.00	0.00	0.00	0.01
	Mean	0.25	0.25	0.25	0.25
	Median	0.17	0.16	0.16	0.16
	Max	0.99	0.98	0.98	0.99
Belgium	Min	0.00	0.00	0.00	0.00
	Mean	0.25	0.25	0.25	0.25
	Median	0.16	0.16	0.17	0.16
	Max	0.99	0.99	0.99	0.98
France	Min	0.00	0.00	0.00	0.00
	Mean	0.25	0.25	0.24	0.26
	Median	0.16	0.15	0.16	0.17
	Max	0.99	0.99	0.98	0.99

Table 2.2: IS price discovery measures for four countries from 2006 till 2015

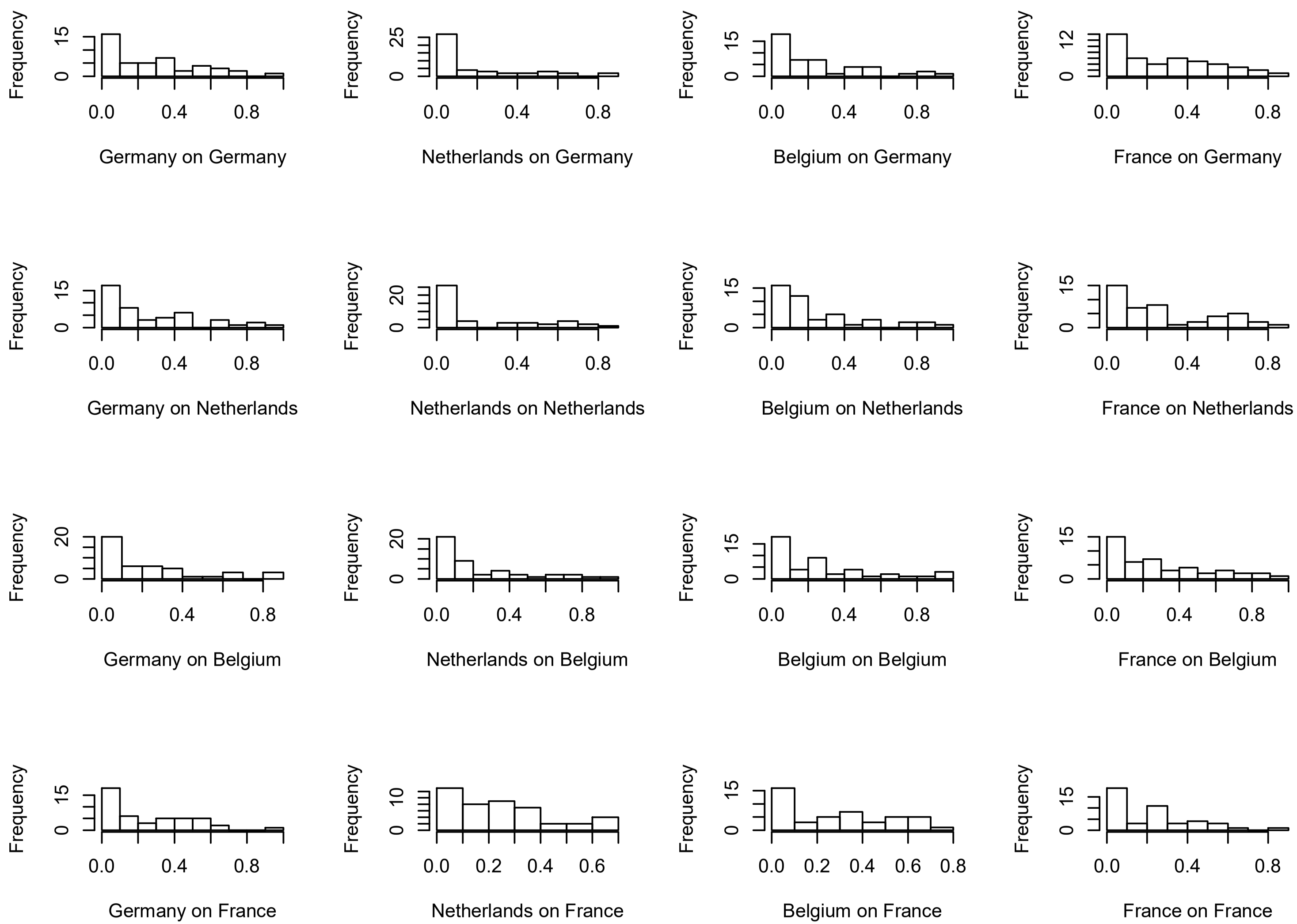


Figure 2.3: Histograms of the IS measure with sign restriction SVAR for four European countries for 2006-2015

Chapter 3

State Space Models and Variance Decomposition

In this chapter we present another approach to the decomposition of time series variance, based on the Dynamic Factor Model (DFM). Over the last three decades, DFM has become a widely used econometric tool to analyze and predict comovement of macroeconomic indicators. One of the reasons is that the method allows for simultaneous estimation of large systems, consisting of many time series.

DFM assumes that in a system, every time series experiences a common influence referred to as a common factor. On the other hand, country specific processes play their role too. These two influences are unobservable and are treated as latent state variables, modeled as autoregressive $AR(p)$ process. DFM are special type of the State Space (SS) models. SS models are general and very flexible structures: any multivariate $ARIMA(p, i, q)$ model can be fit in this framework, which is why they are so widely used. After casting the model of interest in SS form, it can be estimated using the Kalman filter. Afterwards, we use the variance decomposition in AR process to obtain the proportion of the variance that is due to the common factor for each country.

Section 3.1 presents the model and discusses the theory behind its estimation. Section 3.2 applies the method to the analysis of pork meet prices in Europe: in Section 3.2.1 we use the data for twelve countries in 2004-2015, whereas in Section 3.2.2 we concentrate on the dynamic of the integration in a system of seven countries over three time periods.

3.1 The model

The following conceptual model is applied in the analysis. Consider a multivariate system of m time series $y_{it}, i = 1, \ldots, m$ with the time index $t = 1, \ldots, T$. In the present work, we focus on analyzing agricultural prices in m spatial markets. Based on the economic reasoning, we assume that every observable variable y_{it} can be decomposed into two unobservable components: f_t and c_{it}, representing common and country-specific factors or influences, respectively. Common factor f_t influences all the countries in the system, on one hand, but on the other hand, it is also induced by the joint processes of the said countries. On the contrary, c_{it} reflects individual process features. From a statistical standpoint, we also assume that both f_t ans c_{it} are stationary $AR(2)$ processes (lag order $q = 2$ of $AR(q)$ process is chosen due to the weekly nature of the data, to account for the persistency of the influence). The complete model can be then represented as

$$
\begin{aligned}
y_{it} &= \gamma_i f_t + c_{it} \\
f_t &= \phi_1 f_{t-1} + \phi_2 f_{t-2} + \nu_t, \qquad \nu \overset{iid}{\sim} \mathcal{N}(0,1) \\
c_{it} &= \psi_{1,i} c_{i,t-1} + \psi_{2,i} c_{i,t-2} + \eta_{it}, \qquad \eta \overset{iid}{\sim} \mathcal{N}(0, \sigma^2_{\eta_i})
\end{aligned}
\tag{3.1}
$$

for $t = 1, \ldots, T$ and $i = 1, \ldots, m$.

Represented in a State Space (SS) form, model 3.1 consists of a measurement equation, relating the observed data to a state vector α_t, and a Markovian state equation that describes the evolution of the state vector over time. The *measurement equation* is given as

$$\underbrace{\begin{pmatrix} y_{1t} \\ y_{2t} \\ \dots \\ \dots \\ y_{m-1t} \\ y_{mt} \end{pmatrix}}_{Y_t} = \underbrace{\begin{pmatrix} \gamma_1 & 0 & 1 & 0 & 0 & \dots & \dots & \dots & \dots & 0 \\ \gamma_2 & 0 & 0 & 0 & 1 & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \gamma_{m-1} & 0 & \dots & \dots & \dots & \dots & 1 & 0 & 0 & 0 \\ \gamma_m & 0 & 0 & \dots & \dots & \dots & 0 & 0 & 1 & 0 \end{pmatrix}}_{\Gamma} \cdot \underbrace{\begin{pmatrix} f_t \\ f_{t-1} \\ c_{1t} \\ c_{1t-1} \\ \dots \\ c_{mt} \\ c_{mt-1} \end{pmatrix}}_{\alpha_t}, \tag{3.2}$$

where $y_t \in \mathbb{R}_{m\times 1}$, matrix $\Gamma \in \mathbb{R}_{m\times 2(m+1)}$ and vector $\alpha_t \in \mathbb{R}_{2(m+1)\times 1}$.
The *state equation* for the vector α_t is given as the first order Markov process

$$\underbrace{\begin{pmatrix} f_t \\ f_{t-1} \\ c_{1t} \\ c_{1t-1} \\ \dots \\ c_{mt} \\ c_{mt-1} \end{pmatrix}}_{\alpha_t} = \underbrace{\begin{pmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_{11} & \psi_{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_{1m} & \psi_{2m} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}}_{S} \cdot \underbrace{\begin{pmatrix} f_{t-1} \\ f_{t-2} \\ c_{1t-1} \\ c_{1t-2} \\ \dots \\ c_{mt-1} \\ c_{mt-2} \end{pmatrix}}_{\alpha_{t-1}} + \underbrace{\begin{pmatrix} \nu_t \\ 0 \\ \eta_{1t} \\ 0 \\ \dots \\ \eta_{mt} \\ 0 \end{pmatrix}}_{\eta_t}, \tag{3.3}$$

where $S \in \mathbb{R}_{2(m+1)\times 2(m+1)}$ is state matrix and $\eta_t \in \mathbb{R}_{2(m+1)\times 1}$ is normally distributed vector of errors with covariance matrix Σ_η, s.t. $\eta_t \sim \mathcal{N}(0, \Sigma_\eta)$. The matrices Γ, S and Σ_η are referred to as *system matrices* and are non-random. For illustration, for the simplest bivariate case of $m = 2$ we would have the following SS form, given by measurement equation

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \gamma_1 & 0 & 1 & 0 & 0 & 0 \\ \gamma_2 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} f_t \\ f_{t-1} \\ c_{1t} \\ c_{1t-1} \\ c_{2t} \\ c_{2t-1} \end{pmatrix}$$

and state equation

$$\begin{pmatrix} f_t \\ f_{t-1} \\ c_{1t} \\ c_{1t-1} \\ c_{2t} \\ c_{2t-1} \end{pmatrix} = \begin{pmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_{11} & \psi_{21} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_{12} & \psi_{22} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} f_{t-1} \\ f_{t-2} \\ c_{1t-1} \\ c_{1t-2} \\ c_{2t-1} \\ c_{2t-2} \end{pmatrix} + \begin{pmatrix} \nu_t \\ 0 \\ \eta_{1t} \\ 0 \\ \eta_{2t} \\ 0 \end{pmatrix}.$$

3.1.1 Kalman Filter and Maximum Likelihood

The notations under brackets in (3.2) - (3.3) allow for more compact representation of the SS system, namely as

$$\begin{aligned} Y_t &= \Gamma \alpha_t \\ \alpha_t &= S\alpha_{t-1} + \eta_t, \qquad \eta \sim \mathcal{N}(0, \Sigma_\eta). \end{aligned} \tag{3.4}$$

where α_t is unobserved and needs to be estimated, based on the information $(I(t))$ available at the moment $t \in 1, \ldots, T$. This estimation is done via the *Kalman filter* algorithm, outlined below for some initial values fα_0 and P_0.

1. Forecast a new α_t value, conditional on information available in period $t-1$:

$$\alpha_{t|t-1} = E\left[\alpha_t | Y_{t-1}\right] = S\alpha_{t-1|t-1}$$

2. Compute P_t, the variance-covariance matrix of α_t:

$$P_{t|t-1} = \mathbb{E}\left[(\alpha_t - \alpha_{t|t-1})(\alpha_t - \alpha_{t|t-1})'\right] = SP_{t-1|t-1}S' + \Sigma_\eta$$

3. Prediction error is given as:

$$v_{t|t-1} = Y_t - Y_{t-1} = Y_t - \Gamma\alpha_{t|t-1}$$

4. Compute conditional variance of v_t:

$$F_{t|t-1} = \Gamma P_{t|t-1}\Gamma'$$

5. Compute Kalman gain:

$$K_t = P_{t|t-1}\Gamma' F_{t|t-1}^{-1}$$

6. Update values of α_t and P_t:

$$\alpha_{t|t} = \alpha_{t|t-1} + K_t v_{t|t-1}$$

$$P_{t|t} = P_{t|t-1} - K_t \Gamma P_{t|t-1}$$

Kalman filter is a recursive algorithm consisting of a *prediction step* and an *update step*. Steps 1-5 of the algorithm provide prediction, step 6 updates[1].

Initial values of α_0 and P_0 can be computed from the second (state) equation in (3.4). Hence,

$$\alpha_0 = E(\alpha_t) = 0.$$

Similarly, variance of α_0 may be derived analytically from

$$\begin{aligned} P_0 &= var(\alpha_t) = Svar(\alpha_t)S' + var(\eta_t) \\ &= SP_0S' + \Sigma_\eta, \end{aligned}$$

and by using Kronecker product and vectorization of the matrices property $vec(XYZ) = Z' \otimes Xvec(Y)$, we get that at steady state

$$\begin{aligned} vec(P_0) &= vec(SP_0S') + vec(\Sigma_\eta) \\ vec(P_0) &= (S \otimes S)vec(P_0) + vec(\Sigma_\eta) \\ vec(P_0) &= (I_{2(m+1)^2} - S \otimes S)^{-1}vec(\Sigma_\eta), \end{aligned}$$

where $vec(P_0)$ and $vec(Q)$ are the stacked columns of the matrices P_0 and Q, respectively, and $I_{2(m+1)^2}$ is the square identity matrix.

Kalman filter algorithm, however, is based on the assumption that coefficient matrices Γ and S and covariance matrix Σ_η in (3.4) are known. In our model this is not the case, so the matrices need to be estimated alongside with state vector α_t. The vector of unknown parameters of the model (3.2) - (3.3), embedded in the system matrices, is $\theta \in \mathbb{R}_{(4m+2)\times 1}$,

$$\theta = \left(\gamma_1, \dots, \gamma_m, \phi_1, \phi_2, \alpha_1, \beta_1, \dots, \alpha_m, \beta_m, \sigma^2_{\eta_1}, \dots, \sigma^2_{\eta_m}\right). \tag{3.5}$$

[1]Detailed explanation and derivation of the algorithm steps can be found in eg. [Durbin and Koopman, 2012]

The estimation of θ is quite involved. There are two major methods available, Maximum Likelihood (ML) algorithm and Bayesian methodology, based on the distribution of the prior. For the latter approach, see eg. [Koop and Korobilis, 2010]. One must note though that Bayesian approach is computationally very challenging, especially given the fact that in multivariate case dimension of θ is already high. In the present work we make use of the ML algorithm.

To apply the ML estimator, an objective likelihood function, derived from the probability density function of the data, is required. Note, that since innovations in (3.4) are normally distributed, the distribution of Y_t can be written as

$$\begin{aligned} Y_{t|t-1} &\sim \mathcal{N}\left(\Gamma\alpha_{t|t-1}, \Gamma P_{t|t-1}\Gamma'\right) \\ &\sim \mathcal{N}\left(\Gamma\alpha_{t|t-1}, F_{t|t-1}\right), \end{aligned}$$

and its log-likelihood function is then:

$$\ell(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\ln\left(2\pi\cdot|F_{t|t-1}|\right) - \frac{1}{2}\sum_{t=1}^{T}\left(v'_{t|t-1}\cdot F^{-1}_{t|t-1}\cdot v_{t|t-1}\right). \tag{3.6}$$

Now the structure of the estimation algorithm can be summarized as follows:

1. Assign starting values for each entry of vector θ in (3.5).
2. Given θ, set matrices Γ, S, Q of the SS form as in (3.2) - (3.3).
3. Set the initial values of α_0 and P_0.
4. Do the steps (1)-(6) of the Kalman filter algorithm above.
5. Compute log-likelihood using (3.6).

The function $\ell(\theta)$ is then maximized (or equivalently, $-\ell(\theta)$ is minimized), and the optimal values of the parameter vector $\hat{\theta}_{ML}$ are computed as the arguments of ℓ.

The optimization problem (3.6) is unconstrained. However, in (3.2) - (3.3) there are two sources for restrictions for parameter θ. First, the $AR(2)$ coefficients $\phi_1, \phi_2, \alpha_1, \beta_1, \ldots, \alpha_m, \beta_m$ need to ensure that the time series are stationary. Second, variances $\sigma^2_{\eta_1}, \ldots, \sigma^2_{\eta_m}$ must be strictly positive. The latter is easily ensured using the following transformation

$$\tilde{\sigma}^2_{\eta_i} = exp(-\sigma^2_{\eta_1}) > 0. \tag{3.7}$$

For the $AR(2)$ process to be stationary we must have the characteristic roots to lie inside the unit circle $(1 - \phi_1 L - \phi_2 L^2) = 0$ (see eg. [Hamilton, 1994]). It can be proven that the values of the coefficients that make $AR(2)$ process stationary are those included in the triangle region (cf. Figure 3.1)

$$\begin{aligned} &\phi_1 + \phi_2 < 1 \\ &\phi_2 - \phi_1 < 1 \\ &-1 < \phi_2 < 1. \end{aligned} \tag{3.8}$$

Following [Morley, 1999], we therefore use the following reparametrizazion of AR parameters

$$\begin{aligned} a &= \frac{\phi_1}{1 + |\phi_1|} \\ b &= \frac{(1 - |a|) \cdot \phi_2}{1 + |\phi_2|} + |a| - a^2 \\ \tilde{\phi}_1 &= 2a \\ \tilde{\phi}_2 &= -(a^2 + b). \end{aligned} \tag{3.9}$$

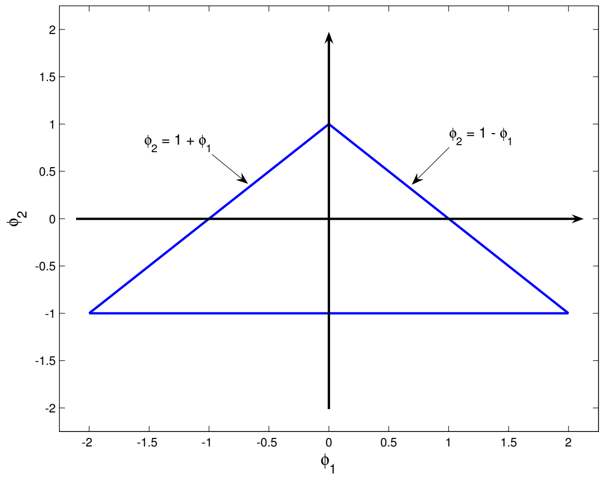


Figure 3.1: Admissible area for stationary AR(2) coefficients.

3.1.2 Variance decomposition

Once the model (3.2) - (3.3) has been estimated and vector $\hat{\theta}$ computed, we can decompose the variance of y_i for each $i = 1, \ldots, m$ into the common and idiosyncratic components. Recall that

$$y_{it} = \gamma_i f_t + c_{it},$$

and hence

$$var(y_{it}) = \gamma_i^2 var(f_t) + var(c_{it}).$$

Since both f_t and c_{it} are $AR(2)$ processes, we can use the properties of autoregressive time series to compute the variances as

$$\begin{aligned} var(f_t) &= \frac{(1-\phi_2)\sigma_\nu^2}{(1+\phi_2)(1-\phi_1-\phi_2)(1+\phi_1-\phi_2)} \\ &= \frac{(1-\phi_2)}{(1+\phi_2)(1-\phi_1-\phi_2)(1+\phi_1-\phi_2)}, \end{aligned} \tag{3.10}$$

since $\sigma_\nu^2 = 1$, and analogously

$$var(c_{it}) = \frac{(1-\beta_i)\sigma_{\eta_i}^2}{(1+\beta_i)(1-\alpha_i-\beta_i)(1+\alpha_i-\beta_i)} \tag{3.11}$$

for $i = 1, \dots, m$.

For country i, the fraction of the variance explained by the common factor f_t is defined as

$$R_i = \frac{\gamma_i^2 var(f_t)}{var(y_{it})} = \frac{\gamma_i^2 var(f_t)}{\gamma_i^2 var(f_t) + var(c_{it})}. \tag{3.12}$$

By plugging (3.10) and (3.11) into (3.12) for $var(f_t)$ and $var(c_{it})$ we can calculate R_i measures for $i = 1, \dots, m$. As stated above, common factor f_t serves as a measure of integration or level of involvement for a certain country i into the whole system that is, in turn, created by all the m counties in question.

3.2 Data analysis

3.2.1 Static variance decomposition ($m = 12$)

For practical application of DFM algorithm we once again turn to the multivariate analysis of the European pork meat market. However, we now analyze processes in 12 countries: Germany, Belgium, France, Great Britain, Greece, Netherlands, Portugal, Spain, Finland, Sweden, Austria and Poland.

If we were to analyze such a large multivariate system with common price discovery measures, we would have up to 11 cointegration relations and hence, just as many adjustment parameters. As we have seen in Section 2.1 there is no guarantee that the PT vector would be interpretale. IS measure as in Section 2.2 would render 12×12 matrix of IS coefficients that is difficult to interpret.

One of the advantages of the SS approach is that it allows to analyze multivariate dependencies simultaneously, but unlike the above methods, also provides easily interpretable results: proportion of how strong each time series is related to the common factor. In fact, from an econometric standpoint the larger the number of countries m, the more information we have on the shape and the structure of the common factor, and hence, the more precise is the estimation.

As before, we analyze the weekly price data from the European Union market, but the observational period is now from June 2004 till March 2015. We choose this period to account for as many countries as possible, to include East European countries (cf. [Holst and von Cramon-Taubadel, 2013]) but also, for technical reasons, to make sure that there are no missing observations[2]. Figure 3.2 demonstrates the time series in the analysis, however, given the high number of variables involved, at this stage it is difficult to make any statements about the system. Table 3.1 contains the descriptive statistics of the data.

	Mean	Std. Dev.	Min	Max
Belgium	141.24	14.06	112.90	183.50
Germany	155.00	14.99	118.32	197.88
Greece	178.10	18.26	118.33	213.85
Spain	156.20	21.07	118.14	217.10
France	141.89	15.42	116.00	190.00
Netherlands	138.33	14.49	110.92	179.01
Portugal	159.41	17.04	127.00	201.00
Great Britain	164.19	17.66	132.35	201.49
Finland	149.45	14.37	126.03	184.78
Sweden	153.47	20.04	118.14	198.16
Austria	151.85	15.16	122.26	195.26
Poland	148.01	19.71	112.18	197.47

Table 3.1: Descriptive statistics for 12 countries.

Dynamic Factor model assumes that the price process in every country follows stationary $AR(2)$ pattern. To ensure its correct specification we first test every y_{it} for a structural break with unknown date. The null hypothesis is the absence of a

[2]We had to exclude some European countries such as Italy, Hungary etc. from the analysis due to lack of or missing observations

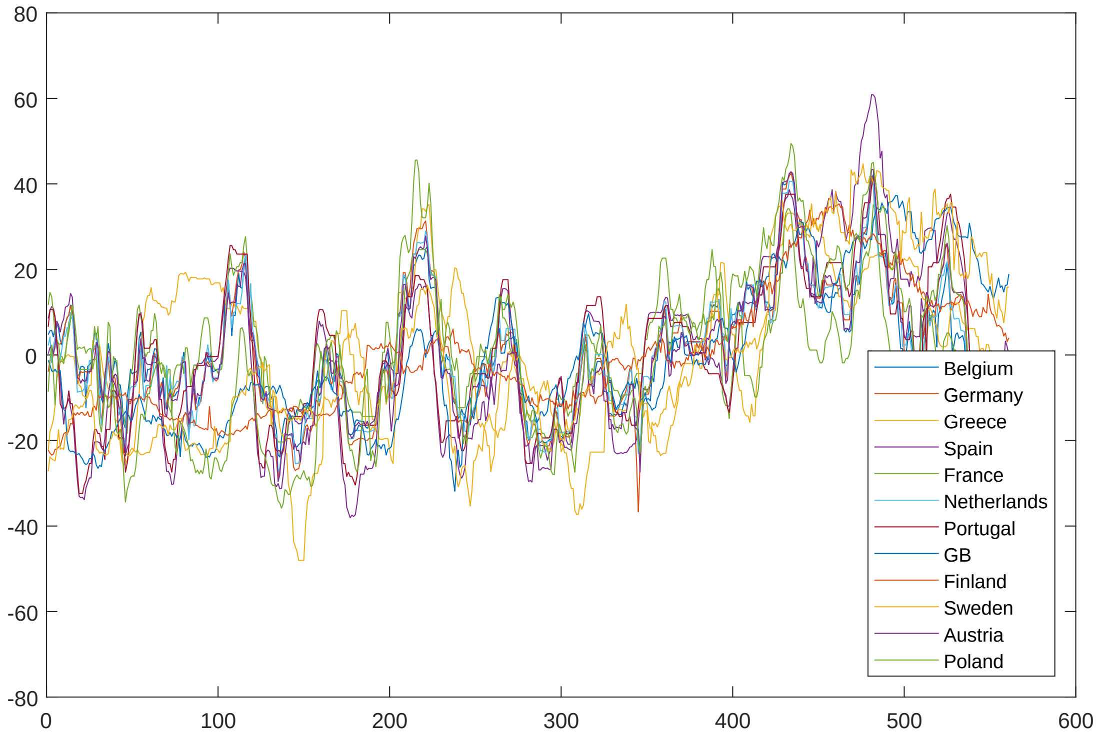


Figure 3.2: Pork meat price data for 12 European countries

structural break. Such tests are known as supremum tests, since the test statistic is the maximum value of the statistics obtained from a series of (individual) Wald or LR tests over a range of possible break dates in the sample. If we denote a possible break date as $b \in \mathcal{B}$, then the resulting test statistic S may be represented as

$$S = \sup_{b \in \mathcal{B}} s(b),$$

with $s(b)$ denoting the individual test statistics.

We applied both Wald and LR (likelihood ratio) tests to each time series. Since the test decisions were similar in every case, we only report LR test statistics and p-values in Table 3.3. For all countries except France we could not reject the null hypothesis and hence assume that there are no structural breaks present in the data. For France, structural break was estimated for January 2012.

	Test Statistic	p-value
Belgium	10.69	0.17
Germany	9.33	0.26
Greece	2.31	1.00
Spain	13.11	0.07
France	32.41	0.00
Netherlands	10.52	0.18
Portugal	13.53	0.06
Great Britain	7.95	0.40
Finland	4.77	0.83
Sweden	3.00	0.98
Austria	5.00	0.80
Poland	9.03	0.29

Table 3.2: Results of LR structural break test for 12 countries.

For further analysis, we demean the data, to get the time series representation similar to (3.1), i.e. without the constant term. For France, as the only country with structural break, we sustract the subset mean before and after the estimated break. For all other countries, total sample mean is subtracted. According to Schwarz Infromation Criteria (SIC), optimal lag length is 2 for all twelve time series, justifying the choice of model representation. Table 3.3 shows the estimated $AR(2)$ coefficients for the data. Those are in agreement with the stationarity conditions (3.8), however, the sum of coefficients is very close to 1, meaning that data is highly persistent (so called hump-shaped), which is however not uncommon for macroeconomic variables (cf. [Perron, 1993]).

After this preliminary analysis, we can estimate the SS form (3.2)-(3.3). To initialize the ML Kalman filter algorithm, for starting values of the parameter vector $\theta \in \mathbb{R}_{50\times 1}$ (3.5) we used 1 for factor loadings γ_i, 0.4 for the $AR(2)$ coefficients $\phi_{1,2}, \alpha_i, \beta_i$ (before their reparametrization according to (3.9)) and neg-

	Lag $-1, L$	Lag $-2, L^2$	Variance
Belgium	1.24	-0.24	9.96
Germany	1.50	-0.53	6.48
Greece	1.19	-0.21	16.47
Spain	1.52	-0.53	6.37
France	1.31	-0.35	8.85
Netherlands	1.23	-0.26	10.36
Portugal	1.41	-0.44	7.98
Great Britain	1.32	-0.33	2.84
Finland	0.70	0.29	1.57
Sweden	0.97	0.02	8.14
Austria	1.28	-0.31	9.29
Poland	1.45	-0.48	9.08

Table 3.3: Results of $AR(2)$ estimation for 12 countries.

ative logarithm of variance estimators of the AR time series for the variances $\sigma^2_{\eta_i}, i = 1, \dots, 12$ (before their reparametrization according to (3.7)). The algorithm converged (based on the size of the gradient condition) after 243 iterations. The estimation results can be found in Table 3.4. These values were then used to decompose the variances of every time series and compute its proportion explained be the common factor as in (3.12), see Table 3.5.

It should be noted that the present variance decomposition is difference from the one implied in Chapters 2 and 3 for IS price discovery measure. Here, $R_i, i = 1, \dots, 12$ serves as an integration measure showing how dependent price of the country i is on the common factor. In other words, it depicts how deep the country i is involved in the "global" process (that is of course defined only by the m countries considered together in a system). Countries such as Germany, Belgium, Netherlands and Austria are the leaders in this regard, since more than 90% of the price variation in these countries is explained by the common factor. It means that they play the most important role in the market spanned by the system. France is up to 48% involved in this European market, Poland up to 28%.

Some countries show very low values for R_i, eg. Finland or Great Britain with less than 1% of the total variance due to common factor. To illustrate, cf. Figure 3.3 comparing the country time series y_{it} and the common factor f_t for Germany and Great Britain.

	Factor Loading γ	AR Coefficients		Variance σ_η^2
		Lag $-1, L$	Lag $-2, L^2$	
Belgium	2.38	0.63	0.32	2.18
Germany	2.32	0.79	-0.28	1.94
Greece	0.41	1.17	-0.20	16.29
Spain	0.87	1.40	-0.42	5.77
France	1.22	1.13	-0.16	7.44
Netherlands	2.36	0.55	0.32	2.34
Portugal	0.69	1.28	-0.30	7.68
Great Britain	0.12	1.33	-0.34	2.84
Finland	0.02	0.72	0.26	1.62
Sweden	0.26	0.96	0.021	8.03
Austria	2.34	0.68	0.29	2.25
Poland	1.18	1.34	-0.37	7.71
common factor		1.50	-0.52	

Table 3.4: Results of DFM estimation for 12 countries.

	R_i
Belgium	0.96
Germany	0.99
Greece	0.03
Spain	0.16
France	0.48
Netherlands	0.98
Portugal	0.12
Great Britain	0.01
Finland	0.00
Sweden	0.03
Austria	0.94
Poland	0.28

Table 3.5: Variance decomposition due to common factor for 12 countries.

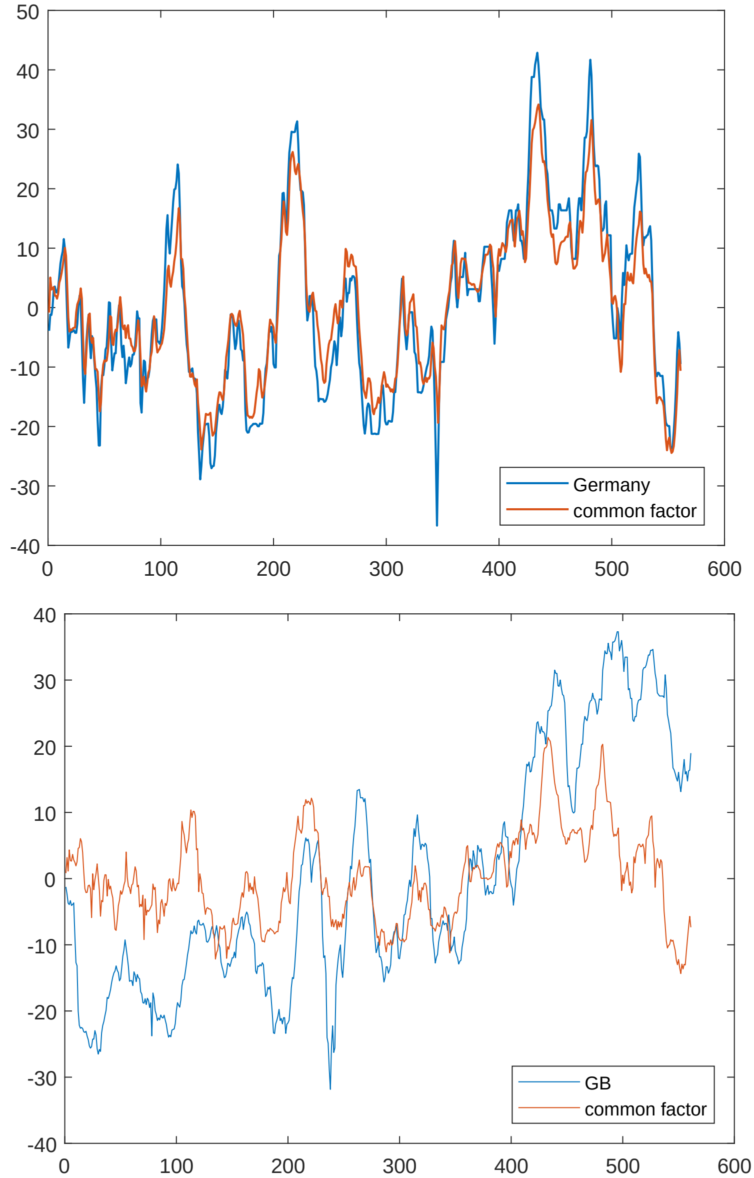


Figure 3.3: Common factor and Germany (above) and Great Britain (below) price data.

3.2.2 Dynamic variance decomposition ($m = 7$)

In previous section, we computed static variance decomposition based on the DFM. The results can be interpreted as showing how strongly the price in a particular country is integrated into the common system. Integration is often considered to be a dynamic process, cf. eg. [Berger et al., 2017], [Kim et al., 2005], [Wessels, 1997], and so it is of interest to analyse how did the involvement of particular country into the common system change over the time. To do so, we consider a longer period of time T divided into subsets $\{T_1, \ldots, T_k\} \in T$ representing different time periods, and compute variance decompositions $R_i(T_j) \forall T_j \in T, j = 1, \ldots, k, i = 1, \ldots, m$ to then analyse its dynamic.

	Period 1: 1987-1996		Period 2: 1996-2006		Period 3: 2006-2015	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Belgium	159.73	22.46	137.13	26.07	141.61	14.92
Germany	148.99	22.21	144.38	25.43	155.82	15.82
Greece	177.88	25.32	168.02	29.17	178.99	19.00
Spain	157.02	25.13	141.83	26.11	158.44	21.38
France	152.503	22.29	136.30	21.78	142.85	16.22
Netherlands	138.41	22.30	125.13	24.05	139.15	15.29
Great Britain	153.55	23.57	146.66	18.47	166.99	17.38

Table 3.6: Descriptive statistics for 7 countries in three time periods.

Due to data availability we limit our analysis to 7 countries: Belgium, Germany, Greece, Spain, France, Netherlands and Great Britain and three almost equal time periods: March 1987-July 1996 ($n_1 = 480$), August 1996-December 2005 ($n_2 = 478$), January 2006-March 2015 ($n_3 = 481$).

Table 3.6 shows descriptive statistics of the data, Table 3.7 contains the results of the structural break test. All data sets have no indication of a possible structural break, so following the logic of the Section 3.2.1 we then proceed with demeaning the data and preliminary AR modelling. SIC again indicates that 2 is the optimal lag length for all variables in all three time periods. Further, time series are all stationary, according to the coefficients analysis in Table 3.8, so we apply the DFM algorithm.

The algorithm converged in 56-93 iterations, depending on the time period considered. The R_i estimates are given in Table 3.10 and in Figure 3.4.

For countries such as Germany, Belgium and Netherlands the degree of integration into European market (spanned by the 7 countries under consideration) has increased over time, though the starting values in the first period were already near 80%. For Spain, the degree of integration has dropped significantly: from 71% to 8%. With less drastic reduction, but the same can be said about Greece and Great Britain. France numbers reduced also, from more than 90% in period 1 to around 60% in period 2, where they stabilized.

Another option to visualize the change of the role of the common factor over time is to look at the comovements between it and the country price data. Figure 3.5 shows three plots for Germany over the time periods, together with the estimated common factor. We see that in the first period there are discrepancies in the movements of two time series, whereas for periods 2 and 3, when German integration increased, the lines are almost identical.

The time span that we study here is the time of EU expansion: Spain and Portugal joined in 1986, Finland, Sweden and Austria in 1996, and then there were the big Eastern expansion in 2004. Though the periods considered in the analysis do not exactly coincide with this timeline, they are very close and allow us to gather insights in the price processes in each time period, as with every EU expansion the change in common factor would happen. The increasing integration finding is consistent with that from other studies, eg. [Holst and von Cramon-Taubadel, 2013].

	Period 1: 1987-1996		Period 2: 1996-2006		Period 3: 2006-2015	
	Test Statistic	p-value	Test Statistic	p-value	Test Statistic	p-value
Belgium	6.87	0.54	7.82	0.10	6.87	0.54
Germany	10.99	0.15	13.68	0.06	10.99	0.15
Greece	2.90	0.99	9.06	0.29	2.90	0.99
Spain	12.41	0.09	3.56	0.77	12.42	0.09
France	11.13	0.14	8.91	0.30	11.13	0.14
Netherlands	7.69	0.44	7.05	0.03	7.69	0.44
Great Britain	1.01	1.00	6.99	0.53	1.01	1.00

Table 3.7: Results of LR structural break test for 7 countries in three time periods.

	Period 1: 1987-1996			Period 2: 1996-2006			Period 3: 2006-2015		
	Lag $-1, L$	Lag $-2, L^2$	Variance	Lag $-1, L$	Lag $-2, L^2$	Variance	Lag $-1, L$	Lag $-2, L^2$	Variance
Belgium	0.85	0.12	39.77	1.22	-0.25	25.18	1.25	-0.28	9.67
Germany	1.14	-0.16	18.18	1.48	-0.51	15.32	1.55	-0.58	6.22
Greece	0.74	0.24	31.16	1.34	-0.36	17.75	1.17	-0.19	18.12
Spain	1.05	-0.08	36.06	1.42	-0.45	19.78	1.49	-0.50	6.47
France	1.17	-0.19	17.83	1.39	-0.43	17.67	1.43	-0.45	6.23
Netherlands	0.86	0.11	41.10	1.26	-0.30	30.01	1.26	-0.29	9.77
Great Britain	0.80	0.17	30.42	1.44	-0.46	5.58	1.36	-0.37	2.77

Table 3.8: Results of $AR(2)$ estimation for 7 countries in three time periods.

Period 1: 1987-1996				
	Factor Loading γ	AR Coefficients		Variance σ^2_η
		Lag $-1, L$	Lag $-2, L^2$	
Belgium	2.74	0.56	0.20	26.83
Germany	2.64	0.84	0.01	11.50
Greece	1.58	0.78	0.08	34.88
Spain	2.85	0.87	-0.02	24.82
France	2.85	0.84	-0.26	7.03
Netherlands	2.78	0.45	0.07	21.45
Great Britain	1.67	0.82	0.06	29.55
common factor		1.54	-0.58	
Period 2: 1996-2006				
	Factor Loading γ	AR Coefficients		Variance σ^2_η
		Lag $-1, L$	Lag $-2, L^2$	
Belgium	3.05	0.71	0.07	9.65
Germany	3.13	0.62	-0.32	2.59
Greece	1.10	1.50	-0.57	18.23
Spain	1.68	1.46	-0.53	16.80
France	2.06	1.24	-0.40	13.88
Netherlands	3.02	0.84	-0.04	12.35
Great Britain	0.56	1.54	-0.59	6.17
common factor		1.58	-0.62	
Period 3: 2006-2015				
	Factor Loading γ	AR Coefficients		Variance σ^2_η
		Lag $-1, L$	Lag $-2, L^2$	
Belgium	2.09	0.58	0.24	2.43
Germany	2.09	1.02	-0.17	1.91
Greece	0.64	1.19	-0.27	18.20
Spain	0.59	1.54	-0.58	6.31
France	1.27	1.26	-0.35	5.83
Netherlands	2.13	0.15	0.01	1.51
Great Britain	0.10	1.64	-0.67	3.09
common factor		1.64	-0.67	

Table 3.9: Results of DFM estimation for 7 countries in three time periods.

	Period 1: 1987-1996	Period 2: 1996-2006	Period 3: 2006 - 2015
Belgium	0.79	0.93	0.97
Germany	0.82	0.99	0.97
Greece	0.36	0.10	0.12
Spain	0.71	0.24	0.08
France	0.94	0.63	0.62
Netherlands	0.88	0.89	0.99
Great Britain	0.38	0.06	0.00

Table 3.10: Variance decomposition due to common factor for 7 countries in three time periods.

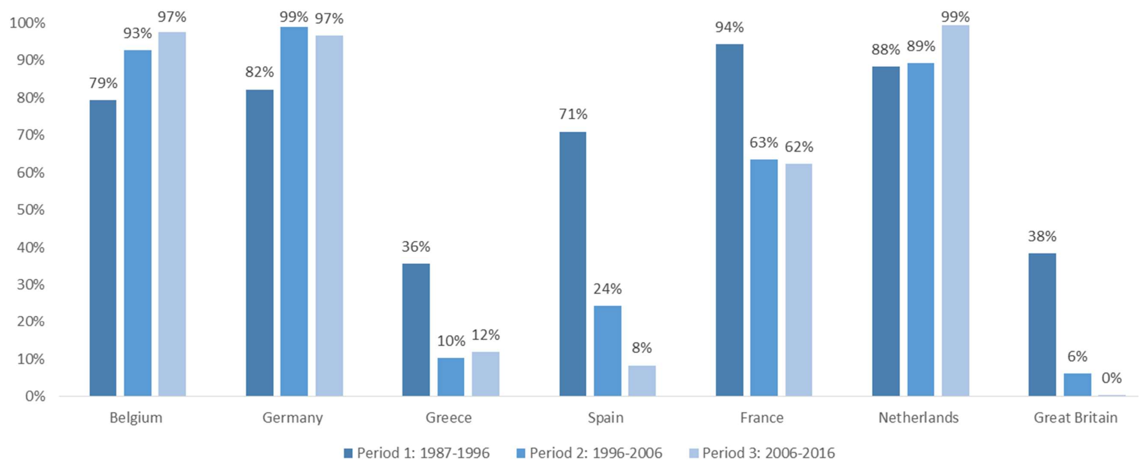


Figure 3.4: Dynamic of integration for 7 countries in three time periods.

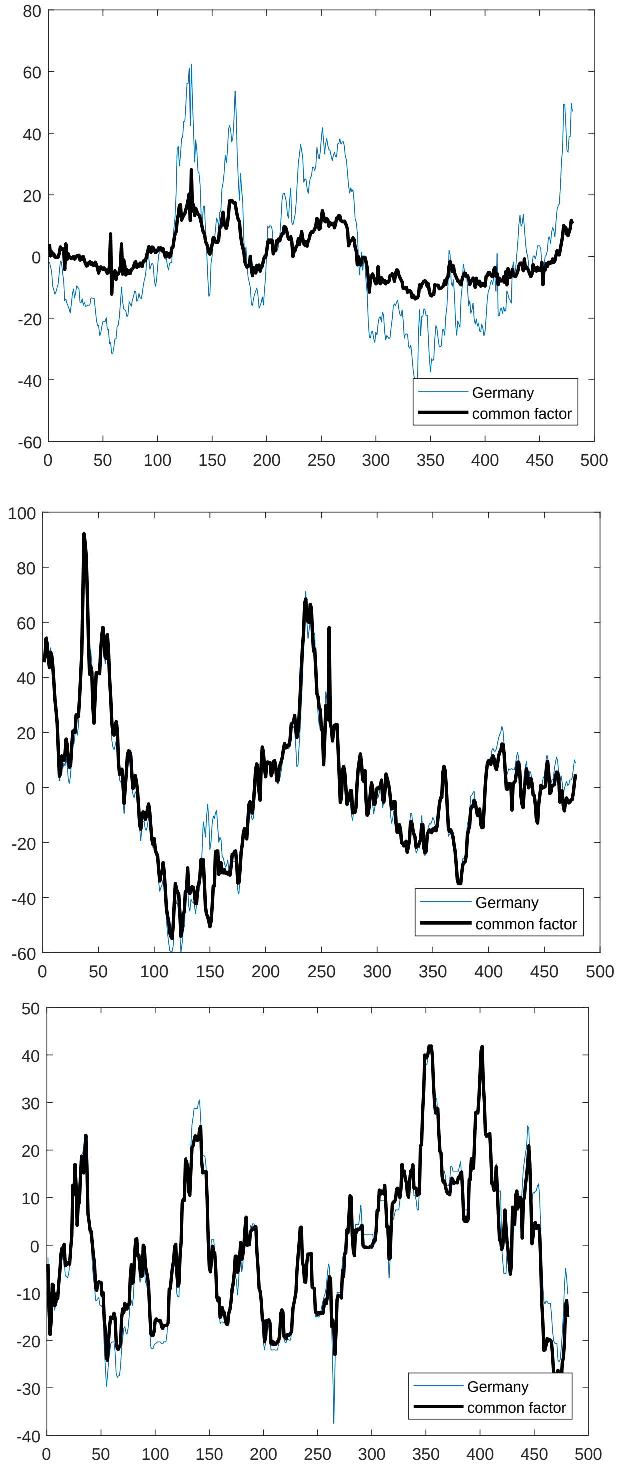


Figure 3.5: Common factor and Germany price data in three time periods: Period 1, 1987-1996 (above), Period 2, 1996-2006 (middle) and Period 3, 2006-2016 (below).

Conclusion

Price discovery is the process of uncovering the fundamental "true" value of an asset/commodity by the markets that are involved in their trade. It has historically been considered one of the central functions of the secondary markets, and the majority of the literature on the subject concentrates on the financial assets and / or spot and future prices. There are, to our knowledge, very few studies that apply the price discovery methodology to the analysis of physical prices of the agricultural commodities ([Arnade and Vocke, 2016] as one of the examples), though this problem is not only very interesting from a theoretical standpoint, but the insights can also be extremely valuable for eq. agricultural policy makers.

One of the ways to define price discovery is by question: "Where is the market?" ([Peter, 2011]). Among spatially different trading venues (geographical regions, countries, regions of the same country etc.), which one(s) dominates the pricing process? From this perspective, we studied the applicability of two classical price discovery measures: Permanent-Transitory decomposition (PT) and Information Shares (IS), to the analysis of agricultural markets. Empirical analysis was carried out with the pork meat prices in European Union in 1987-2015. Pork meat is the most produced and consumed meat in the region, therefore playing a crucial role in the agricultural economic. Also, the considered time period covers the EU expansion, that of course changed the market and had an influence on the price discovery as well.

Chapter 1 concentrates on the bivariate case. We studied the ways in which agricultural commodities differ from the financial assets. We find that these discrepancies are considerable and influence how the price discovery metrics are calculated. We proposed modification of the IS measure by means of the SVAR, to account for the fact that IS is, in general, not uniquely identified. Our identification is based on the sign restrictions. It is a rather flexible approach to identify a SVAR model, since it produces not a point, but a set of estimators. It is more challenging to impose identifications on the innovations in our case than, say, when analyzing demand and supply functions. In the latter case solid economic theory provides the source for the identification. For prices of the same commodity, making similar assumptions is more challenging. For the same reason, zero restrictions on SVAR (meaning that some shocks do not have impacts on some variables at all) may also be too exclusive and hence not realistic.

Chapter 2 expands the analysis to the multivariate case. We provided the methodology to calculate the PT measure for the system of dimension $m > 2$, and showed that in many cases this metric is not applicable, since the resulting values are negative and cannot be viewed as a percentage of the price discovery due to this market. In terms of the IS metric, we expanded the SVAR model with sign restrictions and discussed technical aspects of its implementation, such as QR decomposition. Again, identification of the structural model is an open issue here. Empirical results suggest that the sign restriction approach may not render unequivocal estimators of the price discovery shares. In future research it is therefore of interest to consider not only the economically driven, but also data driven restrictions, such as non-homogeneity or non-normality. Structural VAR is a very promising approach to the variance decomposition that is the basis of the IS measure, and alternative identification approaches may indeed help to produce more definitive estimators.

However, one should note that with the increasing dimension of the time series system, IS measure becomes difficult to interpret. Eg. in the case of 10 countries one would have 100 IS shares, so the answer to the "Where is the market?' question might turn to be intricate.

In Chapter 3 we applied different approach to the decomposition of the time series variance, the Dynamic Factor Model (DFM). DFM is a special case of a broader set of the State Space models, that became a very popular tool in econometric analysis over the last decade. DFM models assume that the system of variables has one common factor. This factor is, on one hand, created by the variables in the system, and on the other hand, influences all of them. Variance decompositions in DFM is defined as the percentage of the total variance for each time series (each country, in our case) that is due to this common factor. It can thus also be viewed as a degree of market integration. One of the advantages of the DFM is that one would have m estimators for the system of m variables, making interpretation of the results more straightforward than in the IS case, for example. We applied the DFM model to the European market of pork meat, once for the whole observation period from 1987 to 2015, and then also for the three subperiods that mark important milestones in the EU expansion. We state that the degree of market intergation in the EU increased over time.

There are, of course, many different extensions of the DFM. One of the directions that could be used in the further research is building time-varying parameters into the model. In this case, one would account for possible changes in the market integration without the necessity to subset the time series.

To sum up, though the price discovery theory was not historically developed for the analysis of the agricultural markets, it is a very interesting and promising field in the analysis. Despite certain limitations, we could outline methods allowing to tackle the issue and provide useful insights, as the directions for further investigations of the matter.

Appendices

Appendix A

ILS measure: analysis and simulations

Consider bivariate time series $y_t = (y_{1t}, y_{2t})$ with the following price discovery mesures: $PT_1 = x, PT_2 = 1-x, IS_1 = y, IS_2 = 1-y,\ x, y \in [0,1]$. Then from (1.27)

$$IL_1 = \left| \frac{y}{1-y} \cdot \frac{1-x}{x} \right|, \qquad IL_2 = \left| \frac{1-y}{y} \cdot \frac{x}{1-x} \right|.$$

Further,

$$IL_1 + IL_2 = \frac{y(1-x)}{x(1-y)} + \frac{x(1-y)}{y(1-x)} = \frac{y^2(1-x)^2 + x^2(1-y)^2}{xy(1-x)(1-y)},$$

and from (1.26)

$$ILS_1 = \frac{y^2(1-x)^2}{y^2(1-x)^2 + x^2(1-y)^2}, \qquad ILS_2 = \frac{x^2(1-y)^2}{y^2(1-x)^2 + x^2(1-y)^2}.$$

Both PT and IS are measures for price discovery, but it is not unusual for them to provide different estimators. Let us account for that by assuming $IS = k \cdot PT$, or $y = k \cdot x$, $k \in \mathbb{R}_+$[1]. When the measures are very close to each other, $k \approx 1$. Under

[1] We exclude the case $k = 0$ as trivial one

this assumption, for the ILS measure we have

$$ILS_1 = \frac{k^2(1-x)^2}{k^2(1-x)^2 + (1-kx)^2}, \qquad ILS_2 = \frac{(1-kx)^2}{k^2(1-x)^2 + (1-kx)^2}.$$

Further,

$$\begin{aligned} ILS_1 &= \frac{k^2(1-x)^2}{k^2(1-x)^2 + (1-kx)^2} = \left(\frac{k^2(1-x)^2 + (1-kx)^2}{k^2(1-x)^2}\right)^{-1} \\ &= \left(1 + \left(\frac{1-kx}{k(1-x)}\right)^2\right)^{-1} = (1+a^2)^{-1} = \frac{1}{1+a^2}, \end{aligned}$$

where

$$a = \frac{1-kx}{k(1-x)}.$$

We can then write

$$ILS_1 = \frac{1}{1+a^2}, \qquad ILS_2 = \frac{a^2}{1+a^2} = 1 - ILS_1.$$

Now for a, by dividing both its numerator and denominator by $k \neq 0$, we get

$$a = \frac{1-kx}{k(1-x)} = \frac{\frac{1}{k} - x}{1-x} = \frac{x - 1/k}{x-1}. \tag{A.1}$$

Of course, values of x, y and k are interdependent. Higher k values are associated with y closer to 1 and x closer to 0. In this case, $a \to 0$ and $ILS_1 \to 1$. For small k, $ILS_1 \to 0$.

To further investigate and visualize our findings, we simulated $n = 10^6$ samples of $x \in [0,1]$ and k such that $y := kx \in [0,1]$. We also bounden $k \in (0,2]$ to exclude the exteme cases as above (eg. values of 0.5 and 0.8 for PT and IS, respectively, are rather common). We only present the result of the simulation for ILS_1, since $ILS_2 = 1 - ILS_1$ by construction.

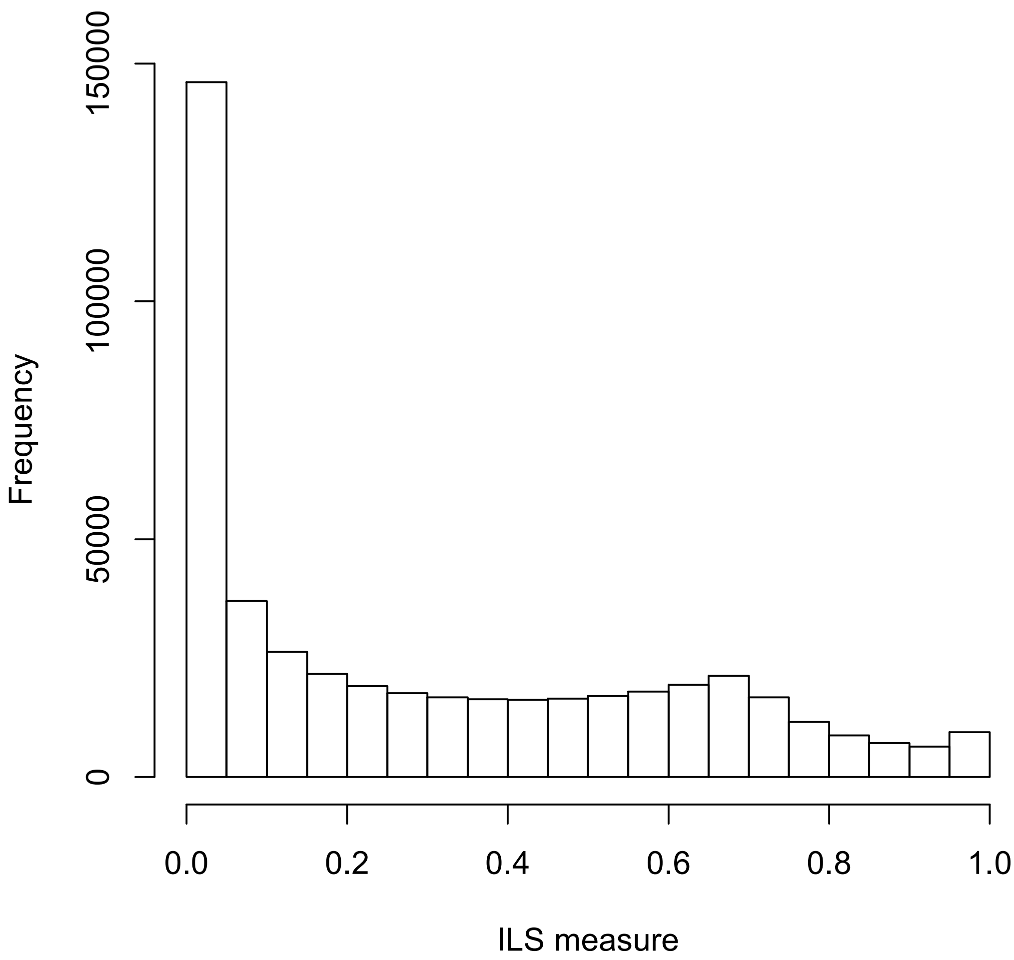


Figure A.1: Histogram of the simulated ILS_1 measure, $n = 10^6$.

Figure A.1 shows the histogram of the values of the simulates ILS_1 measure: values equal to or close to zero are the most frequent ones: for example, between 0 and 0.1 there are 32.1

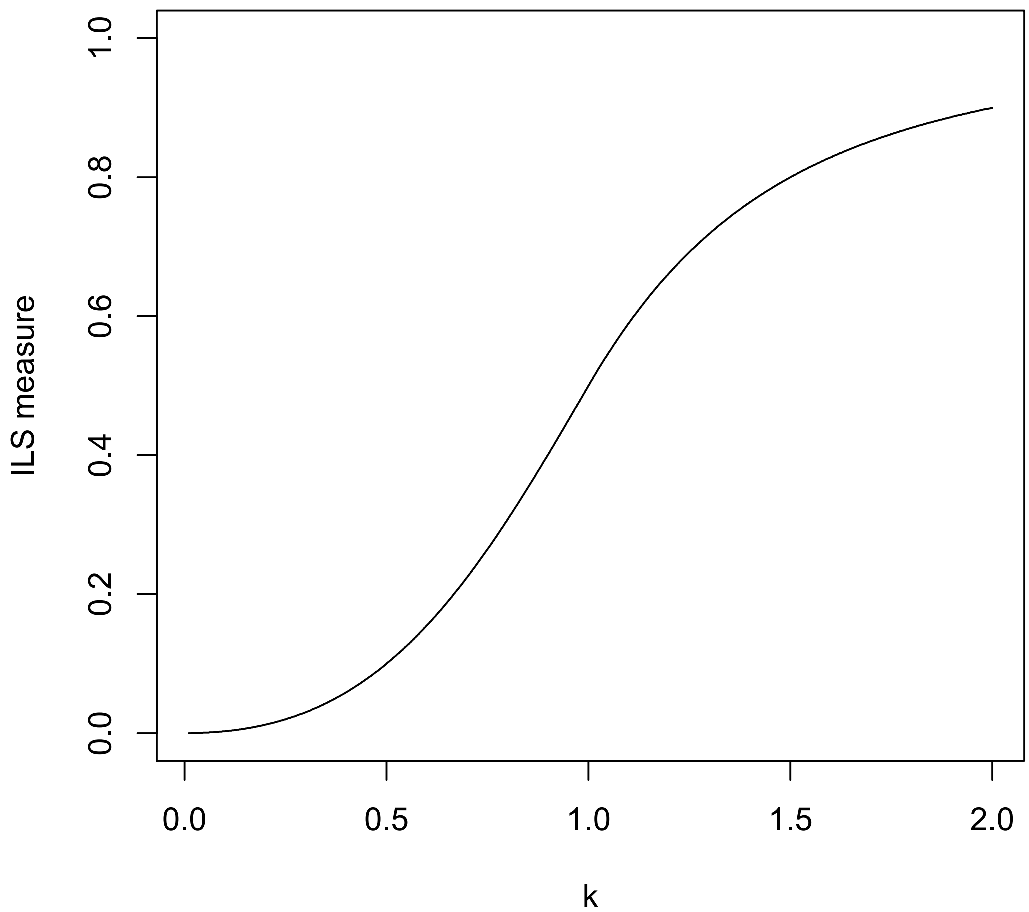


Figure A.2: Plot of the simulated ILS_1 measure as a function of k, $n = 10^6$.

Appendix B

Results of Testing for Unit Root in the Data

Test	Details (in R)	Test statistic	Decision	Result
ADF Test	Model with "drift", BIC	-4.5871	Reject H_0, accept H_1	Stationary, $I(0)$
PP Test	Z-alpha, short lags	-40.9579	Reject H_0, accept H_1	Stationary, $I(0)$
ERS Test	Model with "constant"	-4.7729	Reject H_0, accept H_1	Stationary,$I(0)$
KPSS Test	Model "mu", short lags	0.6843	Reject H_0, accept H_1	Unit root, $I(1)$

Table B.1: Testing for unit roots, Germany, full data set

Test	Details (in R)	Test statistic	Decision	Result
ADF Test	Model with "drift", BIC	-5.0456	Reject H_0, accept H_1	Stationary, $I(0)$
PP Test	Z-alpha, short lags	-45.7075	Reject H_0, accept H_1	Stationary, $I(0)$
ERS Test	Model with "constant"	-3.7165	Reject H_0, accept H_1	Stationary,$I(0)$
KPSS Test	Model "mu", short lags	0.9415	Reject H_0, accept H_1	Unit root, $I(1)$

Table B.2: Testing for unit roots, Netherlands, full data set.

Test	Details (in R)	Test statistic	Decision	Result
ADF Test	Model with "drift", BIC	-4.2295	Reject H_0, accept H_1	Stationary, $I(0)$
PP Test	Z-alpha, short lags	-38.6677	Reject H_0, accept H_1	Stationary, $I(0)$
ERS Test	Model with "constant"	-3.1673	Reject H_0, accept H_1	Stationary,$I(0)$
KPSS Test	Model "mu", short lags	2.9266	Reject H_0, accept H_1	Unit root, $I(1)$

Table B.3: Testing for unit roots, Belgium, full data set.

Test	Details (in R)	Test statistic	Decision	Result
ADF Test	Model with "drift", BIC	-5.0491	Reject H_0, accept H_1	Stationary, $I(0)$
PP Test	Z-alpha, short lags	-40.4179	Reject H_0, accept H_1	Stationary, $I(0)$
ERS Test	Model with "constant"	-3.8854	Reject H_0, accept H_1	Stationary,$I(0)$
KPSS Test	Model "mu", short lags	1.7152	Reject H_0, accept H_1	Unit root, $I(1)$

Table B.4: Testing for unit roots, France, full data set.

	Test	Test statistic	Decision	Result
Period 1	ADF	-2.0825	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.5326	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.3705	Under 5% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	0.6499	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 2	ADF	-2.8024	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.2436	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.0662	Cannot reject H_0	Unit root, $I(1)$
	KPSS	0.5114	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 3	ADF	-2.8169	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.0935	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.1564	Under 5% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	1.752	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$

Table B.5: Testing for unit roots, Germany, three periods.

	Test	Test statistic	Decision	Result
Period 1	ADF	-2.5352	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.2109	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.0988	Under 1% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	1.7979	Reject H_0, accept H_1	Unit root, $I(1)$
Period 2	ADF	-3.4966	Reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.0824	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.497	Cannot reject H_0	Unit root, $I(1)$
	KPSS	0.4802	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 3	ADF	-3.0038	Reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.1155	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.8558	Under 5% significance cannot reject H_0	Unit root, $I(1)$
	KPSS	1.5617	Reject H_0, accept H_1	Unit root, $I(1)$

Table B.6: Testing for unit roots, Netherlands, three periods.

	Test	Test statistic	Decision	Result
Period 1	ADF	-2.4956	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.2589	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.3059	Under 5% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	1.6635	Reject H_0, accept H_1	Unit root, $I(1)$
Period 2	ADF	-2.9294	Reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.32	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.1315	Cannot reject H_0	Unit root, $I(1)$
	KPSS	0.635	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 3	ADF	-2.8624	Under 5% significance cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.1703	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.8636	Under 5% significance cannot reject H_0	Unit root, $I(1)$
	KPSS	1.2754	Reject H_0, accept H_1	Unit root, $I(1)$

Table B.7: Testing for unit roots, Belgium, three periods.

	Test	**Test statistic**	**Decision**	**Result**
Period 1	ADF	-2.1794	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.5638	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.9765	Under 5% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	1.9458	Reject H_0, accept H_1	Unit root, $I(1)$
Period 2	ADF	-3.4149	Under 1% significance reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.0767	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.2051	Cannot reject H_0	Unit root, $I(1)$
	KPSS	0.5989	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 3	ADF	-3.4054	Under 1% significance reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.089	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.8658	Reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	2.1757	Reject H_0, accept H_1	Unit root, $I(1)$

Table B.8: Testing for unit roots, France, three periods.

	critical values		
Test	**1%**	**5%**	**10%**
ADF	-3.43	-2.86	-2.57
ERS	-2.57	-1.94	-1.62
KPSS	0.739	0.463	0.347

Table B.9: Critical values for ADF, ERS and KPSS tests.

Appendix C

PT Measure: Multivariate and Bivariate Approach

We show here that the in the bivariate case of $m = 2$ the standard PT formula (1.5) and the eigenvector method described in Section 2.1 provide identical results.

For $m = 2$ the matrix $I - \alpha\left(\alpha'\alpha\right)^{-1}\alpha'$ can be computed as follows:

1.
$$\alpha'\alpha = \begin{pmatrix}\alpha_1\alpha_2\end{pmatrix}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = \alpha_1^2 + \alpha_2^2 \Longrightarrow \left(\alpha'\alpha\right)^{-1} = \frac{1}{\alpha_1^2 + \alpha_2^2}$$

2.
$$\alpha\alpha' = \begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}\begin{pmatrix}\alpha_1\alpha_2\end{pmatrix} = \begin{pmatrix}\alpha_1^2 & \alpha_1\alpha_2\\ \alpha_1\alpha_2 & \alpha_2^2\end{pmatrix} \Longrightarrow$$
$$\alpha\left(\alpha'\alpha\right)^{-1}\alpha' = \frac{1}{\alpha_1^2 + \alpha_2^2}\begin{pmatrix}\alpha_1^2 & \alpha_1\alpha_2\\ \alpha_1\alpha_2 & \alpha_2^2\end{pmatrix}$$

3.

$$I - \alpha\left(\alpha'\alpha\right)^{-1}\alpha' = \begin{pmatrix} \frac{\alpha_2^2}{\alpha_1^2+\alpha_2^2} & \frac{-\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2} \\ \frac{-\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2} & \frac{\alpha_1^2}{\alpha_1^2+\alpha_2^2} \end{pmatrix} := A.$$

Now we need to find the eigenvalues of the above matrix A, which can be accomplisched in several steps:

1.

$$A - \lambda I = \begin{pmatrix} \frac{\alpha_2^2}{\alpha_1^2+\alpha_2^2} - \lambda & \frac{-\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2} \\ \frac{-\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2} & \frac{\alpha_1^2}{\alpha_1^2+\alpha_2^2} - \lambda \end{pmatrix} =$$

$$\frac{1}{\alpha_1^2+\alpha_2^2}\begin{pmatrix} \alpha_2^2 - \lambda(\alpha_1^2+\alpha_2^2) & -\alpha_1\alpha_2 \\ -\alpha_1\alpha_2 & \alpha_1^2 - \lambda(\alpha_1^2+\alpha_2^2) \end{pmatrix}$$

2.

$$\begin{aligned} \left|\begin{pmatrix} \alpha_2^2 - \lambda(\alpha_1^2+\alpha_2^2) & -\alpha_1\alpha_2 \\ -\alpha_1\alpha_2 & \alpha_1^2 - \lambda(\alpha_1^2+\alpha_2^2) \end{pmatrix}\right| &= -\lambda \cdot (\alpha_1^2+\alpha_2^2)^2 + \lambda^2 \cdot (\alpha_1^2+\alpha_2^2)^2 \\ &= (\lambda^2 - \lambda) \cdot (\alpha_1^2+\alpha_2^2)^2 \end{aligned}$$

3. It then follows, that

$$det(A - \lambda I) = 0 \Leftrightarrow \lambda^2 - \lambda = 0 \Leftrightarrow \lambda_1 = 0, \lambda_2 = 1.$$

As outlined in Section 2.1, we are only interested in the eigenvector associated with the unit eigenvalue of the matrix A, i.e. in the solution of

$$\begin{pmatrix} \alpha_1^2 & \alpha_1\alpha_2 \\ \alpha_1\alpha_2 & \alpha_2^2 \end{pmatrix},$$

which is given as the set of vectors of the form $\left(k, -\frac{\alpha_1}{\alpha_2} \cdot k\right), k \in \mathbb{R}$. By imposing additional restriction that the sum of the coefficients of the vector must equal 1 (to ensure the interpretation of the PT measure), we get that

$$k = \frac{\alpha_2}{\alpha_2 - \alpha_1} \quad \Longrightarrow \quad \text{the vector becomes} \quad \left(\frac{\alpha_2}{\alpha_2 - \alpha_1}, \frac{\alpha_1}{\alpha_1 - \alpha_2}\right),$$

which is exactly the standard PT formula (1.5).

Bibliography

[Arnade and Vocke, 2016] Arnade, C. and Vocke, G. (2016). Seasonal variation in the price discovery process of international wheat. *Agribusiness*, 32(1):16–32.

[Baillie et al., 2002] Baillie, R., Booth, G., Tse, Y., and Zabotina, T. (2002). Price discovery and common factor models. *Journal of Financial Markets*, 5:309–321.

[Bakucs and Ferto, 2005] Bakucs, L. and Ferto, I. (2005). Marketing margins and price transmission on the hungarian pork meat market. *Agribus*, 21(2):273–286.

[Bannerjee et al., 1993] Bannerjee, A., Dolado, L., Galbraith, J., and Hendry, D. (1993). *Integration, Error Correction, and the Econometric Analysis of Non-Stationary Data.* Oxford: Oxford University Press.

[Beck, 1993] Beck, N. (1993). Comparing dynamic specifications: The case of presidential approval. *Political Analysis*, (3):27–50.

[Berger, 2011] Berger, T. (2011). Estimating europe natural rates. *Empirical Economics*, 40(2):521–536.

[Berger et al., 2017] Berger, T., Grabert, S., and Kempa, B. (2017). Global macroeconomic uncertainty. *Journal of Macroeconomics*, 53:42–56.

[Berger and Pozzi, 2013] Berger, T. and Pozzi, L. (2013). Measuring time-varying financial market integration: An unobserved components approach. *Journal of Banking & Finance*, 37(2):463–473.

[Binning, 2014] Binning, A. (2013-2014). Underidentified svar models: A framework for combining short and long-run restrictions with sign-restrictions.

[Campbell and Hendry,] Campbell, B. and Hendry, S. Price discovery in canadian and u.s. 10-year government bond markets. Working Paper 2007-43, Bank of Canada, August 2007.

[Cao et al., 2009] Cao, C., Hansch, O., and Wang, X. (2009). The information content of an open limit-order book. *Journal of Futures Markets*, (29):16–41.

[Davidson and MacKinnon, 1993] Davidson, R. and MacKinnon, J. (1993). *Estimation and Inference in Econometrics*. New York: Oxford University Press.

[de Jong, 2002] de Jong, F. (2002). Measures of contributions to price discovery: a comparison. *Journal of Financial Markets*, 5:323–327.

[DeJong et al., 1992] DeJong, D., Nankervis, J., Savin, N., and Whiteman, C. (1992). The power problems of unit root test in time series with autoregressive errors. *Journal of Econometrics*, 53:323–343.

[Del Negro and Otrok, 2008] Del Negro, M. and Otrok, C. (2008). Dynamic factor models with time-varying parameters: Measuring changes in international business cycles.

[Durbin and Koopman, 2012] Durbin, J. and Koopman, S. (2012). *Time Series Analysis by State Space Methods*. Oxford Statistical Science Series.

[Engle and Granger, 1987] Engle, R. and Granger, C. W. J. (1987). Co-integration and error correction: Representation, estimation and testing. *Econom*, 55:251–276.

[Figuerola-Ferreti and Gonzalo, 2010] Figuerola-Ferreti, I. and Gonzalo, J. (2010). Modelling and measuring price discovery in commodity markets. *Journal of Econometrics*, 158:95–107.

[Fry and Pagan, 2009] Fry, R. and Pagan, A. (2009). Sign restrictions in structural vector autoregressions: A critical review.

[Gambetti,] Gambetti, L. Structural var: Theory.

[Garbade and Silber, 1983] Garbade, K. and Silber, W. (1983). Price movements and price discovery in futures and cash markets. *The Review of Economics and Statistics*, 65:289–297.

[Gonzalo and Granger, 1995] Gonzalo, J. and Granger, C. (1995). Estimation of common long-memory components in cointegrated systems. *Journal of Business and Economic Statistics*, 13:27–36.

[Goodwin and Harper, 2000] Goodwin, B. and Harper, D. (2000). Price transmission, threshold behavior, and asymmetric adjustment in the u.s. pork sector. *Journal of Agricltural and Applied Economics*, 32(3):543–553.

[Gourieroux et al., 2016] Gourieroux, C., Monfort, A., and Renne, J.-P. (2016). Statistical inference for independent component analysis: Application to structural var models.

[Grebitus et al., 2011] Grebitus, C., Menapace, L., and Bruhn, M. (2011). Consumers' use of seals of approval and origin information: evidence from the german pork market. *Agribusiness*, 27(4):478–492.

[Hamilton, 1994] Hamilton, J. (1994). *Time Series Analysis*. Princeton University Press.

[Hansen and Seo, 2002] Hansen, B. and Seo, B. (2002). Testing for two-regime threshold cointegration in vector error-correction models. *Journal of Econometrics*, 110:293–318.

[Harris et al., 2002a] Harris, F., McInish, T., and Wood, R. (2002a). Common factor components versus information shares: a reply. *Journal of Financial Markets*, 5:341–348.

[Harris et al., 2002b] Harris, F., McInish, T., and Wood, R. (2002b). The dynamics price adjustment across exchanges: an investigation of common factor components for dow stocks.

[Hasbrouck, 1995] Hasbrouck, J. (1995). One security, many markets: Determining the contributions to price discovery. *Journal of Finance*, 50:1175–1199.

[Hasbrouck, 2002] Hasbrouck, J. (2002). Stalking the "efficient price" in market microstructure specifications: an overview. *Journal of Financial Markets*, 5:329–339.

[Hassouneh et al., 2012] Hassouneh, I., von, Serra, T., and Gil, J. (2012). Recent developments in the econometric analysis of price transmission. *Working Paper No. 2, Transparency of Food Pricing TRANSFOP*.

[Hernandes and Torero, 2010] Hernandes, M. and Torero, M. (2010). Examining the dynamic relationship between spot and future prices of agricultural commodities. In *IFPRI Discussion Paper 00988*.

[Herwartz and Plodt, 2016] Herwartz, H. and Plodt, M. (2016). The macroeconomic effects of oil price shocks: Evidence from a statistical identification approach. *Journal of International Money and Finance*, 61:30–44.

[Hjalmarsson and Oesterholm, 2007] Hjalmarsson, E. and Oesterholm, P. (2007). *Testing for Cointegration Using the Johansen Methodology When Variables Are Near-Integrated*. International Monetary Fund.

[Holst and von Cramon-Taubadel, 2011] Holst, C. and von Cramon-Taubadel, S. (2011). International synchronisation of the pork cycle. In *EAAE Congress Change and Uncertainty: Challenges for Agriculture, Food and Natural Resources*.

[Holst and von Cramon-Taubadel, 2013] Holst, C. and von Cramon-Taubadel, S. (2013). Trade, market integration and spatial price transmission on eu pork markets following eastern enlargement. *Diskussion Paper, Department of Agricultural Economics und Rurale Development, University Goettingen, No. 1307.*

[Johansen, 1988] Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12(2-3231-254).

[Johansen, 1991] Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in gaussian vector autoregressive models. *Econometrica*, 59:1551–1580.

[Keele and De Boef, 2004] Keele, L. and De Boef, S. (2004). Not just for cointegration: Error correction models with stationary data. *Documento de Trabajo. Departamento de Politica y Relaciones Internacionales, Nuffield College y Oxford University.*

[Kilian, 2009] Kilian, L. (2009). Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market. *American Economic Review*, 99(3):1053–1069.

[Kilian and Lutkepohl, 2017] Kilian, L. and Lutkepohl, H. (2017). *Structural Vector Autoregressive Analysis*. Cambridge University Press.

[Kim et al., 2005] Kim, S., Moshirian, F., and Wu, E. (2005). Dynamic stock market integration driven by the european monetary union: An empirical analysis. *Journal of Banking and Finance*, 29(10):2475–2502.

[Koop and Korobilis, 2010] Koop, G. and Korobilis, D. (2010). *Bayesian Multivariate Time Series Methods for Empirical Macroeconomics*. Business & Economics.

[Kumar, 2004] Kumar, S. (2004). Price discovery and market efficiency: Evidence from agricultural commodities futures markets. *South Asian Jornal of Management*, 11(2):32–47.

[Lanne et al., 2010] Lanne, M., Lutkepohl, H.tkepohl, H., and Maciejowska, K. (2010). Structural vector autoregressions with markov switching. *Journal of Economic Dynamics and Control*, 34(2):121–131.

[Lehmann, 2002] Lehmann, B. (2002). Some desiderata for the measurement of price discovery across markets. *Journal of Financial Markets*, 5:259–276.

[McNew and Fackler, 1997] McNew, K. and Fackler, P. (1997). Testing market equilibrium: Is cointegration informative? *Journal of Agricultural nd Resourse Economics*, 22(2):191–207.

[Moneta et al., 2013] Moneta, A., Entner, D., Hoyer, P., and Coad, A. (2013). Causal inference by independent component analysis: Theory and applications. *Oxford Bulletin of Economics and Statistics*, 75(5):705–730.

[Morley, 1999] Morley, J. (1999). A note on constraining ar(2) parameters in estimation.

[Mountford and Uhlig, 2009] Mountford, A. and Uhlig, H. (2009). What are the effects of fiscal policy shocks? *Journal of Applied Econometrics*, 24(6):960–992.

[Perron, 1993] Perron, P. (1993). The hump-shaped behavior of marcoeconomic fluctuations. *Empirical Economics*, 18:707–727.

[Peter, 2011] Peter, F. (2011). *Where is the Market? Three Econometric Approaches to Measure Contributions to Price Discovery.* PhD thesis, Eberhard-Karls-University Tuebingen.

[Pozo et al., 2016] Pozo, V., Bejan, V., and Tejeda, H. (2016). Disentangling supply and demand shocks to identify changes in the live cattle market structure post livestock mandatory price reporting act. In *Agricultural & Applied Economics Association Annual Meeting.*

[Putnins, 2013] Putnins, T. (2013). What do price discovery metrics really measure? *Journal of Empirical Finance*, 23:68–83.

[Rubio-Ramirez et al., 2010] Rubio-Ramirez, J., Waggoner, D., and Zha, T. (2010). Structural vector autoregressiauto theory of identification and algorithms for inference. *The Review of Economic Studies*, 77(2):665–696.

[Sanjuan and Gil, 2001] Sanjuan, A. and Gil, J. (2001). A note on tests for market integration in a multivariate non-stationary framework. *Journal of Agricultural Economics*, 52(2):113–121.

[Sekhar, 2012] Sekhar, C. (2012). Agricultural market integration in india: An analysis of select commodities. *Food Policy*, 37(3):309–322.

[Stock and Watson, 1988] Stock, J. and Watson, M. (1988). Testing for common trends. *Journal of the American Statistical Association*, 831:1097–1107.

[Trefethen and Bau, 1997] Trefethen, L. N. and Bau, D. I. (1997). *Numerical Linear Algebra*. SIAM.

[Uhlig, 2005] Uhlig, H. (2005). What are the effects of monetary policy on output? resul ts from an agnostic identification procedure. *Journal of Monetary Economics*, 52:381–419.

[Verbeck, 2000] Verbeck, M. (2000). *A Guide to Modern Econometrics.* New York: John Wiley and Sons.

[von Cramon-Taubadel, 1998] von Cramon-Taubadel, S. (1998). Estimating asymmetric price transmission with the error correction representation: An application to the german pork market. *European Review of Agricultural Economics*, 25(1):1–18.

[Wessels, 1997] Wessels, W. (1997). An ever closer fusion? a dynamic macropolitical view on integration processes. *Journal of Common Market Studies*, 35(2):267–299.

[Williams, 1993] Williams, J. (1993). What goes around comes around: Unit root tests and cointegration. *Political Analysis*, (4):229–236.

[Yan and Zivot, 2010] Yan, B. and Zivot, E. (2010). A structural analysis of price discovery measures. *Journal of Financial Markets*, 13:1–19.

[Zang et al., 2004] Zang, C., Friswell, M. I., and Imregun, M. (2004). Structural damage detection using independent component analysis. *Structural Health Monitoring*, 3(1):69–83.

[Zivot, 2000] Zivot, E. (2000). Notes on structural var modeling.