Doctoral Thesis

Business and Financial Cycles in a Globalized World

A thesis submitted in fulfillment of the requirements for the doctoral degree of the Faculty of Business and Economics of the Georg-August-University Göttingen

Julia Richter

2022

Supervisors:
Prof. Dr. Tino Berger
Prof. Dr. Lorenzo Pozzi
Prof. Dr. Helmut Herwartz

Acknowledgements

It's been quite a long way finishing this thesis and it took me almost eight years finally getting to this point. In particular the last two years since I left the university – and since working on the thesis became a thing for the weekends – have been challenging. Having said this, I want to acknowledge that this achievement would not have been possible without the magnificent support by many people who accompanied my way either partly or during the complete almost eight years.

First and foremost, I am thankful to my first supervisor Tino Berger for enabling me to write this dissertation. His endless patience with myself and his confidence in my work were motivating even during difficult times. He taught me to bear the ups and downs of doing research, to be patient and most of all to do the single small steps before the big jumps. And I want to thank Tino for making it possible for me to travel to many conferences and workshops, which taught me how to sell my research but also created tonnes of nice memories.

Then I want to thank Lorenzo Pozzi and Helmut Herwartz, my second and third supervisors, for their guidance and always constructive feedback on my work. I am grateful that I had the chance discussing with and learning from them.

Furthermore, I want to thank my former colleagues and office mates at Göttingen university for their support, patience and countless nights in bars. Thank you Oliver Saffran, Markus Ahlborn, Marcus Wortmann, Markus Stahl and Tore Dubbert for the time we shared. I want to emphasize a big thanks to Maren Ulm, who always lifted me up during difficult times, gave me confidence and supported me with proofreading and great feedback.

I also want to thank my new colleagues at the German Council of Economic Experts and the institution as a whole which allowed me to spent some time on finishing this thesis. In particular, I want to thank Wolf Heinrich Reuter who pushed me over the finishing line and Lars Other for extensive proofreading and lots of motivating conversations.

Last but not least, I want to thank Stefan Bien for his incredible patience and appreciation of my situation and for being my tower of strength. Thanks for being in my life, believing in me and having encouraged me to go on.

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1 | General Introduction

Jerome Powell (2019), chair of the U.S. Federal Reserve, said in a speech in 2019: *"The global nature of the financial crisis and the channels through which it spread sharply highlight the interconnectedness of our economic, financial and policy environments. US economic developments affect the rest of the world, and the reverse is also true."* With this statement, Powell highlights the importance of two aspects which should be at the center of this thesis, namely the co-movement of business cycles across countries as well as the interaction of the general economic activity with financial markets. Along these lines, this dissertation consists of three main chapters, reflecting three self-contained papers, all of which are dedicated to the linkages of business cycles either across countries (chapter 1), the linkages of business cycles with financial variables (chapter 3), or both (chapter 2).

Recessions in most advanced economies (and also major emerging markets) over the past decades were synchronized, like the two oil crisis in the 1970s and the global financial crisis and following recession of 2008-09. But the analysis of business cycle similarities across countries has even started long before these global recessions. Moore and Zarnowitz (1986) summarize, that Mitchell (1927) in one of his early work on business cycle dating at the National Bureau of Economic Research (NBER) already documented a "close correspondence" between the recession dates in the United States and England in the 19th century. Since then, many studies have documented similarities of business cycles across countries. While these studies first focused on the co-movement across advanced economies (see e.g. Backus and Kehoe, 1992; Baxter, 1995; Gregory, Head, and Raynauld, 1997; Gregory and Head, 1999), the proceeding globalization and the intensification of trade linkages with emerging market economies has shifted this focus to a broader set of countries analyzing the synchronization of macroeconomic aggregates between different country groups (Kose, Otrok, and Whiteman, 2003, 2008; Kose, Otrok, and Prasad, 2012). The process of globalization and financial liberalization has intensified the linkages across countries raising the question whether this has increased the synchronization of their business cycles, too. From a theoretical point of view, the answer to this question is not straightforward as there are opposing theories whether intensified trade and financial linkages result in higher or lower international synchronization of business cycles (see Kose, Otrok, and Prasad, 2012, for an overview). However, empirical evidence suggests that business cycle synchronization has not increased in light of increasing globalization. For example, Stock and Watson (2005) show that business cycles across major advanced economies have not become more synchronized during the 1980s and 1990s, a period where globalization and financial liberalization increased pace, because the magnitude of common international shocks has decreased as compared to previous decades. Kose, Otrok, and Prasad (2012) come to a similar conclusion in a sub-sample analysis of the relative importance of global, group-specific, and country-specific fluctuations in macroeconomic aggregates for the variation of these time series in a certain country. While they document that global fluctuations have become less important post-1985, fluctuations specific to a group of countries (advanced, emerging and developing countries) have become more important instead. These results shaped the notion of "business cycle decoupling".

The *second chapter* of this dissertation, which is joint work with Tino Berger, contribute to the discussion on the changing patterns in business cycle synchronization over time. In this study, we examine why the global business cycle has lost its relative importance for the business cycle of individual countries over time. In other words, we ask why business cycle synchronization across countries appears to decrease over time. There are two main reasons. On the one hand, a given country may become less sensitive to global fluctuations over time, i.e. to changes in the global business cycle. A decline in the sensitivity to global changes would be counter-intuitive at first sight. In the light of increasing global trade and complex global value chains, one would expect that countries become more integrated and their economies react more sensitive to changes in global dynamics. On the other hand, the frequency and magnitude of global shocks may have decreased over time, such that the volatility of the global business cycle declines as suggested by Stock and Watson (2005) among others. We use a hierarchical factor model with time-varying factor loadings and time-varying variances, to decompose the growth rates of macroeconomic variables of a large group of countries into global, group-specific, country-specific and idiosyncratic cycles. Thereby, time-varying factor loadings can account for changes in the sensitivity to a factor, for example the global business cycle, and time-varying variances can account for changes in the magnitude and frequency of, e.g., common shocks. But instead of claiming the existence of time-variation in the parameters, we apply a statistical method to test whether the parameters of interest are actually time-varying or whether they rather remain constant over time. Our results clearly show the need for modeling time-varying variance parameters and suggest that the volatility of the global business cycle has decreased over time. However, we find no evidence for time-variation in the factor loadings. These results imply that the observed decline in the synchronization of business cycles across countries are not linked to patterns of de-globalization or less integration in global markets. On the contrary: in the cases where our model confirms that a sensitivity parameter, i.e. a factor loading, has changed over time, the results suggest that sensitivity to fluctuations in the global business cycle has increased. Therefore, we link the decline in international business cycle synchronization to a decline in the magnitude of global shocks, a result in line with the broader literature on the Great Moderation.

As Powell indicates in his speech, not only the cross-country interconnectedness of macroeconomic aggregates is important for the economic developments within an economy. Indeed, also the cross-country dynamics in financial markets and the interaction between the real economy and financial markets may play a major role. The global financial crisis of 2008-09 was a reminder of how close these ties can be and how large fluctuations in asset prices can deteriorate the balance sheets of households and corporations with a negative impact on their creditworthiness and hence on private consumption and investment decisions. In a world with a high degree of financial integration and globally operating banks and financial intermediaries, these effects can rapidly spill-over across countries. A large literature analyzes these linkages between the real economy and financial markets, commonly known as macro-financial linkages. These linkages in a broader sense denote the two-way interactions between the macroeconomy and the financial sector, usually described by financial asset prices, house prices, credit or short-term interest rates. Over the recent past, an increasing number of studies shifted focus towards international macro-financial linkages (see Claessens and Kose, 2018, for a comprehensive overview of the theoretical and empirical literature). The analysis by Helbling, Huidrom, Kose, and Otrok (2011), for example, shows that global credit shocks had a sizable impact on the dynamics of the global business cycle during the global recession of 2007-09. Similarly, Ha, Kose, Otrok, and Prasad (2020) document for the G7 countries the existence of spill-overs from common equity and house price cycles onto fluctuations in macroeconomic aggregates like output and investment.

This is the starting point for the *third chapter* of this thesis, which is also joint work with Tino Berger. In this study, we examine the importance of international macro-financial linkages for the business cycles in the countries of the G7 group.

Building on the broader literature, we define macro-financial linkages in terms of the joint cyclical fluctuations in macroeconomic aggregates, credit and real estate prices, often used to define the financial cycle (e.g. Drehmann, Borio, and Tsatsaronis, 2012; Hiebert, Peltonen, and Schüler, 2015; Galati, Hindrayanto, Koopman, and Vlekke, 2016). As compared to previous studies analyzing the international co-movement of cyclical fluctuations, our empirical model builds on a joint trend-cycle decomposition and decomposes several real activity and financial variables into their trends and short-run deviations from these trends. Put differently, instead of analyzing common fluctuations in the growth rates of variables, we investigate the gaps, like for example the output gap and gaps in credit and house prices, which are commonly used as the business cycle and financial cycles, respectively. The joint trend-cycle decomposition allows us to model different common and country-specific factors in the gaps of the variables by means of a hierarchical factor model. As common in factor model literature, we can therefore quantify the relative importance of different factors – for example of common macro-financial linkages – for the output gap in each country. While our analysis confirms the existence of common cycles in real activity and financial variables separately, we find no evidence for common cross-country fluctuations across real activity and financial variables simultaneously. This finding suggests that there is no common shock driving the dynamics of gaps in our sample, a result similar to the findings documented by Ha, Kose, Otrok, and Prasad (2020).

In chapter three, we focused on analyzing common fluctuations across real activity and financial variables resulting from common shocks. This is by definition entirely analysed in reduced form, because the origin of the shocks is a priori not clear and is not the subject of this study. Hence, with a factor model applied in the previous chapters, we measure the correlation of macro and financial variables, which represents a first step in understanding macro-financial linkages. In a second step, in order to understand the sources of fluctuations in business cycles, one might be interested, for example, in quantifying the role of financial shocks.

In *chapter four*, which is joint work with Tino Berger and Benjamin Wong and is *published in the Journal of Economic Dynamics and Control*, we examine the role of financial variables for the U.S. output gap through the lens of a structural model. The main contribution is this paper, is that we jointly estimate business and financial cycles in the U.S. from a Beveridge-Nelson decomposition within a medium-scale Bayesian Vector Autoregression (BVAR). With this BVAR, we are able to make structural interference the role of financial shocks for the output gap. We document three main findings. First, from the reduced form analysis of the BVAR, we find an increasing role of financial variables for the U.S. output gap since the 2000s. In particular, the results suggest that loose financial conditions in the 2000s, as measured by the excess credit spread constructed by Gilchrist and Zakrajšek (2012), contributed to the overheating of the U.S. business cycle pre-Great Recession. In the structural analysis of the BVAR, we moreover find that the identified financial shock contributed 2 % to 4 % to the size of the output gap during the 2000s. The third result takes the perspective of macroprudential policies aiming to take guidance from developments in financial variables, usually related to credit, for imbalances in the real economy. We therefore analyze the correlation of the business and the credit cycle lagged by four quarters for two cases, conditionally and unconditionally on the occurrence of a financial shock. In the unconditional case, we find that the business cycle and the lagged credit cycle are positively correlated meaning a boom in the credit cycle is followed by a boom in the business cycle a few quarters later. However, if we condition these cross-correlations on a financial shock, the correlation becomes negative, meaning a boom in the credit cycle is followed by bust a in the business cycle. While we argue that these results call for a deeper analysis of the forces that generate the boom-bust cycles between the business and the credit cycle, we think that the results at least suggest that not every boom in the credit cycle justifies the need of stabilization policies, because not every boom in the credit cycle eventually results in a bust in the business cycle.

Overall, this dissertation on the synchronization of business and financial cycles across countries and, more broadly, the linkages between macroeconomic and financial variables contributes to the growing literature in these areas. I use joint empirical modeling approaches to verify and understand several empirical findings. In the context of analyzing time-varying business cycle synchronization, the findings in the second chapter help to understand why we observe a decoupling of business cycles across countries. The third chapter is a starting point for jointly analyzing macro-financial linkages across advanced economies based on the gaps of real activity and financial variables instead of their growth rates. Finally, the joint empirical approach in the fourth chapter contributes to our understanding of the role financial factors may play for the business cycle in a structural sense. Moreover, it is a starting point for future work on understanding the interaction between business and credit cycles from the perspective of macroprudential stabilization policies.

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2 What Has Caused Global Business Cycle Decoupling: Smaller Shocks or Reduced Sensitivity?

with Tino Berger

Abstract

In this paper we ask the question what has caused the decline in the importance of the global business cycle for national cycles: smaller global shocks or reduced sensitivities of national cycles to these shocks? We use a dynamic factor model with time-varying loadings and stochastic volatilities to decompose macroeconomic fluctuations over 1961–2014 into a global, group-, and country-specific factors and use a stochastic model selection procedure to test for time-variation in all parameters. Results show that the size of global shocks has become smaller, whereas the sensitivity to these shocks remains constant for most countries. We conclude that the smaller size of global shocks has driven the decrease in the importance of the global business cycle.

JEL Classification: F44, C52, C32

Keywords: global business cycle, dynamic factor model, time-varying parameter, stochastic volatility, model selection

2.1 Introduction

Over the past decades, economic linkages between countries have increased dramatically. The reduction of trade barriers and capital controls have led to a substantial increase in world trade and international capital flows and thus a high level of interconnectedness among countries. These growing linkages among countries have reinforced research on the global business cycle and raised the question whether its importance for national business cycles has changed over time (see e.g. Kose, Otrok, and Prasad, 2012).

A common approach to analyze the relative importance of international and domestic factors is to estimate a dynamic factor model and calculate the variance share of international and domestic factors with respect to the business cycle variance. Changes in the relative importance of a factor, as measured by its variance share, are analyzed either by estimating the model for different sub-periods or by estimating a time-varying parameter model. Most of the empirical studies using dynamic factor models indeed find that the importance of the global business cycle has changed over time (see e.g. Ductor and Leiva-Leon, 2016; Mumtaz, Simonelli, and Surico, 2011; Kose, Otrok, and Prasad, 2012). A change in the importance of the global business cycle shows whether a country has become more or less synchronized with the rest of the world. This does, however, not show whether this pattern is due to a change in the country's sensitivity to the global business cycle or due to a change in the size of global shocks relative to domestic shocks.

In this paper, we shed light on this issue and disentangle the sources of changes in the importance of the global business cycle. In particular, we ask the following question. Are changes in the importance of the global factor the result from changes in the countries' sensitivity to global shocks, or changes in the volatility of the global factor? We address this question by analyzing the commonality of macroeconomic growth rates for a large panel of 106 industrialized, emerging market, and developing countries over the period 1961-2014. To this end, we employ a Bayesian dynamic factor model with time-varying factor loadings and stochastic volatility to the factor innovation variances. Following Frühwirth-Schnatter and Wagner (2010), we explicitly test for the time variation in the factor loadings and the factor variances by means of a Bayesian stochastic model specification search. As such, the model allows for time variation in the parameters, but does not force the parameters to change. This enables us to obtain time-varying variance shares with endogenously determined time variation without deciding a priori which parameters are constant and which are time-varying. Hence, particular attention is paid to parameter uncertainty.

Changes in factor loadings reflect changes in how sensitive a country is to that factor. Thus, a decline in the loading to the global factor reflects a reduction in the sensitivity to global shocks, while an increase in the factor loading to regional or group specific factors, points to an increased sensitivity to the respective regional or group-specific shocks. In this case, the relative importance of the global factor would decrease, while the relative importance of the group factor would increase. In contrast, changes in a factor's variance reflect changes in the size of the shocks. Allowing for heteroscedasticity is particularly important, as it is a potential reason for changes in the variance decomposition. Suppose the volatility of the global factor declines more than the business cycle volatility of a given country. The fraction of the country's variance that is explained by the global business cycle would decline too, even when its sensitivity, i.e., the factor loading to the global factor, remains constant.

Our results can be summarized as follows. We do find strong evidence for changes in the volatilities of the global and all three group factors: their volatility has steadily declined from 1960 until the early 2000s. Evidence for time variation in the factor loadings is much weaker. Though there is some heterogeneity across countries, the overall picture is that only industrialized countries have exhibited changes in their sensitivity to the global factor. Emerging market economies and developing countries have, on average, constant factor loadings with respect to the global factor. Similarly, the sensitivity to the group-specific factors is found to be constant for the majority of countries in each country group. As a consequence, changes in the variance decomposition are primarily driven by changes in the volatilities. Finally, though we find a significant role of the group-specific factors for national business cycles, the increase in the importance of the group factors relative to the global factor is weaker as compared to previous findings.

The remainder of this paper is organized as follows. Section 2.2 provides an overview over the most important literature related to our paper. Section 2.3 introduces our empirical approach, including the DFM with time-varying factor loadings and stochastic volatilities, and explains the Bayesian model selection procedure to test for time variation in the parameters. The results are presented in Section 2.4 and, finally, Section 5 concludes.

2.2 Literature review

While there is a large body of empirical literature analyzing the synchronization of business cycles at the international, group or regional level, three studies are closest to our paper.

First, Kose, Otrok, and Prasad (2012) (henceforth KOP) were the first who systematically documented the convergence of business cycles within and decoupling of business cycles between different country groups for 106 countries. By calculating variance decomposition for two periods, they show that the global business cycle has become less important for the average of industrial and emerging economies during the globalization period as compared to the pre-globalization period.

Second, Del Negro and Otrok (2008) use a time-varying parameter dynamic factor model similar to the model in this paper to analyze changes in the shock volatility and the sensitivity to shocks for 19 advanced economies in the post-Bretton Woods period. They provide evidence for a decline in the variance of international factors as well as a heterogeneous evolution of countries' sensitivities to the international factor, but without particularly using the model for addressing the 'decoupling' of business cycles as their sample only covers advanced economies.

Third, Karadimitropoulou and León-Ledesma (2013) examine the change in business cycle synchronization between two periods across different sectors among the G7 countries in a dynamic factor model. One of the authors' question is closely related to the research question of this paper. In order to explain the changes in the variance decomposition from the pre-globalization to the globalization period, the variance of the relevant business cycle variable is decomposed into different sources. First, into changes in the factor loadings that reflect the degree of covariation between the business cycle variable and the factors. Second, into changes in the variance of these factors which are further split into changes in the persistence of the factors, and changes in the variance of the factors' innovations. By means of this decomposition, they show that for the G7 countries, a sharp decline in the variance of the global factor innovation drives most of the change in the variance of all series. Moreover, the contribution of the factor loadings is small, but positive for the most countries.

Economically, it is important to differentiate between the two reasons of changes in the variance decomposition. First, changes in the sensitivity to a factor as measured by the factor loadings are the result of domestic policies targeting, for example, the external sector or the financial sector of a particular country, and hence are country-specific. However, the effect of reducing trade and financial barriers on the synchronization of business cycles is ambiguous from a theoretical point of view. On the one hand, increasing trade flows should increase output comovements between countries, since both demand- and supply-side spillovers are generated. On the other hand, output comovements might decrease if stronger trade linkages result in increased specialization according to each country's comparative advantage and if industry-specific shocks are dominant (Baxter and Kouparitsas, 2005). The effect of stronger financial linkages on output comovements could take both directions as well. For example, output synchronization between countries could decrease if stronger financial linkages facilitate the efficient reallocation of capital and hence result in an increased specialization of production according to the countries' comparative advantages. However, increased financial linkages could also result in higher international business cycle comovement if contagion effects, transmitted through financial markets, are present (Kose and Prasad, 2010).

Second, changes in the variances of the factors are affected by the magnitude of the shocks to international and national business cycles. As widely discussed for the case of the U.S. regarding the Great Moderation, there is no consensus on what has caused the reduction in the volatility of output. Hence, this phenomenon could have been the outcome of better policies (Blanchard and Simon, 2001), good luck in the sense of no major adverse shocks (Stock and Watson, 2003), or structural changes in the economy (Kahn, McConnell, and Perez-Quiros, 2002). Even though this discussion has focused on the U.S. economy, these arguments can be applied to the decreasing volatility of international business cycles as well.

2.3 Empirical approach

This section explains our econometric approach. First, it lays out a DFM with time-varying factor loadings and stochastic volatilities. Second, the Bayesian stochastic model selection approach is explained, followed by a description of the Markov Chain Monte Carlo (MCMC) algorithm employed to estimate the model.

2.3.1 A DFM with time-varying loadings and stochastic volatilities

We follow KOP and construct a multivariate DFM that decomposes the real GDP, private consumption, and investment growth into a global factor F_t^g, which is common to all variables in all countries, three group-specific factors, denoted F_t^{IC}, F_t^{EM}, and F_t^{DC}, which are common to all variables and all countries belonging to either the group of industrial countries (ICs), emerging market economies (EMs), or other developing countries (DCs), a country-specific factor $F_{i,t}^c$ that is common to all variables within a country i, and idiosyncratic factors $\varepsilon_{i,t}^j$, which are specific to each variable.

More specifically, let $y_{i,t}^j$ denote the annual growth rate of variable j in country i at time t. The model is given by

$$y_{i,t}^j = \alpha_{i,t}^j F_t^g + \beta_{i,t}^j F_t^r + \delta_{i,t}^j F_{i,t}^c + \varepsilon_{i,t}^j, \tag{2.1}$$

where the group-specific factor F_t^r equals either F_t^{IC}, F_t^{EM}, or F_t^{DC}, depending on the group affiliation of country i. All factors in Equation (2.1) are assumed to follow independent AR(3) processes with stochastic volatility in the innovations,

$$D_t = \sum_{l=1}^{3} \theta_l^D D_{t-l} + \exp(h_t^D)\psi_t^D, \qquad \psi_t^D \overset{iid}{\sim} \mathcal{N}(0,1), \tag{2.2}$$

where $D_t = \left\{F_t^g, F_t^{IC}, F_t^{EM}, F_t^{DC}, F_{i,t}^c\right\}$. The terms h_t^D represent the stochastic volatility parts accounting for changes in the factors' variances and follow random walk processes,[1]

$$h_t^D = h_{t-1}^D + \eta_t^D, \qquad \eta_t^D \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\eta,D}^2). \tag{2.3}$$

Similarly, the idiosyncratic factors follow AR(3) processes,[2]

$$\varepsilon_{i,t}^j = \sum_{l=1}^{3} \phi_{l,i}^j \varepsilon_{i,t-l}^j + \nu_{i,t}^j, \qquad \nu_{i,t}^j \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\nu,i,j}^2). \tag{2.4}$$

In order to take into account possible changes in the sensitivity to the factors, we model all factor loadings as random walks,

$$\zeta_{i,t}^j = \zeta_{i,t-1}^j + \kappa_{\zeta,i,t}^j, \qquad \kappa_{\zeta,i,t}^j \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\kappa,\zeta,i,j}^2), \tag{2.5}$$

where $\zeta = \{\alpha, \beta, \delta\}$. The innovations to the factor loadings are orthogonal, implying that changes in the factor loadings are uncorrelated across countries.

[1] For a detailed description of stochastic volatility processes, we refer to Kim, Shephard, and Chib (1998) and Omori, Chib, Shephard, and Nakajima (2007).

[2] Given that the model is fitted to annual data, the AR(3) assumption is sufficient to capture the dynamics in output, consumption and investment growth. Furthermore, it allows us to directly compare our results to KOP, as they also model all factors as AR(3) processes.

Identification

The model in Equations (2.1)–(2.5) exhibits two well known identification problems present in all DFM, even with constant parameters. First, we cannot separately identify the factor loadings and the factor variances, as it is possible to multiply the terms $\zeta^j_{i,t} D_t$ by any constant, which results in different decompositions of the observed time series $y^j_{i,t}$. This is referred to as the scale problem in dynamic factor models. To overcome this problem, we follow Del Negro and Otrok (2008) and fix the initial volatility h^D_0 of each factor D to a constant. The second problem is that the signs of the factor loadings and the factors are not jointly identified, since the likelihood remains the same if we multiply $\zeta^j_{i,t}$ and D_t by -1. We identify the sign of the global factor by restricting the initial value of the time-varying loading to the global factor for U.S. output growth to be positive, i.e., $\alpha^Y_{US,0} > 0$. Likewise, to identify the signs of the group-specific factors, we restrict the loading for the first country listed in each group (see Appendix A) to be larger than zero for output growth. Finally, country factors are identified by means of positive loadings for the output growth of each country.

Time-varying variance decompositions

We use variance decompositions to measure the relative importance of each factor. Since, by construction, all factors are orthogonal, the variance decompositions can be calculated based on Equation (2.1). For instance, the variance share (VS) of the global factor for GDP growth (Y) in country i is given by

$$VS^{g,Y}_{i,t} = \frac{(\alpha^Y_{i,t})^2 var_t(F^g_t)}{var_t(Y_{i,t})}, \tag{2.6}$$

where $var_t(Y_{i,t}) = (\alpha^Y_{i,t})^2 var_t(F^g_t) + (\beta^Y_{i,t})^2 var_t(F^r_t) + (\delta^Y_{i,t})^2 var_t(F^c_{i,t}) + \sigma^2_{\nu,i,Y}$. The factors' variances can be calculated based on their autoregressive dynamics and time-varying volatilities, i.e.,

$$var_t(F^D_t) = \left(\exp\left(h^D_t\right)\right)^2 \left[I_{3^2} - (\Theta \otimes \Theta)\right]^{-1}. \tag{2.7}$$

with $\otimes$ denoting the Kronecker product, I the identity matrix, and Θ the companion form of the AR coefficients in Equation (2.2). The variance shares in Equation (2.6) are time-varying due to the time-varying factor loadings and the stochastic volatilities. This allows us to analyze changes over time in the relative importance of factors without splitting the sample at an arbitrary point in time. Further, it allows for heterogeneity across countries since the timing of the changes in the variance shares can be different in each country. Changes in the variance share of, e.g., the global or a group factor can be caused by changes in the loadings, changes in the volatilities, or both.

A drawback of the model outlined so far is that it forces the factor loadings and the volatilities, and thus the variance shares, to change over time. The dynamics of the factor loadings and the (log) volatilities are given by random walk processes, which are driven by their innovation variance parameters, $\sigma^2_{\kappa,\zeta,i,j}$ and $\sigma^2_{\eta,D}$. Bayesian estimation techniques typically assume that the prior for a variance parameter follows an inverse Gamma distribution, which has no probability mass at zero. However, the inverse Gamma prior for $\sigma^2_{\kappa,\zeta,i,j}$ has two undesirable properties. First, consider the question whether a loading $\zeta^j_{i,t}$ is time-varying or constant. This implies testing the null hypothesis of $\sigma^2_{\kappa,\zeta,i,j} = 0$ against the alternative $\sigma^2_{\kappa,\zeta,i,j} > 0$ in Equation (2.5), which is a non-regular testing problem since the null hypothesis lies at the boundary of the parameter space for the variance parameter. The same problem arises when testing whether the factors' conditional variances are time-varying or constant. As such, using the conventional inverse Gamma prior does not allow testing the null of constant parameters and variances. Second, as shown by Frühwirth-Schnatter and Wagner (2010), using an inverse Gamma prior can lead to a substantial overestimation of the variances, even in cases where the true innovation variance is positive but small. As a consequence, it can overstate changes in the variance decomposition. To deal with these problems, we rewrite the random walk processes in a non-centered parametrization form, which allows us to estimate the innovation standard errors instead of variances. Additionally, we use a stochastic model specification search to test the hypothesis of constant factor loadings and volatilities.

2.3.2 Stochastic model specification search

The Bayesian stochastic model specification search is based on Frühwirth-Schnatter and Wagner (2010) and extends Bayesian variable selection in standard regression models to state space models. The model selection relies on a non-centered parametrization of the model in which (i) binary stochastic indicators for each of the model components are sampled together with the parameters and (ii) the standard inverse Gamma prior for the variances of innovations to the components is replaced by a Gaussian prior centered at zero for the square root of these variances.

Non-Centered parametrization

The first piece of information on the hypothesis whether a variance parameter is zero or not can be obtained by considering a non-centered parametrization. For the variances of the innovations to the factor loadings, i.e., $\sigma^2_{\kappa,\zeta,i,j}$, this implies rearranging Equation (2.5) to

$$\zeta^j_{i,t} = \zeta^j_{i,0} + \sigma^\zeta_{\kappa,i,j}\widetilde{\zeta}^j_{i,t}, \tag{2.8}$$

$$\text{with} \quad \widetilde{\zeta}^j_{i,t} = \widetilde{\zeta}^j_{i,t-1} + \widetilde{\kappa}^{j,\zeta}_{i,t}, \qquad \widetilde{\zeta}^j_{i,0} = 0, \qquad \widetilde{\kappa}^{j,\zeta}_{i,t} \overset{iid}{\sim} \mathcal{N}(0,1), \tag{2.9}$$

where $\zeta^j_{i,0}$ is the initial value of the level of $\zeta^j_{i,t}$. A crucial aspect of a non-centered parametrization is that it is not identified, i.e., the signs of $\sigma^\zeta_{\kappa,i,j}$ and $\widetilde{\zeta}^j_{i,t}$ can be changed by multiplying both with -1 without changing their product in Equation (2.8). As a result of this non-identification, the likelihood is symmetric around 0 along the $\sigma^\zeta_{\kappa,i,j}$ dimension and therefore multimodal. If the factor loading is time-varying, i.e., $\sigma^2_{\kappa,\zeta,i,j} > 0$, the likelihood function will concentrate around the two modes $-\sigma^\zeta_{\kappa,i,j}$ and $\sigma^\zeta_{\kappa,i,j}$. For $\sigma^2_{\kappa,\zeta,i,j} = 0$, the likelihood function will become unimodal around zero. As such, allowing for a non-identification of $\sigma^\zeta_{\kappa,i,j}$ provides useful information on whether $\sigma^2_{\kappa,\zeta,i,j} > 0$.

Likewise, the non-centered parametrization of the stochastic volatility terms in Equation (2.3) is given by

$$h^D_t = h^D_0 + \sigma_{\eta,D}\widetilde{h}^D_t, \tag{2.10}$$

$$\text{with} \quad \widetilde{h}^D_t = \widetilde{h}^D_{t-1} + \widetilde{\eta}^D_t, \qquad \widetilde{h}^D_0 = 0, \qquad \widetilde{\eta}^D_t \overset{iid}{\sim} \mathcal{N}(0,1), \tag{2.11}$$

where h^D_0 is the initial value of the level of h^D_t.[3]

Parsimonious specification

A second advantage of the non-centered parametrization is that when, e.g., $\sigma^\zeta_{\kappa,i,j} = 0$, the transformed component $\widetilde{\zeta}^j_{i,t}$, in contrast to $\zeta^j_{i,t}$, does not degenerate to a time-invariant factor loading, as this is now represented by $\zeta^j_{i,0}$. As such, the question whether the factor loadings are time-varying or not can be expressed by a variable selection problem in Equation (2.5). Consider the following parsimonious specification,

$$\zeta^j_{i,t} = \zeta^j_{i,0} + \lambda^\zeta_{i,j}\sigma^\zeta_{\kappa,i,j}\widetilde{\zeta}^j_{i,t}, \tag{2.12}$$

[3] As mentioned before, h^D_0 is fixed to be a constant due to an identification restriction.

where $\lambda_{i,j}^{\zeta}$ is a binary indicator which is either 0 or 1. If $\lambda_{i,j}^{\zeta} = 0$, the component $\widetilde{\zeta}_{i,t}^{j}$ drops out of the model, so that $\zeta_{i,0}^{j}$ represents a constant factor loading. If $\lambda_{i,j}^{\zeta} = 1$, then $\widetilde{\zeta}_{i,t}^{j}$ is included in the model, and $\sigma_{\kappa,i,j}^{\zeta}$ is estimated from the data. In this case, $\zeta_{i,0}^{j}$ is the initial value of the time-varying factor loading.

Likewise, the parsimonious non-centered specification of the stochastic volatility terms in Equation (2.3) is given by

$$h_t^D = h_0^D + \rho^D \sigma_{\eta,D} \widetilde{h}_t^D, \tag{2.13}$$

where ρ^D is again a binary indicator. If $\rho^D = 0$, the component $\widetilde{h}_t^D$ drops out of the model, so that $\left(\exp\{h_0^D\}\right)^2$ is the constant variance of ψ_t^D. If $\rho^D = 1$, then $\widetilde{h}_t^D$ is included in the model and $\sigma_{\eta,D}$ is estimated from the data. In this case, $\left(\exp\{h_0^D\}\right)^2$ is the initial value of the time-varying variance of ψ_t^D. We collect the binary indicators into the vector $\mathcal{M} = (\lambda_{i,j}^{\zeta}, \rho^D)$.

Gaussian prior centered at zero

It is well-known that when using an inverse Gamma prior distribution for the variance parameters, the choice of the shape and scale hyperparameters that define this distribution have a strong influence on the posterior when the true value of the variance is close to zero. More specifically, as the inverse Gamma distribution does not have any probability mass at zero, using it as a prior distribution tends to push the posterior density away from zero. This is of particular importance when estimating the variances of the innovations to the time-varying factor loadings and to the stochastic volatilities, because for these components we want to decide whether they are relevant or not. A further important advantage of the non-centered parametrization is therefore that it allows us to replace the standard inverse Gamma prior on a variance parameter σ^2 by a Gaussian prior centered at zero on σ. Centering the prior distribution at zero makes sense, since for both $\sigma^2 = 0$ and $\sigma^2 > 0$, σ is symmetric around zero. Frühwirth-Schnatter and Wagner (2010) show that, compared to using an inverse Gamma prior for σ^2, the posterior density of σ is much less sensitive to the hyperparameters of the Gaussian distribution and is not pushed away from zero when $\sigma^2 = 0$.

As such, we choose a Gaussian prior distribution centered at zero for σ_κ and σ_η, which are the standard deviations of the innovations to the time-varying factor loadings and to the stochastic volatilities. Specifically, we choose $\mathcal{N}(0, 5^2)$ for both σ_κ and σ_η. Similarly, a flat prior is used for the time-invariant components of the factor loadings, i.e., $\zeta_{i,0}^{j} \sim \mathcal{N}(0.5, 10^2)$. For each binary indicator in $\mathcal{M}$, we choose a uniform prior distribution such that $p_0 = 0.5$ is the prior probability for each time-varying component to be included in the model.

For the variance parameters of the innovations to the idiosyncratic factors σ^2_ν, we use the standard inverse Gamma prior $\mathcal{IG}(c_0, C_0)$, where c_0 and C_0 are the shape and scale parameters, respectively. The calculation of c_0 and C_0 is explained in greater detail in 2.B. Finally, the priors for the autoregressive coefficients are assumed to be Gaussian with mean zero and unit variance.

2.3.3 MCMC algorithm

The inclusion of time-varying factor loadings $\zeta^j_{i,t}$, stochastic volatilities h^D_t, and the use of a stochastic model specification search, confronts us with a highly non-linear estimation problem. We estimate the model using a Gibbs sampler, which is a Markov chain Monte Carlo (MCMC) method to simulate draws from the intractable joint and marginal posterior distributions of the unknown parameters and the unobserved factors and states using only tractable conditional distributions. Intuitively, this amounts to reducing the complex non-linear model to a sequence of blocks for subsets of the parameters and states that are tractable, conditional on the other blocks in the sequence.

As such, given initial values for the factors and parameters, we can summarize the algorithm by the following steps:[4]

1. Sample each factor conditional on all other factors, factor loadings, stochastic volatility processes, and all constant parameters.

2. Sample the binary indicators, the initial values in the non-centered form of the factor loadings, and the unrestricted standard deviations conditional on all unobserved states.

3. Sample the time-varying factor loadings for global, group, and country factors successively while conditioning on the respective other loadings, all factors, stochastic volatility terms, the binary indicators, and constant parameters.

4. Sample the mixture indicators and the stochastic volatility processes.

5. Perform a random sign switch for the unrestricted standard deviations and the time-varying part of the factor loadings in the non-centered form.

[4]Our MCMC scheme follows Del Negro and Otrok (2008) and Frühwirth-Schnatter and Wagner (2010) for the model selection part.

Sampling from these blocks is iterated $J = 20,000$ times and, after a sufficiently long burn-in period of $B = 10,000$, the sequence of draws $(B+1, ..., J)$ approximates a sample from the desired posterior distribution. Details of the exact implementation of the Gibbs sampler are given in 2.B. In each iteration of the sampling process, we calculate the (potentially time-varying) variance decomposition for output, consumption, and investment growth in each country.

2.4 Estimation results

2.4.1 Data

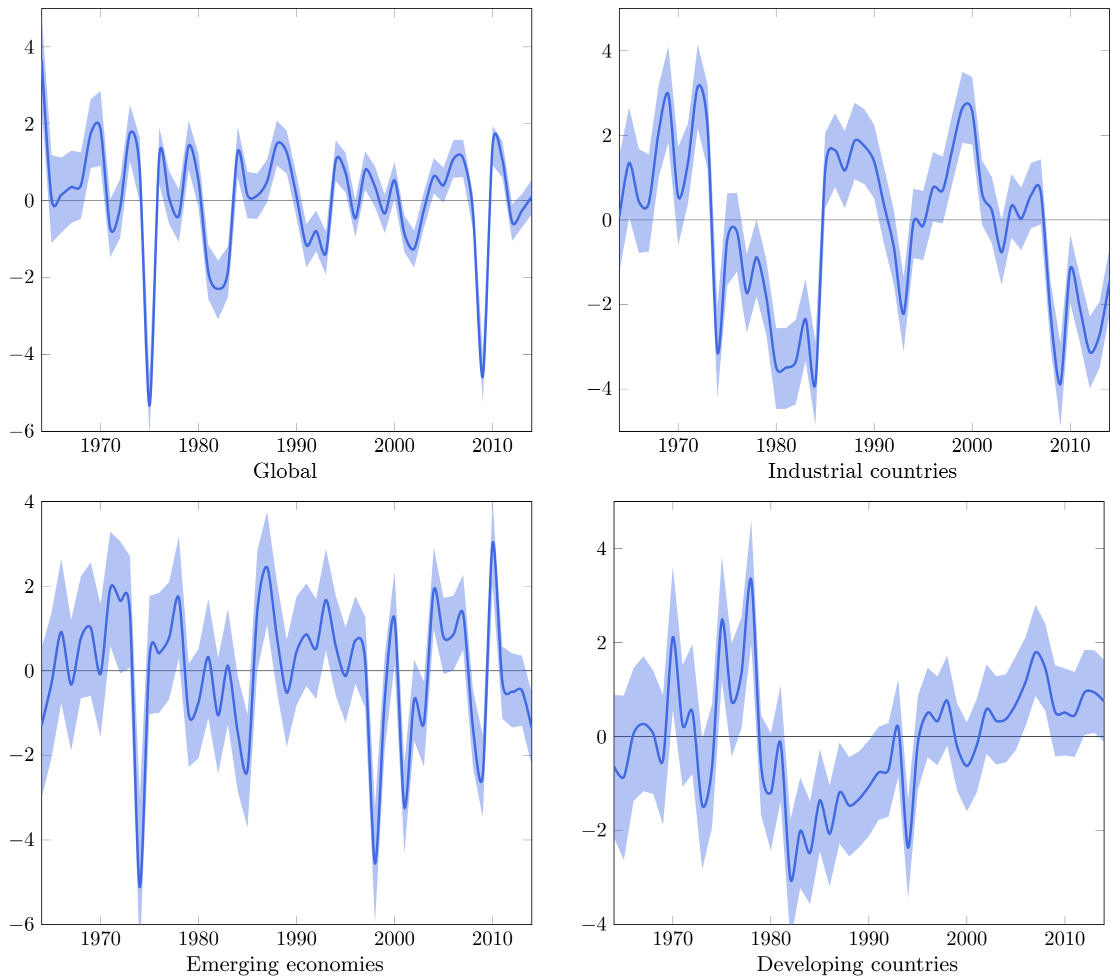
The data set is drawn from *Penn World Tables 9.0* and includes annual observations from 1960–2014 for 106 countries. We use National Accounts data for real GDP, real private consumption, and real investment at constant prices in local currencies, and compute the demeaned growth rate of each series. Following KOP, countries are grouped according to their level of development as classified by the IMF. Each country belongs either to the group of industrial countries (23 ICs), emerging market economies (24 EMs), or developing countries (59 DCs). 2.A provides a list of the countries and their group affiliation.

2.4.2 Estimated global and group-specific factors

This section presents the estimates of the global and group-specific factors obtained from estimating the model with endogenously determined time variation. Figure 2.1 plots the posterior median of the global and the group-specific factors along with the 90% probability coverage intervals. The global business cycle, displayed in the upper left panel, captures some major economic events of the last decades. The first global recession is found in the mid-1970s, the time of the first oil crisis induced by a sharp increase in the price of oil in 1973/74. In 1979, the second surge in oil prices caused a global downturn in macroeconomic growth rates in the early 1980s. To a lesser extend, the factor also picks up the 1990s recession after the collapse of the U.S. stock market, and a small drop in global activity after the burst of the dot-com bubble. The recent Great Recession was clearly a global recession as indicated by the sharp decline of the global factor in 2008/09. The factor moreover reflects the large volatility during the 1970s followed by the Great Moderation period from the mid-1980s.

The upper right panel in Figure 2.1 shows the evolution of the industrial countries (ICs) factor. Note that the adjustment dynamics captured by the ICs factor in

Figure 2.1: Global and group-specific factors



Note: Reported are the median, the 5th and the 95th percentile of the posterior distribution of the global and the group-specific factors.

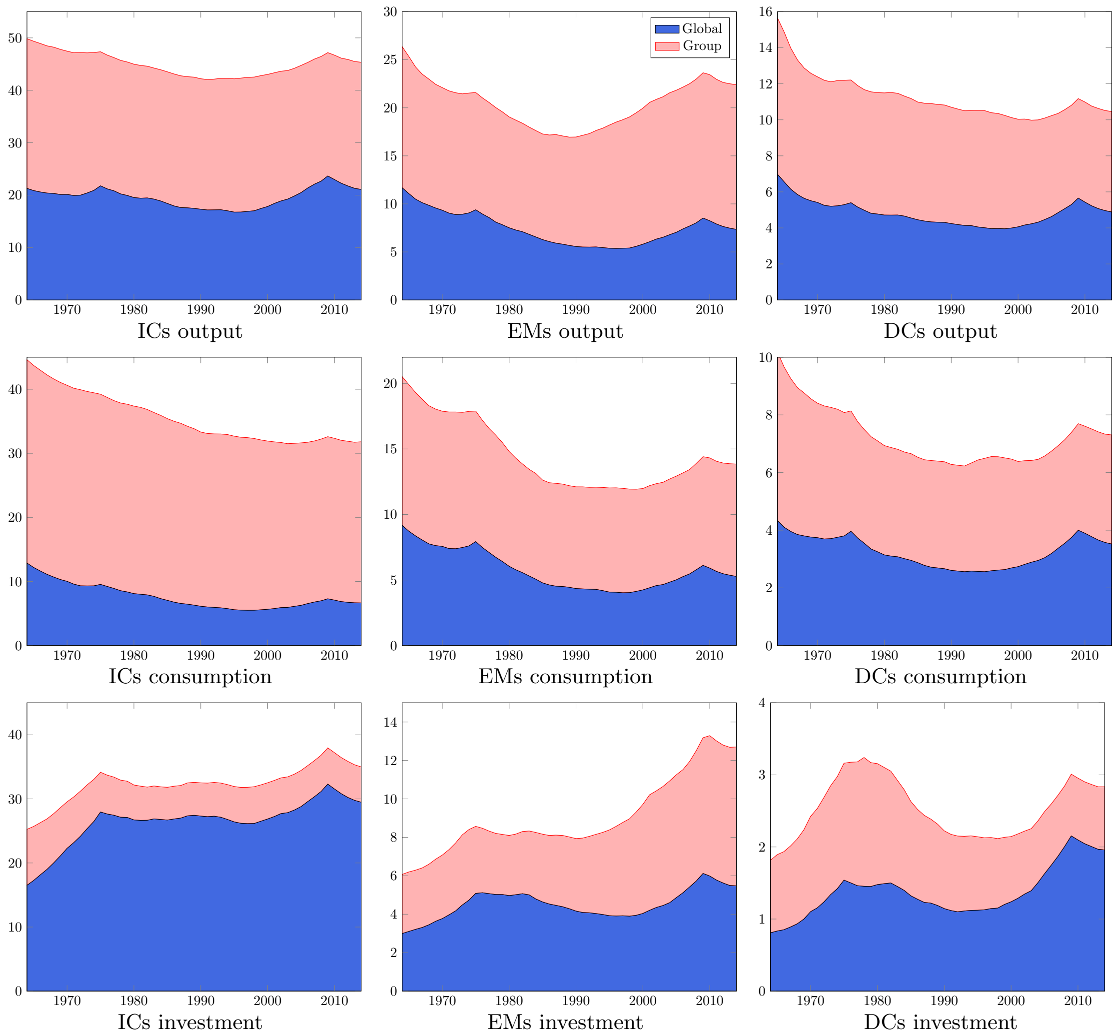
response to the oil price shocks, the crash in U.S. stock market, and the financial crisis are different as compared to global factor. In particular, the early 1980s recession is deeper and longer lasting than the perturbation of the global factor. A similar characteristic can be seen in the Great Recession, where the ICs factor shows a very slow recovery and exhibits the pattern of a double dip recession as experienced particularly in industrialized countries in Europe and North America. The estimates for the emerging market (EMs) factor and the developing countries (DCs) factor capture relevant economic events specific to these groups. For example, the EMs factor shown in the lower left panel picks up the Asian financial crisis in the late 1990s and the quick recovery from the recent financial crisis.

The innovations to the global and the three group-specific factors are uncorrelated by construction, i.e., variances to these cycles are mutually orthogonal. However, specific events such as the recession in the mid-1970s and the Great Recession are visible in the global as well as in the group factors for the ICs and the EMs, but the factors remain distinct. In particular, the timing, depth, and length of the recessions are different across country groups and also to the dynamics that are common for all countries. When calculating the correlation of the estimated factors, we find only a very moderate correlation between the factors.

2.4.3 Time-varying variance decompositions

This section examines the evolution of the time-varying variance decompositions allowing for endogenously determined time variation as described in Section 2.3. Figure 2.2 shows the group-specific average of the time-varying variance decomposi-

Figure 2.2: Time-varying variance decompositions in percent. Group-specific averages.



tions for the global and group-specific factors for all country groups and variables.[5] Our variance decomposition reveals several regularities similar to previous results in the literature. First, we find the global factor, to be most important for the industrialized countries and least important for the developing countries. International factors, as measured by the joint contribution of the global and the group-specific factors, explain a substantial fraction of the vast majority of variances of all three variables in industrial and emerging market economies, while these factors are of less importance for developing countries. For example, in the group of industrialized countries, the global and group factors together explain roughly 50% of the output growth variances. In emerging market economies, international factors still have a share of more than 20% of the output variances for most of the sample period. The average variance share of international factors with respect to output growth in developing countries is between 10% and 12%. International business cycles also play a more important role for consumption and investment growth in industrial countries than in emerging market economies and developing countries.

Second, the importance of the global factor for output fluctuations has decreased over time for all groups of countries, but this decline is particularly pronounced for emerging market economies. In addition, we find that the importance of the group-specific factor has increased over time for the group of emerging market economies.

2.4.4 Testing for time variation in the factor variances and factor loadings

Given our results for the variance decomposition, we now address the key question of this paper: what has caused the decline in the importance of the global factor? As discussed in the introduction, we argue that it is important to distinguish between changes in the factor loadings and changes in the factor variances as both may drive changes in the variance decomposition. Economically, changes in the countries' factor loadings imply changes in the sensitivity to the respective factors, for example induced by changes in domestic policies or trade and financial integration. Changes in the factor variances are, however, the result of changes in the size of shocks at the global, group-specific or national level.

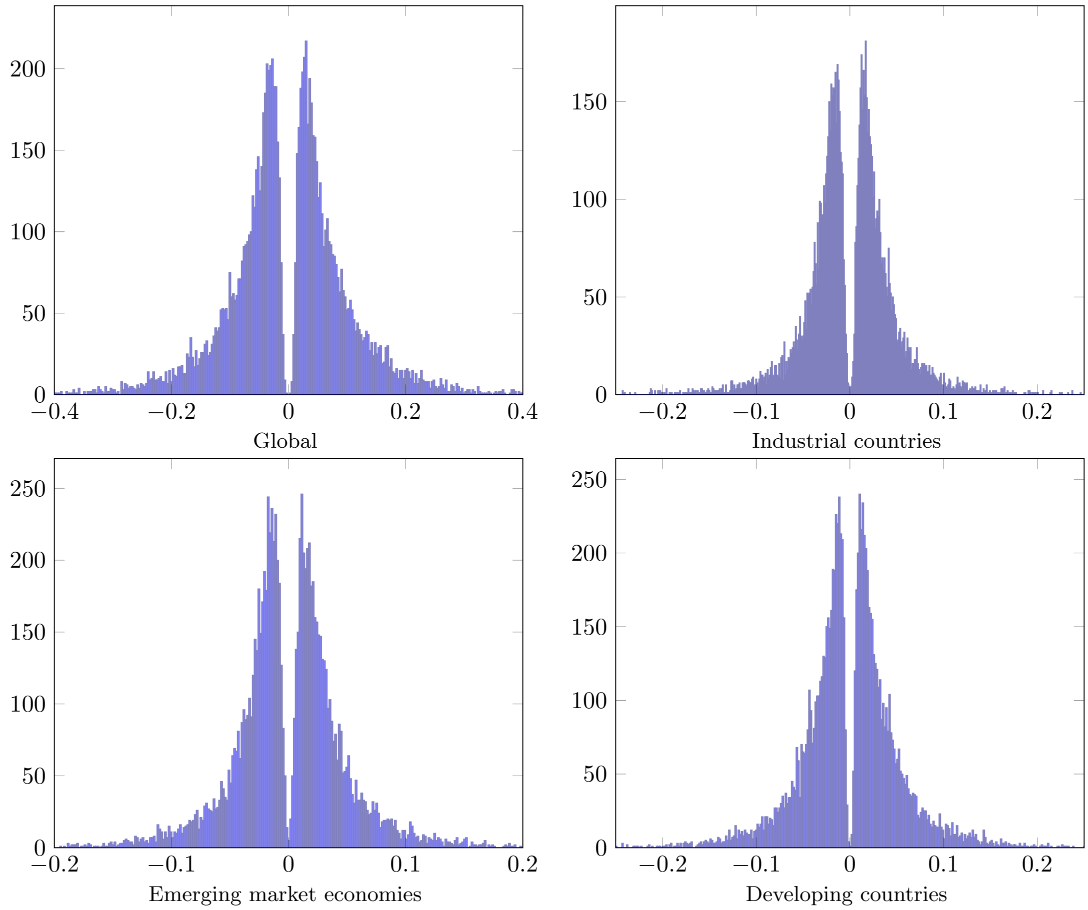
By means of the Bayesian stochastic model selection described in Section **??**, we are able to test for the time variation in the factor loadings and the factor variances and hence evaluate whether changes in the variance decomposition are driven by loadings, variances, or both.

[5] Country-specific and idiosyncratic cycles are available upon request.

Factor variances

This section presents the results of the stochastic model selection. Preliminary evidence for potential time variation in the parameters is obtained by estimating the model with all binary indicators set equal to one. The resulting posterior distributions for the standard deviations σ_η in the volatility equation Equation (2.10) of the international factors are shown in Figure 2.3. The posterior distributions of

Figure 2.3: Standard deviations of the stochastic volatilities of the global and group-specific factors



Note: Reported are the posterior distributions of the standard deviations for the non-centered stochastic volatilities in the unrestricted model, i.e., σ_η (all binary indicators set to 1).

these standard deviations show clear bi-modality with very little probability mass at zero.[6] This is taken as preliminary evidence of time variation in the variances of the global and the group-specific factors, i.e., it indicates that $\sigma_\eta^2 > 0$.

[6]The volatilities of the country-specific factors present a more mixed picture. Graphs of the posterior distributions of the country factors are not shown, due to space constraints, but are available upon request.

A more formal test for time variation in the volatilities is provided by estimating the binary indicators in Equation (2.13) together with the other parameters in the model. Table 2.1 displays the posterior inclusion probabilities for the time-varying component in the non-centered stochastic volatility terms of each factor. The inclusion probabilities are calculated as the average selection frequencies of all retained iterations of the Gibbs sampler. From Table 2.1, we conclude that the model clearly supports time-varying variances of the global and all group-specific factors, as the corresponding inclusion probabilities all are equal to one.

Table 2.1: Posterior inclusion probabilities for the binary indicators for stochastic volatilities

Common factors				**Country factors**		
Global	IC	EM	DC	IC	EM	DC
1	1	1	1	0.88	0.52	0.44

Note: The reported probabilities are calculated as the average of the binary indicators over the $10,000$ iterations of the Gibbs sampler.

Concerning the time variation in the variances of the country-specific factors, the results are less clear and differ considerably for different country groups. The average inclusion probabilities across countries in each group suggest that the volatilities of the country-specific factors are time-varying in most industrial countries. The cross-country average of the inclusion probability is close to 0.5 for the remaining two groups. In particular, 21 out of the 23 industrial countries in the sample exhibit an inclusion probability above 0.5, indeed, 19 countries even have an inclusion probability larger than 0.9. In the group of emerging market economies, the fraction of countries with an inclusion probability higher than 0.5 is still 50%, but only 26 out of 59 developing countries have an inclusion probability for the binary indicator of above 0.5.

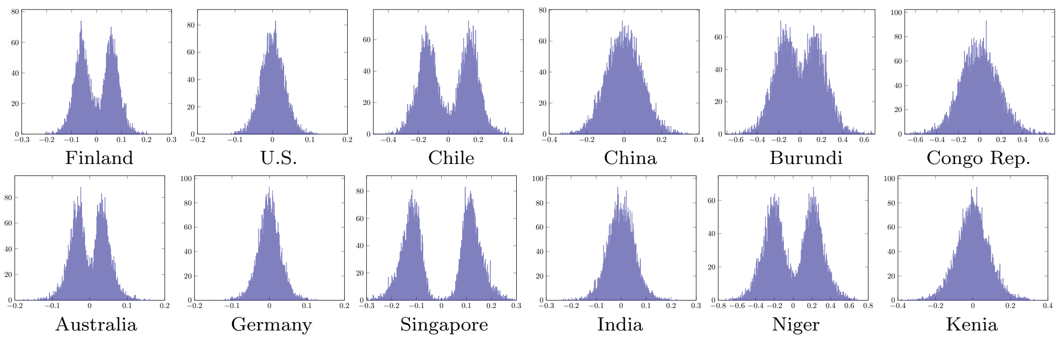
Factor loadings

For the factor loadings, we proceed in the same manner as for the factor variances and first of all look at the posterior distributions of the standard deviations from the non-centered factor loadings in Equation (2.8). We find considerable heterogeneity across countries shown in Figure 2.4 for the factor loadings of output growth in some selected countries from all groups.[7]

[7]Because of the available space, we cannot shows the results for all factor loadings. These are available upon request.

The upper panel shows the results for the loadings of output to the global factor. The distributions are bi-modal for Finland, Chile, and Burundi, which suggests that the factor loadings of output to the global factor are time-varying for these countries, i.e., $\sigma_\kappa^2 > 0$, whereas the uni-modal distributions of the standard deviations for the United States, China, and Congo Republic suggest that these factor loadings did not change over time, i.e., $\sigma_\kappa^2 = 0$. Similarly, the lower panel of Figure 2.4 shows evidence for time-varying factor loadings of output to the group-specific factors for Australia, Singapore, and Niger, but constant factor loadings for the group factors for Germany, India, and Kenya.

Figure 2.4: Standard deviations of the factor loadings of output to the global (upper panel) and group-specific (lower panel) factors

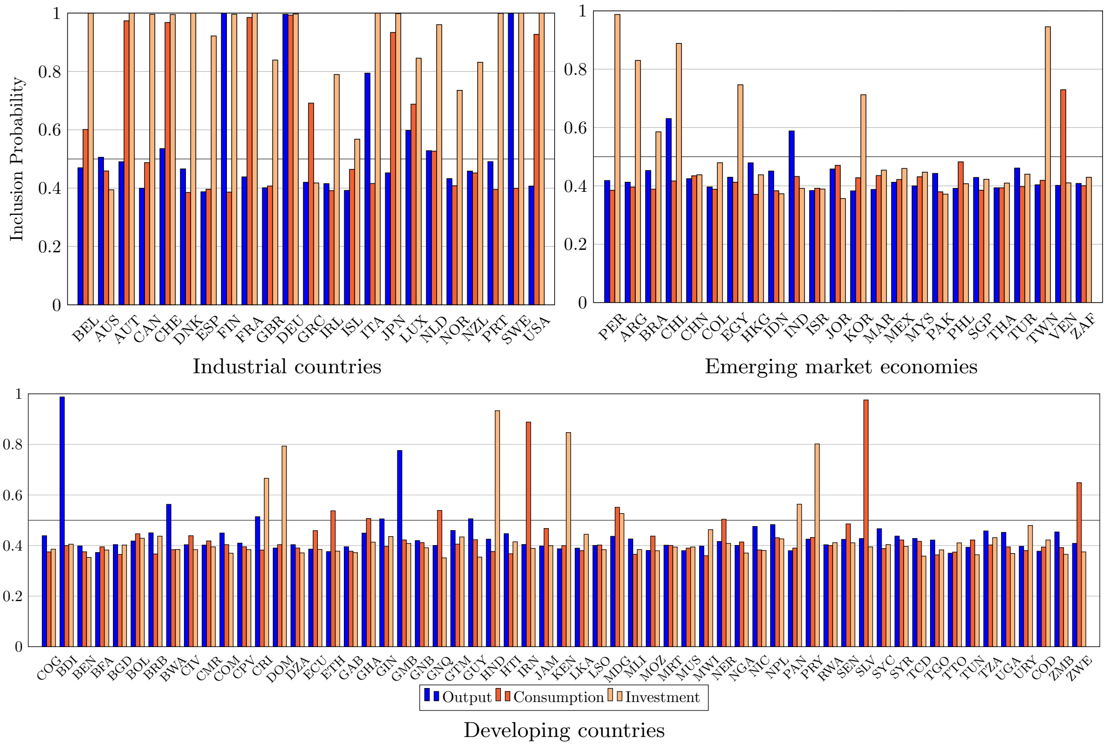


Note: Reported are the posterior distributions of the standard deviations for the non-centered factor loadings of output to the global and group-specific factors for selected countries in each group in the unrestricted model, i.e., σ_κ (all binary indicators set to 1).

Turning to the formal tests for time variation in the factor loadings, Figure 2.5 displays the inclusion probabilities for the time-varying component in the loadings to the global factor. In the group of industrialized countries, only three out of 23 countries show very strong evidence of time-varying loadings of output to the global factor, i.e., an inclusion probability higher than 95%: namely, Finland, Germany, and Sweden. For Italy, the model detects time variation in about 80% of the retained Gibbs iterations. Consumption growth is found to have time-varying loadings to the global factor in nine industrialized countries. For investment growth, we find evidence that the majority of industrialized countries exhibit time variation in the loadings to the global factor, with most countries' inclusion probability being above 90%. In the group of emerging markets, evidence for time variation in the loadings to the global factor is even weaker. For output and consumption, almost all emerging market economies in our sample have a probability below 50% for including the time-varying part in their loadings to the global business cycle. For investment growth, there is evidence for time variation in seven out of 24 countries.

The picture remains the same in the group of developing countries, i.e., the model selection procedure rejects time variation in the loadings to the global factor for the vast majority of countries.

Figure 2.5: Probability for time-varying loadings to the global factor

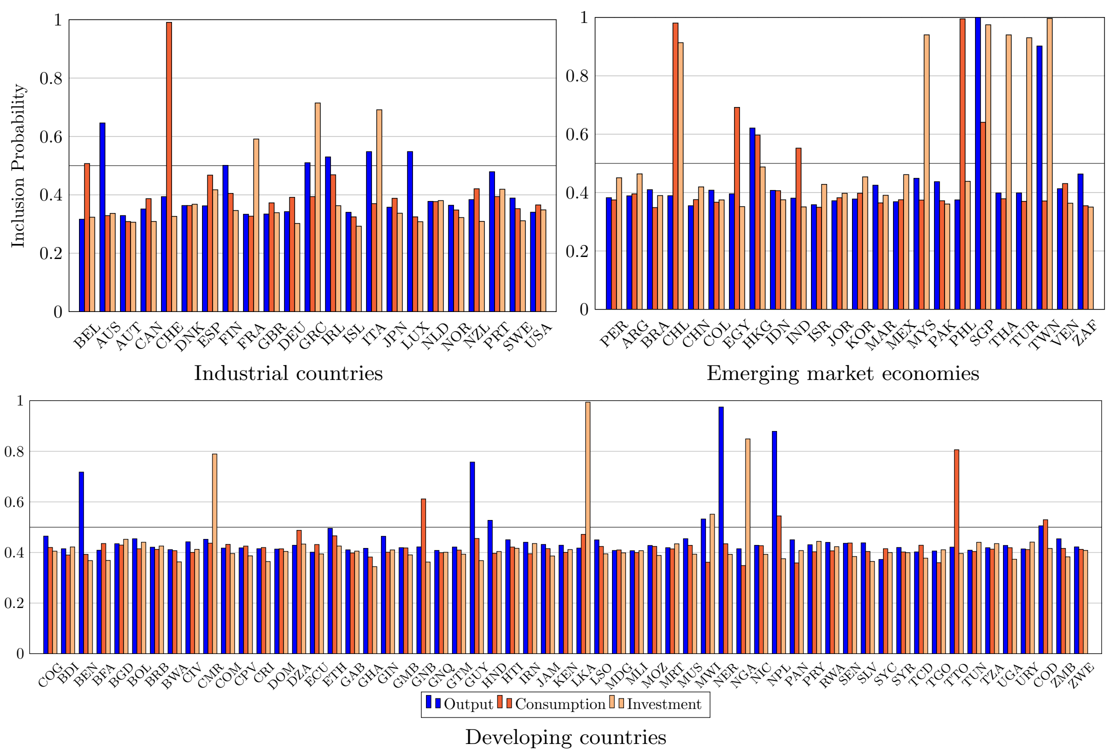


Note: Reported are the inclusion probabilities of the time-varying part in the factor loadings computed as the average selection frequencies over the retained iterations of the Gibbs sampler.

Evidence for time-varying factor loadings to the group-specific factors is even weaker, as shown in Figure 2.6. In most countries across all country groups, the inclusion probability of the binary indicator is well below 40% for output, consumption and investment.[8] Our threshold for time variation is an inclusion probability of 50%, indicated by the thick horizontal lines in Figures 2.5 and 2.6, i.e. we treat all factor loadings and variances as random walks, if their inclusion probability exceeds this threshold, while loadings and variances are treated as constant parameters, if their inclusion probability is equal to or below 50%. Overall, it is striking that many of the estimated inclusion probabilities are below 50% but are still around 40%. As a robustness check, we changed the threshold for time variation to 10% and hence treated all loadings with an inclusion probability above 10% as time-varying. The resulting variance decomposition, i.e., the shares attributed to the global, group, country and idiosyncratic factors, changes only very little. Due to the available

[8]Inclusion probabilities for the country factors are not shown but are available upon request.

Figure 2.6: Probability for time-varying loadings to the group-specific factors



Note: Reported are the inclusion probabilities of the time-varying part in the factor loadings computed as the average selection frequencies over the retained iterations of the Gibbs sampler.

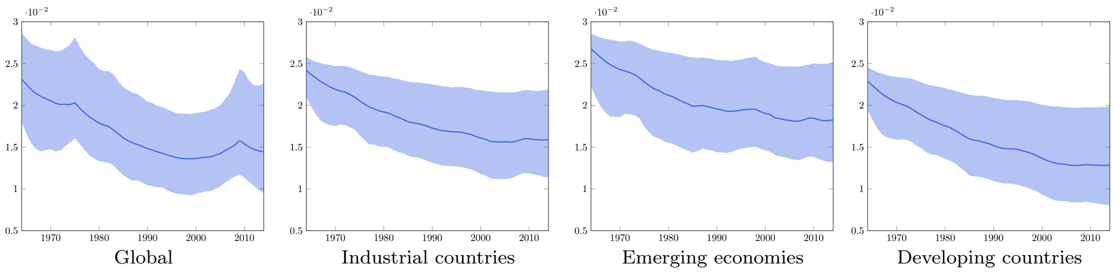
space, these results are not shown here, but are available upon request. The reason for the minor changes in the variance decomposition is that the loadings for which the inclusion probability is below 50% hardly change in magnitude, even when we restrict the binary indicators to one, i.e., force the loading to vary over time. In sum, we find strong evidence for time-varying volatilities in the global and all three group factors. In contrast, the model selection procedure rejects time variation in the loadings for the majority of countries. With respect to the main question addressed in this paper, we find that the decrease in the relative importance of the global business cycle, in particular observed in emerging market economies, is predominantly driven by a decline in the size of global shocks rather than a decrease in the countries' sensitivity to these shocks. These results thus contribute to resolving the puzzle of a decline in global business cycle co-movement during a period of increasing trade and financial globalization. Similar findings are reported by Karadimitropoulou and León-Ledesma (2013) who investigate international business cycle synchronization at the sectoral level. They also report a substantial fall in the variance of the global shock that reduces the relative contribution of international factors to sectoral business cycles.

2.4.5 Estimated factor loadings and volatilities

Given the results from the Bayesian stochastic model selection, we now look at the estimated stochastic volatility terms of the international factors and the factor loadings of some countries for which we find time variation in the sensitivity. The estimated time-varying standard deviations for the common factors are shown in Figure 2.7.

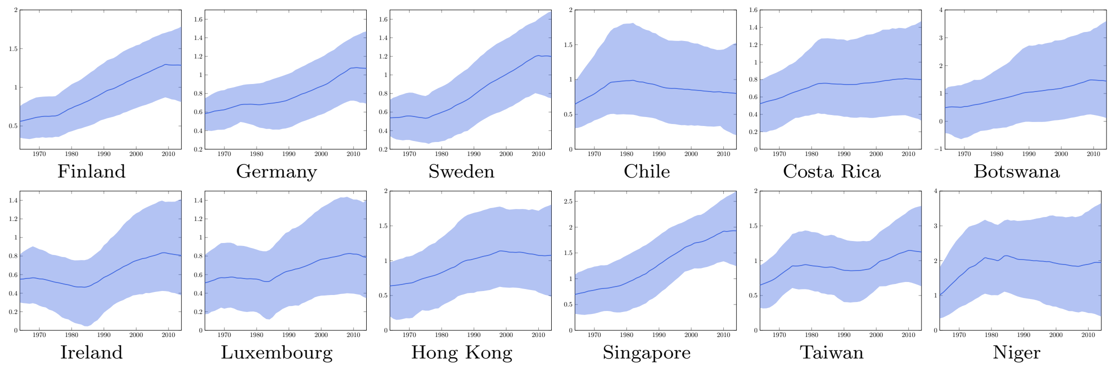
The volatility of all common shocks has decreased over time, whereas periods of economic crisis are associated with a surge in volatility. For example, the volatility of the global factor increased during the first oil price shock and more recently during the Great Recession in the late 2000s. Similarly, the volatilities of the emerging market economies and the developing countries factor rose slightly during the Asian crisis in the late 1990s.

Figure 2.7: Stochastic volatilities of the global and group-specific factors



Note: Reported are the median, the 5th and the 95th percentile of the posterior distribution of the time-varying standard deviations of the global and the group-specific factors.

Figure 2.8: Factor loadings of output to the global and group-specific factors



Note: Reported are the median, the 5th, and the 95th percentile of the posterior distribution of the time-varying factor loadings for output to the global factor (upper panel) and the group-specific factors (lower panel) for randomly selected countries. Time-varying factor loadings are estimated given that posterior inclusion probabilities are above 0.5.

Figure 2.8 shows the factor loadings for some randomly selected countries of all groups where the model selection procedure finds time variation. The upper panel displays the loadings of output to the global factor, which are clearly increasing over time for Finland, Germany, and Sweden and to a lesser extend for Costa Rica and Botswana. The loading for Chile increased until about 1980 but has been slightly decreasing since then. The lower panel of Figure 2.8 shows the loadings of output to the respective group factor. These increase moderately for Ireland, Luxembourg, Hong Kong, and Taiwan, but rather strongly for Singapore. For Niger, the loading of output to its group factor increased substantially until about 1980, but has not changed much since then. Overall, time variation in the factor loadings of output to the global and group factors is detected in 32 countries, of which only eight countries exhibit decreasing loadings over time. Thus, output growth has become more sensitive to the global and the group factor in most of the countries which reflects the increase in economic linkages across countries.[9]

2.5 Conclusions

During recent years, the empirical literature that reports a decrease in the importance of global shocks to national business cycles has grown (Kose, Otrok, and Whiteman, 2003; Kose, Otrok, and Prasad, 2012; Mumtaz, Simonelli, and Surico, 2011). This result is, however, at odds with the fact that countries have become more interlinked, as world trade and international capital flows have increased tremendously. At the same time, shocks dedicated to smaller sub-groups of countries, e.g., industrialized countries, emerging market economies, and developing countries, have been found to be more important in explaining national business cycles.

In this paper, we have focused on whether the changes in the relative importance of global and group-specific shocks have been driven by changes in the sensitivity to common shocks or by changes in the size of the shocks. To this end, a hierarchical DFM with time-varying loadings and stochastic volatilities for output, consumption, and investment growth, for 106 countries 1961–2014 has been estimated. A Bayesian model selection procedure has been used to explicitly test for time variation in the factor loadings and volatilities. As such, the model allows for time variation in the parameters but does not force the parameters to change.

We have found strong evidence of changes in the size of the international shocks, both at the global and the group-specific level. International shocks have become smaller over the past decades. In contrast, the sensitivity of countries to international shocks has been constant for the majority of countries. As a consequence, the

[9] Due to the large number of individual country results, we only report randomly selected results. All remaining results not reported here are available upon request.

finding of a decline in the importance of the global business cycles is driven by the global shocks' having been smaller, and not by a reduction in countries' integration in the global economy.

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Appendix

2.A List of countries

Table 2.A.1: List of Countries

Country Groups			
Industrial Countries	Emerging Economies	Developing Countries	
Belgium	Peru	Congo, Rep.	Sri Lanka
Australia	Argentina	Burundi	Lesotho
Austria	Brazil	Benin	Madagascar
Canada	Chile	Burkina Faso	Mali
Switzerland	China	Bangladesh	Mozambique
Denmark	Colombia	Bolivia	Mauritania
Spain	Egypt	Barbados	Mauritius
Finland	Hong Kong, China	Botswana	Malawi
France	Indonesia	Cote d'Ivoire	Niger
United Kingdom	India	Cameroon	Nigeria
Germany	Israel	Comoros	Nicaragua
Greece	Jordan	Cape Verde	Nepal
Ireland	Korea, Rep.	Costa Rica	Panama
Iceland	Morocco	Dominican Republic	Paraguay
Italy	Mexico	Algeria	Rwanda
Japan	Malaysia	Ecuador	Senegal
Luxembourg	Pakistan	Ethiopia	El Salvador
Netherlands	Philippines	Gabon	Seychelles
Norway	Singapore	Ghana	Syrian Arab Republic
New Zealand	Thailand	Guinea	Chad
Portugal	Turkey	Gambia, The	Togo
Sweden	Taiwan	Guinea-Bissau	Trinidad and Tobago
United States	Venezuela, RB	Equatorial Guinea	Tunisia
	South Africa	Guatemala	Tanzania
		Guyana	Uganda
		Honduras	Uruguay
		Haiti	Congo, Dem. Rep.
		Iran, Islamic Rep.	Zambia
		Jamaica	Zimbabwe
		Kenya	

2.B Gibbs sampling algorithm

In this appendix we provide details on the Gibbs sampling algorithm used in subsection 2.3.3 to jointly sample the binary indicators $\mathcal{M}$, the hyperparameters Ψ, the common and country-specific factors F, the time-varying factor loadings α, β, and δ, the mixture indicators ι, and the stochastic volatilities h. The structure of our Gibbs sampling approach is based on Frühwirth-Schnatter and Wagner (2010).

Block 1: Filtering and sampling the state vectors F

In this block we use the general forward-filtering and backward-sampling approach for all common and country-specific factors F based on a state space model of the general form

$$y_t^* = Z_t s_t + e_t, \quad e_t \overset{iid}{\sim} \mathcal{N}(0, H), \tag{2.14}$$

$$s_t = T s_{t-1} + K_t v_t, \quad v_t \overset{iid}{\sim} \mathcal{N}(0, Q), \quad s_1 \overset{iid}{\sim} \mathcal{N}(a_1, A_1), \tag{2.15}$$

where y_t^* is a vector of observations and s_t an unobserved state vector. The matrices Z_t, T, K_t, H, Q and the expected value a_1 and variance A_1 of the initial state vector s_1 are assumed to be known (conditioned upon) and the error terms e_t and ν_t are assumed to be serially uncorrelated and independent of each other at all points in time. As Equations (2.14)–(2.15) constitute a linear Gaussian state space model, the unknown state variables in s_t can be filtered using the Kalman filter. In particular, we filter and draw the unobserved common and country-specific factors $D = \left\{F_t^g, F_t^{IC}, F_t^{EM}, F_t^{DC}, F_{i,t}^c\right\}$ conditionally on the time-varying factor loadings $\zeta = \left\{\alpha_{i,t}^j, \beta_{i,t}^j, \delta_{i,t}^j\right\}$, the stochastic volatility terms $h = \left\{h_t^D\right\}$ and the hyperparameters collected in Ψ.

Block 1(a): Sampling the global factor F_t^g

In this step of the Gibbs sampler, we filter and sample the common global factor F_t^g conditioning on the group-specific factors $F_t^r = (F_t^{IC}, F_t^{EM}, F_t^{DC})$, the country-specific factors $F_{i,t}^c$, the time-varying factor loadings ζ, the stochastic volatilities h, and the hyperparameters. As we treat the country- and variable-specific idiosyncratic disturbances $\varepsilon_{i,t}^j$ as autocorrelated processes of order three, we transform the endogenous variables $Y_{i,t}$, $C_{i,t}$ and $I_{i,t}$ and express these variables as functions of the global factor F_t^g. The state space representation for the conditional model in this block is given by

$$\underbrace{\begin{bmatrix} Y_{1,t}^* \\ \vdots \\ Y_{N,t}^* \\ C_{1,t}^* \\ \vdots \\ C_{N,t}^* \\ I_{1,t}^* \\ \vdots \\ I_{N,t}^* \end{bmatrix}}_{y_t^*} = \underbrace{\begin{bmatrix} \alpha_{1,t}^Y & -\alpha_{1,t}^Y\phi_{1,1}^Y & -\alpha_{1,t}^Y\phi_{2,1}^Y & -\alpha_{1,t}^Y\phi_{3,1}^Y \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{N,t}^Y & -\alpha_{N,t}^Y\phi_{1,N}^Y & -\alpha_{N,t}^Y\phi_{2,N}^Y & -\alpha_{N,t}^Y\phi_{3,N}^Y \\ \alpha_{1,t}^C & -\alpha_{1,t}^C\phi_{1,1}^C & -\alpha_{1,t}^C\phi_{2,1}^C & -\alpha_{1,t}^C\phi_{3,1}^C \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{N,t}^C & -\alpha_{N,t}^C\phi_{1,N}^C & -\alpha_{N,t}^C\phi_{2,N}^C & -\alpha_{N,t}^C\phi_{3,N}^C \\ \alpha_{1,t}^I & -\alpha_{1,t}^I\phi_{1,1}^I & -\alpha_{1,t}^I\phi_{2,1}^I & -\alpha_{1,t}^I\phi_{3,1}^I \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{N,t}^I & -\alpha_{N,t}^I\phi_{1,N}^I & -\alpha_{N,t}^I\phi_{2,N}^I & -\alpha_{N,t}^I\phi_{3,N}^I \end{bmatrix}}_{Z_t} \underbrace{\begin{bmatrix} F_t^g \\ F_{t-1}^g \\ F_{t-2}^g \\ F_{t-3}^g \end{bmatrix}}_{s_t} + \underbrace{\begin{bmatrix} \nu_{1,t}^Y \\ \vdots \\ \nu_{N,t}^Y \\ \nu_{1,t}^C \\ \vdots \\ \nu_{N,t}^C \\ \nu_{1,t}^I \\ \vdots \\ \nu_{N,t}^I \end{bmatrix}}_{e_t}, \tag{2.16}$$

$$\underbrace{\begin{bmatrix} F_t^g \\ F_{t-1}^g \\ F_{t-2}^g \\ F_{t-3}^g \end{bmatrix}}_{s_t} = \underbrace{\begin{bmatrix} \theta_1^g & \theta_2^g & \theta_3^g & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} F_{t-1}^g \\ F_{t-2}^g \\ F_{t-3}^g \\ F_{t-4}^g \end{bmatrix}}_{s_{t-1}} + \underbrace{\begin{bmatrix} \exp(h_t^g) \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{K_t} \psi_t^g, \tag{2.17}$$

where $H = \text{diag}(\sigma_{\nu,i,Y}^2, \sigma_{\nu,i,C}^2, \sigma_{\nu,i,I}^2)$, $Q = 1$, and $Y_{i,t}^* = \phi_i^Y(L)\left[Y_{i,t} - \beta_{i,t}^Y F_t^r - \delta_{i,t}^Y F_{i,t}^c\right]$, $C_{i,t}^* = \phi_i^C(L)\left[C_{i,t} - \beta_{i,t}^C F_t^r - \delta_{i,t}^C F_{i,t}^c\right]$ and $I_{i,t}^* = \phi_i^I(L)\left[I_{i,t} - \beta_{i,t}^I F_t^r - \delta_{i,t}^I F_{i,t}^c\right]$ with $\phi_i^j(L) = (1 - \phi_{1,i}^j L - \phi_{2,i}^j L^2 - \phi_{3,i}^j L^3)$ and $r = IC, EM, DC$ depending on the group affiliation of country i. Pre-multiplying each term in the observation equation by the lag polynomial $\phi_i^j(L)$ is necessary to take into account the autoregressive structure of the variable specific idiosyncratic error terms $\varepsilon_{i,t}^j$ in Equation (2.4).

The unobserved state vector s_t is extracted using standard forward filtering and backward sampling. Instead of taking the entire observational vector y_t^* as the item for analysis, we follow the univariate treatment of the multivariate series approach of Durbin and Koopman (2012), in which each of the elements $y_{i,t}^*$ in y_t^* is brought into the analysis one at a time. This not only offers significant computational gains, it also avoids the risk that the prediction error variance matrix becomes nonsingular during the Kalman filter procedure.

Block 1(b): Sampling the group-specific factors F_t^r

In order to filter and sample the group-specific factors $F_t^r = (F_t^{IC}, F_t^{EM}, F_t^{DC})$, we split our entire sample into smaller sub-samples, each including the countries belonging to either the group of industrial countries (IC), the emerging market economies (EM), or the developing countries (DC). More specifically, the group of industrial countries includes those indexed from $i = 1, \ldots, 23$, the group of emerging market economies includes countries indexed from $i = 24, \ldots, 47$, and the group of developing countries includes countries indexed from $i = 48, \ldots, 106$. For these smaller subgroups, we sample each group-specific factor separately, similarly to the procedure outlined in Block 1(a), conditioning on the global factor F_t^g, the country-specific factors $F_{i,t}^c$, the time-varying factor loadings α, β, and δ, the stochastic volatilities h, and the corresponding hyperparameters in Ψ. We transform the endogenous variables and express them as functions of the group-specific factor F_t^r, which gives us $Y_{i,t}^* = \phi_i^Y(L)\left[Y_{i,t} - \alpha_{i,t}^Y F_t^g - \delta_{i,t}^Y F_{i,t}^c\right]$, $C_{i,t}^* = \phi_i^C(L)\left[C_{i,t} - \alpha_{i,t}^C F_t^g - \delta_{i,t}^C F_{i,t}^c\right]$ and $I_{i,t}^* = \phi_i^I(L)\left[I_{i,t} - \alpha_{i,t}^I F_t^g - \delta_{i,t}^I F_{i,t}^c\right]$.

Block 1(c): Sampling the country-specific factors $F_{i,t}^c$

In this block of the Gibbs sampler, we filter and sample the country-specific factors $F_{i,t}^c$ separately for each country i, conditioning on the global factor F_t^g, the group-specific factors F_t^r, the time-varying factor loadings α, β, and δ, the stochastic volatilities h, and the corresponding hyperparameters in Ψ. Again, we transform each endogenous variable as in Block 1(a) and (b), and express them as functions of the country-specific factor $F_{i,t}^c$, which gives us $Y_{i,t}^* = \phi_i^Y(L)\left[Y_{i,t} - \alpha_{i,t}^Y F_t^g - \beta_{i,t}^Y F_t^r\right]$, $C_{i,t}^* = \phi_i^C(L)\left[C_{i,t} - \alpha_{i,t}^C F_t^g - \beta_{i,t}^C F_t^r\right]$ and $I_{i,t}^* = \phi_i^I(L)\left[I_{i,t} - \alpha_{i,t}^I F_t^g - \beta_{i,t}^I F_t^r\right]$.

Block 2: Sampling the binary indicators $\mathcal{M}$ and the parameters σ, ζ_0 and h_0

For notational convenience, let us define a general regression model,

$$w = z^{\mathcal{M}} b^{\mathcal{M}} + e, \qquad e \sim \mathcal{N}(0, \Sigma), \tag{2.18}$$

with w a vector including observations on a dependent variable w_t, and z an unrestricted predictor matrix with rows z_t that contain the state processes from the vectors D_t, ζ_t and h_t that are relevant for explaining w_t. The corresponding unrestricted parameter vector with the relevant elements from Ψ is denoted by b. Then, $z^{\mathcal{M}}$ and $b^{\mathcal{M}}$ are the restricted predictor matrix and restricted parameter vector excluding those elements in z and b for which the corresponding indicator in $\mathcal{M}$ is 0. Furthermore, Σ is a diagonal matrix with elements $\sigma_{e,t}^2$ that may vary over time to allow for heteroskedasticity of a known form.

A naive implementation of the Gibbs sampler would be to sample $\mathcal{M}$ from $f(\mathcal{M}\,|D,\zeta,h,\Psi,w)$ and Ψ from $f(\Psi\,|D,\zeta,h,\mathcal{M},w)$. However, this approach does not result in an irreducible Markov chain, since whenever an indicator in $\mathcal{M}$ equals zero, the corresponding coefficient in Ψ is also zero, which implies that the chain has absorbing states. Therefore, as in Frühwirth-Schnatter and Wagner (2010), we marginalize over the parameters in Ψ for which variable selection is carried out when sampling $\mathcal{M}$, and then draw the respective parameters in Ψ conditional on the indicators $\mathcal{M}$. The posterior distribution $f(\mathcal{M}\,|D,\zeta,h,w)$ can be obtained using Bayes' Theorem as

$$f(\mathcal{M}\,|D,\zeta,h,w) \propto f(w\,|\mathcal{M},D,\zeta,h)\,p(\mathcal{M}), \tag{2.19}$$

with $p(\mathcal{M})$ being the prior probability of $\mathcal{M}$ and $f(w\,|\mathcal{M},D,\zeta,h)$ being the marginal likelihood of the regression model given by Equation (2.18) where the effect of the parameters $b^{\mathcal{M}}$ and σ_e^2 has been integrated out. The closed form solution of the marginal likelihood depends on whether the error term e_t is homoskedastic or heteroskedastic. More specifically:

- In the homoskedastic case, $\Sigma = \sigma_e^2 I_T$, under the normal-inverse gamma conjugate prior

$$b^{\mathcal{M}} \sim \mathcal{N}\left(a_0^{\mathcal{M}}, A_0^{\mathcal{M}}\sigma_e^2\right), \qquad \sigma_e^2 \sim IG(c_0, C_0), \tag{2.20}$$

  the closed form solution for $f(w\,|\mathcal{M},D,\zeta,h)$ is

$$f(w\,|\mathcal{M},D,\zeta,h) \propto \frac{\left|A_T^{\mathcal{M}}\right|^{0.5}}{\left|A_0^{\mathcal{M}}\right|^{0.5}}\frac{\Gamma(c_T)\,C_0^{c_0}}{\Gamma(c_0)\,(C_T^{\mathcal{M}})^{c_T}}, \tag{2.21}$$

  and the posterior moments $a_T^{\mathcal{M}}$, $A_T^{\mathcal{M}}$, c_T and $C_T^{\mathcal{M}}$ of $b^{\mathcal{M}}$ and σ_e^2 can be calculated as

$$a_T^{\mathcal{M}} = A_T^{\mathcal{M}}\left(\left(z^{\mathcal{M}}\right)' w + \left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}}\right), \tag{2.22}$$

$$A_T^{\mathcal{M}} = \left(\left(z^{\mathcal{M}}\right)' z^{\mathcal{M}} + \left(A_0^{\mathcal{M}}\right)^{-1}\right)^{-1}, \tag{2.23}$$

$$c_T = c_0 + T/2\,, \tag{2.24}$$

$$C_T^{\mathcal{M}} = C_0 + 0.5\left(w'w + \left(a_0^{\mathcal{M}}\right)'\left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}} - \left(a_T^{\mathcal{M}}\right)'\left(A_T^{\mathcal{M}}\right)^{-1} a_T^{\mathcal{M}}\right). \tag{2.25}$$

- In the heteroskedastic case, $\Sigma = \operatorname{diag}\left(\sigma_{e,1}^2, \ldots, \sigma_{e,T}^2\right)$, under the normal conjugate prior $b^{\mathcal{M}} \sim \mathcal{N}\left(a_0^{\mathcal{M}}, A_0^{\mathcal{M}}\right)$, the closed form solution for the marginal likelihood $f\left(w\,|\mathcal{M}, D, \zeta, h\right)$ is

$$f\left(w\,|\mathcal{M}, D, \zeta, h\right) \propto \frac{|\Sigma|^{-0.5}\left|A_T^{\mathcal{M}}\right|^{0.5}}{\left|A_0^{\mathcal{M}}\right|^{0.5}} \exp\left(-\frac{1}{2}\left(w'\Sigma^{-1}w + \left(a_0^{\mathcal{M}}\right)'\left(A_0^{\mathcal{M}}\right)^{-1}a_0^{\mathcal{M}} - \left(a_T^{\mathcal{M}}\right)'\left(A_T^{\mathcal{M}}\right)^{-1}a_T^{\mathcal{M}}\right)\right), \tag{2.26}$$

with

$$a_T^{\mathcal{M}} = A_T^{\mathcal{M}}\left(\left(z^{\mathcal{M}}\right)'\Sigma^{-1}w + \left(A_0^{\mathcal{M}}\right)^{-1}a_0^{\mathcal{M}}\right), \tag{2.27}$$

$$A_T^{\mathcal{M}} = \left(\left(z^{\mathcal{M}}\right)'\Sigma^{-1}z^{\mathcal{M}} + \left(A_0^{\mathcal{M}}\right)^{-1}\right)^{-1}. \tag{2.28}$$

Following George and McCulloch (1993), instead of using a multi-move sampler in which all the elements in $\mathcal{M}$ are sampled simultaneously, we use a single-move sampler in which each of the binary indicators λ and ρ in $\mathcal{M}$ is sampled from $f\left(\lambda_{i,j}^{\alpha}|\lambda_{i,j}^{\beta}, \lambda_{i,j}^{\delta}, \rho^D, D, \zeta_i^j, h, w\right)$, $f\left(\lambda_{i,j}^{\beta}|\lambda_{i,j}^{\alpha}, \lambda_{i,j}^{\delta}, \rho^D, D, \zeta_i^j, h, w\right)$, $f\left(\lambda_{i,j}^{\delta}|\lambda_{i,j}^{\alpha}, \lambda_{i,j}^{\beta}, \rho^D, D, \zeta_i^j, h, w\right)$, and $f\left(\rho^k|\rho^{\backslash k}, \lambda^{\varsigma}, D, \zeta, h, w\right)$ respectively. Block 2 is therefore split up into the following subblocks:

Block 2(a): Sampling the binary indicators λ and the parameters ζ_0 and σ_κ

In this block we sample the binary indicators $\lambda = \left\{\lambda_{i,j}^{\zeta}\right\}$ and the parameters $\zeta_{i,0}^j$, and $\sigma_\kappa = \left\{\sigma_{\kappa,i,j}^{\zeta}\right\}$ conditional on the states D, ζ, and h.

In order to sample the binary indicators and parameters corresponding to the time-varying loadings to the global factor, we use Equation (2.12) and rewrite each equation in model Equation (2.1) in the general linear regression format of Equation (2.18) as

$$\overbrace{y_{i,t}^{j*}}^{w_t} = \overbrace{\begin{bmatrix} F_t^{g*} & \lambda_{i,j}^{\alpha}\widetilde{\alpha}_{i,t}^{j}F_t^{g*}\end{bmatrix}}^{z_t^{\mathcal{M}}} \overbrace{\begin{bmatrix}\alpha_{i,0}^j \\ \sigma_{\kappa,i,j}^{\alpha}\end{bmatrix}}^{b^{\mathcal{M}}} + \overbrace{\nu_{i,t}^j}^{e_t}, \tag{2.29}$$

with $y_{i,t}^{j*} = \phi_i^j(L)\left[y_{i,t}^j - \beta_{i,t}^j F_t^r - \delta_{i,t}^j F_{i,t}^c\right]$ and $F_t^{g*} = \phi_i^j(L)F_t^g$ for $j = (Y, C, I)$, where r = IC, EM, or DC depending on the corresponding group membership of country

i.

Likewise, in order to sample the binary indicators and parameters corresponding to the time-varying loadings to the group factors, we use Equation (2.12) and rewrite each Equation in (2.1) in the general linear regression format of Equation (2.18) as

$$\overbrace{y_{i,t}^{j*}}^{w_t} = \overbrace{\begin{bmatrix} F_t^{r*} & \lambda_{i,j}^{\beta}\widetilde{\beta}_{i,t}^{j}F_t^{r*} \end{bmatrix}}^{z_t^{\mathcal{M}}} \overbrace{\begin{bmatrix} \beta_{i,0}^{j} \\ \sigma_{\kappa,i,j}^{\beta} \end{bmatrix}}^{b^{\mathcal{M}}} + \overbrace{\nu_{i,t}^{j}}^{e_t}, \tag{2.30}$$

with $y_{i,t}^{j*} = \phi_i^j(L)\left[y_{i,t}^j - \alpha_{i,t}^j F_t^g - \delta_{i,t}^j F_{i,t}^c\right]$ and $F_t^{r*} = \phi_i^j(L)F_t^r$.

Lastly, in order to sample the binary indicators and parameters corresponding to the time-varying loadings to the country-specific factors, we use Equation (2.12) and rewrite each Equation in (2.1) in the general linear regression format of Equation (2.18) as

$$\overbrace{y_{i,t}^{j*}}^{w_t} = \overbrace{\begin{bmatrix} F_{i,t}^{c*} & \lambda_{i,j}^{\delta}\widetilde{\delta}_{i,t}^{j}F_{i,t}^{c*} \end{bmatrix}}^{z_t^{\mathcal{M}}} \overbrace{\begin{bmatrix} \delta_{i,0}^{j} \\ \sigma_{\kappa,i,j}^{\delta} \end{bmatrix}}^{b^{\mathcal{M}}} + \overbrace{\nu_{i,t}^{j}}^{e_t}, \tag{2.31}$$

with $y_{i,t}^{j*} = \phi_i^j(L)\left[y_{i,t}^j - \alpha_{i,t}^j F_t^g - \beta_{i,t}^j F_t^r\right]$ and $F_{i,t}^{c*} = \phi_i^j(L)F_{i,t}^c$.

In all three equations, Equations (2.29)–(2.31), the second term in both the restricted vector $z_t^{\mathcal{M}}$ and the restricted parameter vector $b^{\mathcal{M}}$ is excluded when $\lambda_{i,j}^{\zeta} = 0$. Note that next to the parameters in $b^{\mathcal{M}}$ and σ_e^2, each of the specifications given by Equations (2.29), (2.30), and (2.31) depends only on the transformed data $y_{i,t}^{j*}$, on some of the transformed factors in D_t^*, on the time-varying loadings $\zeta_{i,t}^j$, and on the corresponding binary indicator $\lambda_{i,j}^{\zeta}$. Hence, we can simplify the specification of the posterior from $f\left(\lambda_{i,j}^{\zeta}\left|\lambda_{\backslash i,\backslash j}^{\backslash\zeta}, \rho^D, D, \zeta, h, x\right.\right)$ to $f\left(\lambda_{i,j}^{\zeta}\left|D^*, \zeta_i^j, y_i^{j*}\right.\right)$, for which we have $f\left(\lambda_{i,j}^{\zeta}\left|D^*, \zeta_i^j, y_i^{j*}\right.\right) \propto f\left(y_i^{j*}\left|\lambda_{i,j}^{\zeta}, D^*, \zeta_i^j\right.\right) p\left(\lambda_{i,j}^{\zeta}\right)$.

As the error terms $\nu_{i,t}^j$ in Equations (2.29)–(2.31) are homoskedastic, we have $\Sigma = \sigma_e^2 I_T$ in the general notation of Equation (2.18), so that the marginal likelihood $f\left(\lambda_{i,j}^{\zeta}\left|D^*, \zeta_i^j, y_i^{j*}\right.\right)$ can be calculated as in Equation (2.21). The binary indicator $\lambda_{i,j}^{\zeta}$ can then be sampled from the Bernoulli distribution with probability

$$p\left(\lambda_{i,j}^{\zeta} = 1\left|D^*, \zeta_i^j, y_i^{j*}\right.\right) = \frac{f\left(\lambda_{i,j}^{\zeta} = 1\left|D^*, \zeta_i^j, y_i^{j*}\right.\right)}{f\left(\lambda_{i,j}^{\zeta} = 0\left|D^*, \zeta_i^j, y_i^{j*}\right.\right) + f\left(\lambda_{i,j}^{\zeta} = 1\left|D^*, \zeta_i^j, y_i^{j*}\right.\right)}, \tag{2.32}$$

while $b^{\mathcal{M}}$ can be sampled from $\mathcal{N}\left(a_T^{\mathcal{M}}, A_T^{\mathcal{M}}\sigma^2_{\nu,i,j}\right)$ with $a_T^{\mathcal{M}}$ and $A_T^{\mathcal{M}}$ as defined in Equations (2.22)–(2.28). Note that $b^{\mathcal{M}} = \left(\zeta^j_{i,0}, \sigma^\zeta_{\kappa,i,j}\right)'$ when $\lambda^\zeta_{i,j} = 1$ and $b^{\mathcal{M}} = \zeta^j_{i,0}$ when $\lambda^\zeta_{i,j} = 0$. In the former case, $\sigma^\zeta_{\kappa,i,j}$ is sampled from the posterior, whereas in the latter case we set $\sigma^\zeta_{\kappa,i,j} = 0$.

Block 2(b): Sampling the binary indicators ρ and the parameters h_0 and σ_η

In this block we sample the binary indicators $\rho^D = \left\{\rho^g, \rho^{IC}, \rho^{EM}, \rho^{DC}, \rho^c_i\right\}$ and the parameters $h_0 = \left\{h^g_0, h^{IC}_0, h^{EM}_0, h^{DC}_0, h^c_{i,0}\right\}$ and $\sigma_\eta = \left\{\sigma^g_\eta, \sigma^{IC}_\eta, \sigma^{EM}_\eta, \sigma^{DC}_\eta, \sigma^c_{\eta,i}\right\}$ conditional on the states D and h. Using Equation (2.13), Equation (**??**) can be rewritten in the general linear regression format of Equation (2.18) as

$$\overbrace{g_t^D - \left(m_{\iota_t^D} - 1.2704\right)}^{w_t} = 2\overbrace{\begin{bmatrix} 1 & \rho^D \widetilde{h}_t^D \end{bmatrix}}^{z_t^{\mathcal{M}}} \overbrace{\begin{bmatrix} h_0^D \\ \sigma_{\eta,D} \end{bmatrix}}^{b^{\mathcal{M}}} + \overbrace{\widetilde{\epsilon}_t^D}^{e_t}, \tag{2.33}$$

with $\widetilde{\epsilon}_t^D = \epsilon_t^D - \left(m_{\iota_t^D} - 1.2704\right)$ is ϵ_t^D recentered around zero, and where using Equation (2.2), $g_t^D = \ln\left(\left(\exp\{h_t^D\}\psi_t^D\right)^2 + .001\right)$ can be calculated as

$$g_t^D = \ln\left(\left(D_t - \theta_1^D D_{t-1} - \theta_2^D D_{t-2} - \theta_3^D D_{t-3}\right)^2 + .001\right), \tag{2.34}$$

As specification (2.33) depends only on the data w_t, on the stochastic volatility term h_t^D, and on ρ^D, we can simplify the specification of the posterior from $f\left(\rho^D|\rho^{\setminus D}, \lambda, \zeta, h, w\right)$ to $f\left(\rho^D\left|h^D, w\right.\right)$. Using Bayes' Theorem, we have $f\left(\rho^D\left|h^D, w\right.\right) \propto f\left(w\left|\rho^D, h^D\right.\right)p\left(\rho^D\right)$. Given the mixture distribution of ϵ_t^D defined in Equation (**??**), the error term $\widetilde{\epsilon}_t^D$ in Equation (2.33) has a heteroskedastic variance $v^2_{\iota_t^D}$ such that $\Sigma = \text{diag}\left(v^2_{\iota_1^D}, ..., v^2_{\iota_T^D}\right)$ in the general notation of Equation (2.18). In this case, the marginal likelihood $f\left(w\left|\rho^D, h^D\right.\right)$ can be calculated as in Equation (2.26). The binary indicator ρ^D can then be sampled from the Bernoulli distribution with probability $p\left(\rho^D = 1\left|h^D, w\right.\right)$ calculated from an equation similar to Equation (2.32). Next, $b^{\mathcal{M}}$ can be sampled from $\mathcal{N}\left(a_T^{\mathcal{M}}, A_T^{\mathcal{M}}\right)$ with $a_T^{\mathcal{M}}$ and $A_T^{\mathcal{M}}$ as defined in Equations (2.27) and (2.28). Note that $b^{\mathcal{M}} = \left(h_0^D, \sigma_{\eta,D}\right)'$ when $\rho^D = 1$ and $b^{\mathcal{M}} = h_0^D$ when $\rho^D = 0$. In the latter case, we set $\sigma_{\eta,D} = 0$.

Block 3: Sampling the state vectors ζ and h, and the mixture indicators ι

In this block we use the forward-filtering and backward-sampling approach of Carter and Kohn (1994) and De Jong and Shephard (1995) to sample the states ζ and h based on a general state space model of the form

$$w_t = Z_t s_t + e_t, \qquad e_t \overset{iid}{\sim} \mathcal{N}(0, H_t), \tag{2.35}$$

$$s_t = R_1 s_{t-1} + K_t \mu_t, \qquad \mu_t \overset{iid}{\sim} \mathcal{N}(0, Q_t), \qquad s_1 \overset{iid}{\sim} \mathcal{N}(a_1, A_1), \tag{2.36}$$

where w_t is now a vector of observations and s_t an unobserved state vector. The matrices Z_t, R_1, K_t, H_t, Q_t and the expected value a_1 and variance A_1 of the initial state vector s_1 are assumed to be known (conditioned upon). The error terms e_t and μ_t are assumed to be serially uncorrelated and independent of each other at all points in time. As Equations (2.35)–(2.36) constitute a linear Gaussian state space model, the unknown state variables in s_t can be filtered using the standard Kalman filter. Sampling $s = [s_1, \ldots, s_T]$ from its conditional distribution can then be done using the multimove Gibbs sampler of Carter and Kohn (1994) and De Jong and Shephard (1995).

Block 3(a): Sampling the time-varying parameters ζ

We first filter and draw the time-varying parameters $\zeta_t = (\alpha^j_{i,t}, \beta^j_{i,t}, \delta^j_{i,t})$ conditionally on all factors D, the stochastic volatility terms h, the hyperparameters Ψ, and the binary indicators $\mathcal{M}$. More specifically, using Equation (2.12) for each Equation in (2.1), the unrestricted (i.e., $\lambda = 1$) conditional state space representations for the time-varying factor loadings $\widetilde{\zeta}^j_{i,t}$ are given by

$$\overbrace{\left[\, y^{j*}_{i,t} - \alpha^j_{i,0} F^{g*}_t \,\right]}^{w_t} = \overbrace{\left[\, \sigma^\alpha_{\kappa,i,j} F^{g*}_t \,\right]}^{Z^{\mathcal{M}}_t} \overbrace{\left[\, \widetilde{\alpha}^j_{i,t} \,\right]}^{s^{\mathcal{M}}_t} + \overbrace{\left[\, \nu^j_{i,t} \,\right]}^{e_t}, \tag{2.37}$$

$$\underbrace{\left[\, \widetilde{\alpha}^j_{i,t} \,\right]}_{s_t} = \underbrace{\left[\, 1 \,\right]}_{R_1} \underbrace{\left[\, \widetilde{\alpha}^j_{i,t-1} \,\right]}_{s_{t-1}} + \underbrace{\left[\, 1 \,\right]}_{K_t} \underbrace{\left[\, \widetilde{\kappa}^{j,\alpha}_{i,t} \,\right]}_{\mu_t}, \tag{2.38}$$

with $H_t = \sigma^2_{\nu,i,j}$ and $Q_t = 1$, $y^{j*}_{i,t} = \phi^j_i(L)\left[y^j_{i,t} - \beta^j_{i,t}F^r_t - \delta^j_{i,t}F^c_{i,t}\right]$ and $F^{g*}_t = \phi^j_i(L)F^g_t$ and

$$\overbrace{\left[\, y^{j*}_{i,t} - \beta^j_{i,0}F^{r*}_t \,\right]}^{w_t} = \overbrace{\left[\, \sigma^\beta_{\kappa,i,j}F^{r*}_t \,\right]}^{Z^{\mathcal{M}}_t}\overbrace{\left[\, \widetilde{\beta}^j_{i,t} \,\right]}^{s^{\mathcal{M}}_t} + \overbrace{\left[\, \nu^j_{i,t} \,\right]}^{e_t}, \tag{2.39}$$

$$\underbrace{\left[\, \widetilde{\beta}^j_{i,t} \,\right]}_{s_t} = \underbrace{\left[\, 1 \,\right]}_{R_1}\underbrace{\left[\, \widetilde{\beta}^j_{i,t-1} \,\right]}_{s_{t-1}} + \underbrace{\left[\, 1 \,\right]}_{K_t}\underbrace{\left[\, \widetilde{\kappa}^{j,\beta}_{i,t} \,\right]}_{\mu_t}, \tag{2.40}$$

with $H_t = \sigma^2_{\nu,i,j}$ and $Q_t = 1$, $y^{j*}_{i,t} = \phi^j_i(L)\left[y^j_{i,t} - \alpha^j_{i,t}F^g_t - \delta^j_{i,t}F^c_{i,t}\right]$ and $F^{r*}_t = \phi^j_i(L)F^r_t$ and

$$\overbrace{\left[\, y^{j*}_{i,t} - \delta^j_{i,0}F^{c*}_{i,t} \,\right]}^{w_t} = \overbrace{\left[\, \sigma^\delta_{\kappa,i,j}F^{c*}_{i,t} \,\right]}^{Z^{\mathcal{M}}_t}\overbrace{\left[\, \widetilde{\delta}^j_{i,t} \,\right]}^{s^{\mathcal{M}}_t} + \overbrace{\left[\, \nu^j_{i,t} \,\right]}^{e_t}, \tag{2.41}$$

$$\underbrace{\left[\, \widetilde{\delta}^j_{i,t} \,\right]}_{s_t} = \underbrace{\left[\, 1 \,\right]}_{R_1}\underbrace{\left[\, \widetilde{\delta}^j_{i,t-1} \,\right]}_{s_{t-1}} + \underbrace{\left[\, 1 \,\right]}_{K_t}\underbrace{\left[\, \widetilde{\kappa}^{j,\delta}_{i,t} \,\right]}_{\mu_t}, \tag{2.42}$$

with $H_t = \sigma^2_{\nu,i,j}$ and $Q_t = 1$, $y^{j*}_{i,t} = \phi^j_i(L)\left[y^j_{i,t} - \alpha^j_{i,t}F^g_t - \beta^j_{i,t}F^r_t\right]$ and $F^{c*}_{i,t} = \phi^j_i(L)F^c_{i,t}$. All random walk components $\widetilde{\alpha}^j_{i,t}$, $\widetilde{\beta}^j_{i,t}$ and $\widetilde{\delta}^j_{i,t}$ are initialized by setting $a_1 = 0$ and $A_1 = 1000$.

In the restricted model (i.e., $\lambda = 0$), $Z^{\mathcal{M}}$ and $s^{\mathcal{M}}$ are empty. In this case, no forward-filtering and backward-sampling is needed, and $\widetilde{\zeta}^j_{i,t}$ can be sampled directly from the prior using Equation (2.9). Note that the sampling of the common and country-specific factors in block 1 depends on ζ rather than on $\widetilde{\zeta}$. We therefore reconstruct $\zeta^j_{i,t}$ from Equations (2.8) by using the corresponding parameters from ζ_0 and σ_κ.

Block 3(b): Sampling the mixture indicators ι and the stochastic volatilities h

In this block we draw the mixture indicators $\iota = (\iota^g, \iota^{IC}, \iota^{EM}, \iota^{DC}, \iota^c_i)$ and the stochastic volatilities $h = \left(h^g, h^{IC}, h^{EM}, h^{DC}, h^c_i\right)$ conditionally on the state vector D, the time-varying parameters ζ, the hyperparameters Ψ, and the binary indicators $\mathcal{M}$. Following Del Negro and Primiceri (2014), in this block we first sample the mixture indicator ι^D_t from its conditional probability mass

$$p\left(\iota^D_t = n | h^D_t, \epsilon^D_t\right) \propto q_n f_{\mathcal{N}}\left(\epsilon^D_t | 2h^D_t + m_n - 1.2704, \nu^2_n\right), \tag{2.43}$$

with values for $\{q_n, m_n, \nu_n^2\}$ taken from Table 1 in Omori, Chib, Shephard, and Nakajima (2007).

Next, we filter and sample the stochastic volatility terms $\widetilde{h}_t^D$ conditioning on the transformed states g_t^D defined in Equation (2.34), on the mixture indicators ι_t^D, and on the parameters Ψ. More specifically, the unrestricted (i.e., $\rho^D = 1$) conditional state space representation is given by

$$\overbrace{\left[\; g_t^D - \left(m_{\iota_t^D} - 1.2704\right) - 2h_0^D \;\right]}^{w_t} = \overbrace{\left[\; 2\rho^D\sigma_{\eta,D} \;\right]}^{Z_t^{\mathcal{M}}}\overbrace{\left[\; \widetilde{h}_t^D \;\right]}^{s_t^{\mathcal{M}}} + \overbrace{\left[\; \widetilde{\epsilon}_t^D \;\right]}^{e_t}, \tag{2.44}$$

$$\underbrace{\left[\; \widetilde{h}_t^D \;\right]}_{s_t} = \underbrace{\left[\; 1 \;\right]}_{R_1}\underbrace{\left[\; \widetilde{h}_{t-1}^D \;\right]}_{s_{t-1}} + \underbrace{\left[\; 1 \;\right]}_{K_t}\underbrace{\left[\; \widetilde{\eta}_t^D \;\right]}_{\mu_t}, \tag{2.45}$$

with $H_t = v^2_{\iota_t^D}$, $Q_t = 1$ and where $\widetilde{\epsilon}_t^D = \epsilon_t^D - \left(m_{\iota_t^D} - 1.2704\right)$ is ϵ_t^D recentered around zero. The random walk components $\widetilde{h}_t^D$ are initialized by setting $a_1 = 0$ and $A_1 = 1000$.

In the restricted model (i.e., $\rho^D = 0$), $Z^{\mathcal{M}}$ and $s^{\mathcal{M}}$ are empty. In this case, no forward-filtering and backward-sampling is needed and $\widetilde{h}_t^D$ can be sampled directly from its prior using Equation (2.11). Note that the sampling of the common and country-specific factors F_t in block 1 depends on h_t^D rather than on $\widetilde{h}_t^D$. Using h_0^D, $\sigma_{\eta,D}$ and $\widetilde{h}_t^D$, h_t^D can easily be reconstructed from Equation (2.10).

Block 4: Estimating and sampling the parameters θ, ϕ, and σ_ν^2

In the final block of the Gibbs sampler we estimate and draw the autoregressive parameters θ and ϕ and the variances of the idiosyncratic error terms σ_ν^2 conditioning on the factors D, the time-varying factor loadings ζ and the stochastic volatilities h, and the binary indicators $\mathcal{M}$. We estimate and draw the variances and the AR parameters separately. Therefore, block 4 is split up in the following subblocks:

Block 4(a): Estimating and sampling the variances σ_ν^2

First we estimate and draw the variances $\sigma^2_{\nu,i,j}$ of the iid shocks to the idiosyncratic error terms in Equation (2.4) for each country i and variable j separately. We follow the approach of Kim and Nelson (1999) (pp. 175–177) and draw the variance of the shocks to idiosyncratic errors from an inverted-gamma distribution with prior information

$$\sigma^2|\beta \sim \mathcal{IG}(c_0, C_0), \tag{2.46}$$

where β is a vector of known parameters and is conditioned on. We obtain prior information $c_0 = T * str_0/2$ on the shape parameter, where T (observations) is the prior number of degrees of freedom and str_0 is the strength of the belief about the value of the variance σ^2. Prior information on the scale parameter is given by $C_0 = bel_0 * c_0$, where bel_0 denotes the corresponding prior belief about the value of the variance parameter σ^2. The posterior distribution can be represented by

$$\sigma^2|\beta \sim \mathcal{IG}(c_1, C_1), \tag{2.47}$$

where $c_1 = c_0 + T/2$ and $C_1 = C_0 + (w' * w)/2$, and $w' * w$ are the residual sum of squares yielding an estimate of the variance based on the data. Therefore, the posterior parameters of the inverse gamma distribution consist of prior information and the information in the data.

We sample the variances $\sigma^2_{\nu,i,j}$ of the shocks to the idiosyncratic errors conditioning on the factors D, the time-varying loadings α, β, and δ, and the AR coefficients ϕ from the posterior given in Equation (2.47) for each country i and variable j separately. Therefore, we first compute the residuals for each equation in Equation (2.1) as $\varepsilon^j_{i,t} = y^j_{i,t} - \alpha^j_{i,t} F^g_t - \beta^j_{i,t} F^r_t - \delta^j_{i,t} F^c_{i,t}$ and set $w^j_{i,t} = \varepsilon^j_{i,t} - \phi^j_{1,i}\varepsilon^j_{i,t-1} - \phi^j_{2,i}\varepsilon^j_{i,t-2} - \phi^j_{3,i}\varepsilon^j_{i,t-3}$ to take into account the autoregressive structure of the error terms.

Block 4(b): Estimating and sampling the AR parameters ϕ and θ

Next we estimate and draw the autoregressive parameters ϕ^j_i of the idiosyncratic error terms $\varepsilon^j_{i,t}$. Conditioning on the factors D, the time-varying loadings α, β, and δ, and the error variance of the idiosyncratic error terms σ^2_ν known from the previous step, these are all unknown parameters in the standard linear regression model

$$y = X\beta + e, \quad e \sim \mathcal{N}(0, \sigma^2 I_T), \tag{2.48}$$

where y and e are $T \times 1$ vectors, and X is a $T \times K$ matrix containing the fixed regressors $X_1, ..., X_K$. β is the unknown vector of coefficients and the error variance σ^2 is assumed to be known. We follow the approach outlined in Kim and Nelson

(1999) (pp. 173–174). The prior information of β follows a normal distribution

$$\beta|\sigma^2 \sim \mathcal{N}(\beta_0, \Sigma_0), \tag{2.49}$$

where β_0 is the prior belief of β and Σ_0 is the prior variance regarding this belief indicating the degree of uncertainty on β_0. The parameters β can then be sampled from the posterior distribution

$$\beta|\sigma^2 \sim \mathcal{N}(\beta_1, \Sigma_1), \tag{2.50}$$

with hyperparameters defined by

$$\Sigma_1 = \left(\frac{X'X}{\sigma^2} + \Sigma_0^{-1}\right)^{-1} \tag{2.51}$$

$$\beta_1 = \left(\frac{X'y}{\sigma^2} + \Sigma_0^{-1}\beta_0\right)\Sigma_1, \tag{2.52}$$

where the prior information is combined with the information in the data given by the sample estimate.[10]

The autoregressive parameters can now be sampled as follows:

- We obtain the posterior distribution of $\phi^j_{p,i}$ for each country i and each variable j separately (for $p = 1, 2, 3$ lags). First we compute the variable specific idiosyncratic error terms for each country i:

$$\varepsilon^j_{i,t} = y^j_{i,t} - \alpha^j_{i,t}F^g_t - \beta^j_{i,t}F^r_t - \delta^j_{i,t}F^c_{i,t}. \tag{2.53}$$

  We then set $y = \varepsilon^j_{i,t}$, $X_1 = \varepsilon^j_{i,t-1}$, $X_2 = \varepsilon^j_{i,t-2}$, and $X_3 = \varepsilon^j_{i,t-3}$ for each country i and variable j in Equation (2.48) and sample the AR coefficients $\phi^j_{p,i}$ from Equation (2.50) correspondingly. We accept the draw if $\left|\Sigma^3_{p=1}\left(\phi^j_{p,i}\right)\right| < 1$, ensuring the stationarity of the autoregressive processes.

- We obtain the posterior distribution of the AR coefficients $\theta_{p,D}$ for each factor separately, conditioning on D_t and h^D_t by using Equation (2.50) and setting $y = D_t/\exp(h^D_t)$, $X_1 = D_{t-1}/\exp(h^D_{t-1})$, $X_2 = D_{t-2}/\exp(h^D_{t-2})$, and $X_3 = D_{t-3}/\exp(h^D_{t-3})$, so that Equation (2.48) becomes a GLS-type regression model since the errors in Equation (2.2) are heteroskedastic due to the stochastic volatilities. Again, we then sample the AR coefficients θ^D_p from Equation (2.50). We accept the draw if $\left|\Sigma^3_{p=1}\left(\theta^D_p\right)\right| < 1$, ensuring the stationarity of the autoregressive processes.

[10] We refer to Kim and Nelson (1999) for a more detailed explanation.

3 Estimating Macro-Financial Linkages in Advanced Economies – A Unified Approach

with Tino Berger

Abstract

In this paper, we analyze macro-financial linkages across G7 countries using a multivariate trend-cycle decomposition. We embed a dynamic factor model in a trend-cycle decomposition in order to analyze and quantify the role of common factors in the gaps, or cycles, of real activity and financial variables. Our analysis, first, confirms the result of existing studies, which show the existence of common factors in macroeconomic and financial variables separately. However, in a variance decomposition we find no evidence for a joint common factor in real activity and financial variables across countries. Second, we show that the inclusion of financial variables in output gap estimations does not alter the estimated gap. This indicates that the inclusion of financial variables is not important for the measurement, but rather for the interpretation of the output gap.

JEL Classification: C38, E32, E44, F44
Keywords: finance-adjusted output gap, international business & financial cycle, dynamic factor model

3.1 Introduction

The global financial crisis of 2008-09 has highlighted how financial market developments can spill-over into the real economy and may be a potential source of macroeconomic imbalances. Furthermore, the global dimension of the crisis demonstrated how shocks can be transmitted rapidly across countries in the presence of tight cross-country inter-dependencies. The question of how these interactions, both across countries and between real activity and financial variables, shape a country's business cycle has become an important topic in the empirical literature and in policy circles.

Our paper adds to several strings of empirical literature. First, the global financial crisis led to a renewed debate about the understanding of the linkages between macroeconomic and financial variables. Macro-financial linkages are at the center of this debate. They describe the two-way interaction between the real economy and the financial sector. For example, shocks in the real economy may be transmitted to financial markets via asset prices affecting the balance sheets of households and companies and thus amplifying business cycles dynamics (see, e.g., Bernanke, Gertler, and Gilchrist, 1999; Claessens and Kose, 2018). Claessens, Kose, and Terrones (2012) show that recessions associated with disruptions in financial variables like house and equity prices or credit dynamics are longer and deeper than other recessions and thus provide evidence for the importance of macro-financial linkages for shaping the business cycle. Similar evidence is provided by Prieto, Eickmeier, and Marcellino (2016), who show that the contribution of shocks in financial variables to fluctuations in GDP growth increased from 20 % in normal times up to 50 % during the recent financial crisis. However, few studies analyze macro-financial linkages from an international perspective. For example, Helbling, Huidrom, Kose, and Otrok (2011) investigate the importance of global credit shocks for driving the international co-movement of business cycles. Ha, Kose, Otrok, and Prasad (2020) provide evidence for sizable spill-over effects from shocks to global equity and housing markets on the global business cycle, which have become stronger since the period leading to the financial crisis. We refer to Morley (2016); Cochrane (2017); Claessens and Kose (2018) and the references therein for excellent and comprehensive surveys on the extensive literature on macro-financial linkages.

Second, a growing literature aims to model the linkages between business and financial cycles. For example, the finance-neutral output gap literature uses financial variables as explanatory variables in an extended HP-filter which not only changes the interpretation of the output gap, but also its measurement (e.g. Borio, Disyatat, and Juselius, 2017). Other studies use Unobserved Components (UC) models to jointly decompose macroeconomic and financial variables into trend and cycle and in a second step analyze the relationship between the cycles (Rünstler and Vlekke, 2018; Winter, Koopman, and Hindrayanto, 2022). The analysis of macro-financial linkages on the basis of a decomposition into trend and cycle is an improvement with respect to the analyses based on growth rates, as the latter might discard important information in the variables by taking first differences of the data.

However, the existing literature that analyzes macro-financial linkages within the class of UC models and a trend-cycle decomposition is missing a potential international dimension in these linkages.In fact, a large body of literature has highlighted the close linkages of economic activity across countries. Next to the rich evidence on international co-movement in the growth rates of different macroeconomic variables – or international business cycles – like for instance GDP and its components (see e.g. Gregory and Head, 1999; Kose, Otrok, and Whiteman, 2003, 2008; Kose, Otrok, and Prasad, 2012; Mumtaz, Simonelli, and Surico, 2011), a growing literature provides evidence on common dynamics in house and equity prices and credit markets – or financial cycles. These common dynamics have been particularly strong during the global financial crisis and subsequent Great Recession (see e.g. Claessens, Kose, and Terrones, 2011a; Hirata, Kose, Otrok, and Terrones, 2013; Breitung and Eickmeier, 2014; Miranda-Agrippino and Rey, 2021; Passari and Rey, 2015; Rey, 2015). However, the literature on financial cycles is less clear about the exact definition of what variables best describe the financial cycle. While many studies interpret the financial cycle in terms of cyclical fluctuations of private sector credit and house prices (e.g., see Aikman, Haldane, and Nelson, 2015; Drehmann, Borio, and Tsatsaronis, 2012; Borio, Disyatat, and Juselius, 2017; Rünstler and Vlekke, 2018), in particular studies focusing on the international co-movement of financial variables define the financial cycle in terms of movements in equity prices and/or capital flows (e.g., see Rey, 2015; Miranda-Agrippino and Rey, 2021; Cerutti, Claessens, and Rose, 2019). However, for example, Drehmann, Borio, and Tsatsaronis (2012) argue that equity prices exhibit a higher volatility at short-term frequencies and therefore do not share the dynamics observed in credit and house price cycles.

We note that existing studies analyze the similarities between the size and length of business and financial cycles across countries from a joint trend-cycle decomposition (Rünstler and Vlekke, 2018; Winter, Koopman, and Hindrayanto, 2022), or by means of factor models analyzing the co-movement in growth rates of real activity and financial variables (Ha, Kose, Otrok, and Prasad, 2020). In this paper, we aim to bridge these different parts of the empirical literature and, consequently, take a fully joint approach in modeling macro-financial linkages across the G7 economies. The key contribution of this paper is to pursue a joint empirical modeling approach for estimating business and financial cycles based on a multivariate multi-country trend-cycle decomposition where we integrate a dynamic factor model in order to capture common dynamics in the cycles across and within countries. This modeling approach allows us to quantify the relative importance of macro-financial linkages for the size of a country's output gap.

Our main results are as follows. First, we document the existence of common dynamics in real activity and financial cycles across G7 countries. First, we find a high synchronization of real activity variables at the medium-term frequency and of financial variables which seem to share common dynamics at a longer duration and exhibit a larger amplitude. However, we do not find convincing evidence for the existence of a common macro-financial factor. Second, we find that the inclusion of financial variables in the output gap estimation does not alter the resulting output gap estimates (also see Berger, Richter, and Wong, 2022). This stands in contrast to the finance-neutral output gap literature pioneered by Borio, Disyatat, and Juselius (2017), who argues that financial variables should be included in measures of the output gap, if they add information on the position of the business cycle.

The rest of this paper proceeds as follows. Section 3.2 describes our empirical approach and the estimation strategy. Section 3.3 presents the data and our empirical results, and the section 3.4 concludes and discusses our findings.

3.2 Empirical specification and estimation

This section presents the empirical specification. We first describe the multivariate multi-factor trend-cycle decomposition to jointly decompose macroeconomic and financial variables into their long-run trends and cyclical components. Then we turn to a description of the Markov Chain Monte Carlo (MCMC) algorithm used to estimate the state space model.

3.2.1 Data

To estimate the macro-financial linkages, data on macroeconomic variables as well as on financial variables closely related to the real side of the economy are necessary. Thereby, choice of the variables used in this paper is inspired by different parts of the macroeconomic literature discussed in the previous section. First, the economic activity indicators are real GDP (y^1_{it}), real private consumption expenditures (y^2_{it}) and real private investment (y^3_{it}, as measured by the gross fixed capital formation) typically used in the analysis of international business cycle synchronization (e.g., see Kose, Otrok, and Whiteman, 2003, 2008; Kose, Otrok, and Prasad, 2012). The data for all real activity indicators is taken from the OECD quarterly National Accounts database. For the purpose of analyzing the linkages between macroeconomic and financial variables, we use real credit to the private non-financial sector (y_{it}^5) and a real house prices (y_{it}^6) as financial variables in our model. This choice is also consistent with Leamer (2007) who finds that "housing IS the business cycle" emphasizing the tight linkages between asset price and the macroeconomy. We source nominal credit data from the Bank of International Settlements (BIS) credit statistics and obtain real credit volumes by deflating with the respective CPI of a country after seasonal adjustment. House prices indices are taken from the OECD analytical house price indicators database which are already seasonally adjusted and in real terms.

Our measure of consumer prices is the all items consumer price index (CPI) with the log of this index denoted as p_{it}. The inflation variable that enters the model is the annualized quarterly change in the CPI calculated as $\pi_{it} = (p_{it} - p_{it-1}) * 400$.

The analysis focuses on the G7 countries – the United States, Canada, France, Germany, Italy, Japan, the United Kingdom – all of which are major advanced economies with sizable financial markets. In 2017 the outstanding amount of total credit to the private non-financial sector ranged between about 112 % of GDP in Italy and 213 % of GDP in Canada.[1] The time period under consideration covers quarterly data from 1971Q2 to 2017Q3, which should be sufficiently long to account for the typically longer duration of financial cycles as compared to business cycles (Drehmann, Borio, and Tsatsaronis, 2012).

3.2.2 A multi-factor trend-cycle decomposition

We estimate linkages in macroeconomic and financial variables across and within countries and analyze their role for the business cycle in a joint empirical approach. The empirical specification utilizes unobserved components models from the class of dynamic factor models to estimate several common and country-specific unobserved factors. We embed this factor model in a standard trend-cycle decomposition (an approach used in an extant work on estimating the output gap (e.g. see Kuttner, 1994; Basistha and Nelson, 2007; Blagrave, Zhang, Garcia-Saltos, and Laxton, 2015)) of real activity and financial variables.

In the spirit of Jarociński and Lenza (2018), we use a multivariate trend-cycle decomposition of real activity indicators and financial variables to define the trend

[1] According to the Bank for Internationale Settlements (BIS) credit statistics.

and cycle of a variable. Hence, a time series y_{it}^k is decomposed into a non-stationary trend w_{it}^k and a stationary cycle c_{it}^k such that the first observation equation reads as

$$y_{it}^k = w_{it}^n + c_{it}^k + \varepsilon_{it}^{y,k}, \tag{3.1}$$

where $i = 1, \ldots, C$ denotes the country, $k = 1, \ldots, K$ indexes real (r) and financial (fin) variables, and observations ranging from $t = 1, \ldots, T$. For GDP, we refer to c_{it}^{gdp} as the output gap. The second observation equation of the system is a hybrid Phillips-Curve

$$\pi_{it} = \delta_i \pi_{i,t-1} + (1 - \delta_i)\pi_{i,t-2,4} + \gamma_i c_{it}^{gdp} + \varepsilon_{it}^{\pi}, \tag{3.2}$$

where the inflation rate π_{it} to the cyclical component in real GDP c_{it}^{gdp}, to past inflation $\pi_{i,t-1}$, and inflation expectations $\pi_{i,t-2,4}$, which we approximate by the average of the second to the fourth lag of inflation (see Laubach and Williams, 2003; Holston, Laubach, and Williams, 2017).

$\varepsilon_{it}^{y,k}$ and ε_{it}^{π} in Equations (3.1) and (3.2) are independent Gaussian errors with variances $\sigma_{y,i,k}^2$ and $\sigma_{\pi,i}^2$.

The laws of motion of the unobservable trend components w_{it}^k follow the local linear trend model which allows for a flexible evolution of the trend component because the trend growth rate d_{it}^k may change over time

$$w_{it}^k = w_{i,t-1}^k + d_{i,t-1}^k + \eta_{it}^{w,k}, \tag{3.3}$$
$$d_{it}^k = d_{i,t-1}^k + \eta_{it}^{d,k}, \tag{3.4}$$

where $\eta_{it}^{w,k}$ and $\eta_{it}^{d,k}$ again denote Gaussian errors with variances $\sigma_{w,i,k}^2$ and $\sigma_{d,i,k}^2$.

We assume that each cycle c_{it}^k in Equation (3.1) can be described by a linear combination of several unobserved stationary factors $\{\mathbf{f_t}\}$

$$\begin{aligned} c_{it}^k = {} & \overbrace{b_{i,k}^{MF} f_t^{MF} + b_{i,k}^{M} f_t^{M} + b_{i,k}^{F} f_t^{F}}^{international\ factors} \\ & + \underbrace{b_{i,k}^{MFc} f_{it}^{MFc} + b_{i,k}^{Mc} f_{it}^{Mc} + b_{i,k}^{Fc} f_{it}^{Fc}}_{country-specific\ factors}, \end{aligned} \tag{3.5}$$

such that Equation (3.1) becomes in matrix notation

$$y_{it}^k = w_{it}^k + \underbrace{\mathbf{B_{i,k} f_t}}_{c_{it}^k} + \varepsilon_{it}^{y,k}, \tag{3.1a}$$

which relates the k^{th} variable in country i to a country- and variable-specific trend w_{it}^k and to the international and country-specific factors collected in $\mathbf{f_t}$ which may affect the observable variables via the factors loadings $\mathbf{B_{i,k}}$ that measure the sensitivity of a time series to changes in a factor. In Equation (3.5), f_t^{MF} is the international macro-financial factor, which picks up fluctuations that are common across the cycles of all real activity and financial variables and across all countries, f_t^M denotes the international macro factor, which captures the remaining common dynamics in real activity across all countries and similarly f_t^F labels the international financial factor which accounts for the remaining common dynamics in the financial variables across countries. Co-movement of variables that is not common across countries but within a certain country only, is captured by the country-specific factors f_{it}^{MFc}, f_{it}^{Mc}, and f_{it}^{Fc} defined in a similar manner.

As common for the business cycle literature, the law of motion of the stationary dynamic factors are independent AR(2) processes where the factors f_t^m and factor innovations $\eta_t^{f,m}$ are assumed to be orthogonal and thus uncorrelated across each other.[2],

$$f_t^m = \phi_1^m f_{t-1}^m + \phi_2^m f_{t-2}^m + \eta_t^{f,m}, \tag{3.5}$$

for $m = 1, \ldots, M = \{MF, M, F, MFc, Mc, Fc\}$ labeling the different unobserved factors in Equation (3.5). $\eta_t^{f,m}$ are Gaussian errors with variance $\sigma_{f,m}^2$.

Identification of the latent factors

As described above, we use a large set of unobserved factors in the stationary cycles of the variables under consideration to model linkages between real activity and financial variables across and within countries. As such, the factor model captures the variation in different blocks and sub-blocks of the data also known as a hierarchical or multi-level factor model (e.g., see Bai and Wang, 2015). A distinctive advantage in comparison to standard factor models is that hierarchical factors are easier to interpret in economic terms. For illustration of the hierarchical block structure in our factor model, let $r = \{gdp, consumption, investment\}$ be the index of real activity variables and $fin = \{credit, house\ prices\}$ of financial variables and write equation (3.1a) in matrix notation

[2] Bai and Wang (2015) show that the independence of the dynamic factors is sufficient, but not necessary to achieve identification of the factors. In particular, they show how an unrestricted VAR(p) structure across factors can be used to allow for "spill-over" effects between the factors.

$$\begin{bmatrix} Y_{1t}^{r} \\ Y_{1t}^{fin} \\ \vdots \\ Y_{Ct}^{r} \\ Y_{Ct}^{fin} \end{bmatrix} = \begin{bmatrix} W_{1t}^{r} \\ W_{1t}^{fin} \\ \vdots \\ W_{Ct}^{r} \\ W_{Ct}^{fin} \end{bmatrix} + \begin{bmatrix} B_{1,r}^{MF} & B_{1,r}^{M} & 0 & B_{1,r}^{MFc} & B_{1,r}^{Mc} & 0 & \cdots & \cdots & \cdots & 0 \\ B_{1,fin}^{MF} & 0 & B_{1,fin}^{F} & B_{1,fin}^{MFc} & 0 & B_{1,fin}^{Fc} & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & 0 & 0 & 0 & \ddots & \vdots & \vdots & \vdots \\ B_{C,r}^{MF} & B_{C,r}^{M} & 0 & \vdots & \vdots & \vdots & 0 & B_{C,r}^{MFc} & B_{C,r}^{Mc} & 0 \\ B_{C,fin}^{MF} & 0 & B_{C,fin}^{F} & 0 & 0 & 0 & 0 & B_{C,fin}^{MFc} & 0 & B_{C,fin}^{Fc} \end{bmatrix} \begin{bmatrix} f_t^{MF} \\ f_t^{M} \\ f_t^{F} \\ f_{1t}^{MFc} \\ f_{1t}^{Mc} \\ f_{1t}^{Fc} \\ \vdots \\ f_{Ct}^{MFc} \\ f_{Ct}^{Mc} \\ f_{Ct}^{Fc} \end{bmatrix} + \begin{bmatrix} E_{1t}^{r} \\ E_{1t}^{fin} \\ \vdots \\ E_{Ct}^{r} \\ E_{Ct}^{fin} \end{bmatrix}, \tag{3.6}$$

We obtain the multi-level structure by imposing zero restrictions on certain factor loadings as shown in Equation (3.6), which allows us to have the economic interpretation of the factors discussed above. As such, the international macro-financial factor, f_t^{MF}, is identified by the stationary cycles across all real activity (r) and financial (fin) variables and countries $(1, \ldots, C)$, reflected by the full non-zero entries in the first column of the loadings matrix B. Identification of the international macro factor, f_t^{M}, is obtained by only allowing the cycles of the real activity variables to load on this factor; that is, setting the loadings of the cycles in credit and house prices to this factor equal to zero. Likewise, for the international financial factor, f_t^{F}, we require the loadings of the cycles in real activity to be zero in the third column of the loadings matrix B. The block diagonal structure of the fourth to the last column of B identifies the common dynamics within a country. Within each country-block, we apply similar zero restrictions on the factor loadings to distinguish the different factors. As such, f_t^{MFc}, f_t^{Mc} and f_t^{Fc} capture macro-financial, macro and financial factors at the country-specific level, respectively.

A common identification problem that arises in all factors models, is that the sign and the scale of the factors are not identified without further restrictions; that is, we can multiply and divide F_t and B by any constant without changing their product. To obtain a unique identification, we require the factor innovation variances $\sigma_{\eta,m}^2$ to be diagonal and certain factor loadings to be 1.[3]

[3] Precisely, we require the loading of the US output gap (cycle in GDP) to be 1 for the common macro-financial factor f_t^{MF} and for the common macro factor f_t^{M}, i.e. $b_{US,gdp}^{MF} = b_{US,gdp}^{M} = 1$, and the loading of the US credit gap (cycle in credit) on the common financial factors to be 1, i.e. $b_{US,cr}^{F} = 1$. For sign and scale identification of the country-specific factors we require the loading of a country's output gap to be 1 for its own country-specific macro-financial and macro factor, i.e. $b_{i,gdp}^{MFc} = b_{i,gdp}^{Mc} = 1$ and the loading of a country's credit gap to be 1 for its own country-specific financial factor, i.e. $b_{i,cr}^{Fc} = 1$.

A variance decomposition of the output gap

Applied work in factor analysis has focused on conducting a variance decomposition of the observed time series with respect to the unobserved factors, because this can tell how much a certain factor contributes to the overall variation in the time series relative to other orthogonal factors. For the purpose of this paper, where we are interested in the linkages between real activity and financial variables across and within countries, we decompose the variance of the output gap, which is our measure of the business cycle, in each country into the contribution from all macro-financial and macro factors. First, we calculate the variance of each country's output gap (τ_i^{gdp}).[4]

$$\begin{aligned}\tau_i^{gdp} &= var(c_{it}^{gdp}) \\ &= (b_{i,gdp}^{MF})^2 var(f_t^{MF}) + (b_{i,gdp}^{M})^2 var(f_t^{M}) \\ &+ (b_{i,gdp}^{MFc})^2 var(f_{it}^{MFc}) + (b_{i,gdp}^{Mc})^2 var(f_{it}^{Mc}) + \sigma^2_{\varepsilon,i,gdp},\end{aligned} \tag{3.7}$$

where $var(f_t^m)$ is the unconditional variance of factor m and is obtained by

$$var(f_t^m) = \frac{(1-\phi_2^m)\sigma^2_{\eta,m}}{(1+\phi_2^m)[(1-\phi_2^m)^2 - \phi_1^{m2}]}. \tag{3.8}$$

We then calculate the corresponding variance shares of the output with respect to each factor expressed in percentage shares of the total variance

$$VS_{i,gdp}^{MF} = \frac{(b_{i,gdp}^{MF})^2 var(f_t^{MF})}{\tau_i^{gdp}} \quad \text{and} \quad VS_{i,gdp}^{M} = \frac{(b_{i,gdp}^{M})^2 var(f_t^{M})}{\tau_i^{gdp}},$$

$$VS_{i,gdp}^{MFc} = \frac{(b_{i,gdp}^{MFc})^2 var(f_{it}^{MFc})}{\tau_i^{gdp}} \quad \text{and} \quad VS_{i,gdp}^{Mc} = \frac{(b_{i,gdp}^{Mc})^2 var(f_{it}^{Mc})}{\tau_i^{gdp}}.$$

3.2.3 Estimation

State space representation of the model

The model given by Equations (3.1)-(3.5) can be cast into a linear Gaussian state space model with the observation equations given by Equations (3.1a) and (3.2) and the state or transition equations given by Equations (3.3), (3.4), and (3.5) describing the dynamics of the unobserved non-stationary trends and stationary cycles.[5]

[4]Notice that the output gap does not load on the common and country-specific financial factors by construction.

[5]See Section 3.A for a matrix representation of the state space form.

Given the fairly large number of parameters and unobserved states (non-stationary trends and latent factors), we favor a Bayesian approach over classical estimation by means of maximum likelihood (ML) methods. Bayesian methods allow us to specify prior distributions to down-weight the likelihood function in regions of the parameters that are not consistent with existing empirical evidence or in which there is no reasonable interpretation of the model. Therefore, we analyze the state space form of the model using the Gibbs sampler, which is a Markov Chain Monte Carlo (MCMC) method to approximate the joint distribution of the unknown parameters and the unobserved factors and other states and to simulate draws from the marginal posterior distributions using only conditional distributions. Intuitively, this amounts to reducing the model to a sequence of blocks for subsets of the parameters and states that are estimated, conditional on the other blocks in the sequence.

Gibbs algorithm

We denote $y_i^k = \{y_{it}^k\}_{t=1}^T$ as the vector of observed time series stacked over time, $y_i = \{y_i^k\}_{k=1}^K$ stacked over variables n, and $y = \{y_i\}_{i=1}^C$ stacked over all countries. Similarly, let w, d, and F denote the stacked vectors of unobserved trends, drifts, and dynamic factors, respectively and collect the parameters in the vector $\theta = (b, \delta, \gamma, \phi, \sigma^2)$.

Given an arbitrary set of initial values $(w_{(0)}, d_{(0)}, F_{(0)}, \theta_{(0)})$ we iterate over the following blocks:

1. States
   Sample the unobserved states $(w_{(1)}, d_{(1)}, F_{(1)})$ from $f(w, d, F|y, \theta_{(0)})$ using the simulation smoother of Durbin and Koopman (2002) implemented as explained in Jarociński (2015).

2. Parameters
   Sample the parameters $\theta_{(1)}$ from $f(\theta|y, w_{(1)}, d_{(1)}, F_{(1)})$.

We repeat these steps for 200.000 iterations to approximate realizations from a Markov chain with the joint posterior distribution $f(\mu, \nu, F, \theta, y)$. We discard the first 100.000 iterations as a burn-in sample. To reduce the auto-correlation within the Markov chain, we thin the remaining 100.000 draws and keep every 10th draw such that we obtain 10.000 draws for posterior interference. A detailed description of the sampling steps is given in 3.B. We assess the convergence of the Gibbs sampler in Appendix 3.D.

Prior distributions

Bayesian estimation involves prior information on the unknown parameters by specifying suitable prior distributions. Prior information on the parameters down-weight the likelihood function in regions of the parameter space that are not plausible.[6]

For the variance parameters of the idiosyncratic errors $\sigma^2 = (\sigma_y^2, \sigma_\pi^2, \sigma_w^2, \sigma_d^2, \sigma_f^2)$ we use the inverse Gamma prior $IG(c_0, C_0)$ where the shape $c_0 = \nu_0 T$ and scale $C_0 = c_0 \sigma_0^2$ are calculated from the *prior belief* σ_0^2 about the variance parameter and the *prior strength* ν_0 which is expressed as a fraction of the sample size T.[7] For the remaining parameters, we assume standard independent Gaussian priors $\mathcal{N}(a_0, V_0)$. Prior choices for all parameters are discussed below.

- **Idiosyncratic components,** $\sigma^2_{y,i,k}, \sigma^2_{\pi,i}, \sigma^2_{w,i,k}, \sigma^2_{d,i,k}, \sigma^2_{f,m}$**:** We set prior beliefs to $\sigma_{y,i,k} = 0.1$, $\sigma_{\pi,i} = 1.0$, $\sigma_{w,i,k} = 0.01$, $\sigma_{d,i,k} = 0.001$ and $\sigma_{f,m} = 1$. The strength of all priors is 0.001 which implies that the prior on the variance parameters is basically uninformative.

- **Factor loadings,** b_i^k**:** We set the prior on the factor loadings, other than those that we fix to 1 for identification, to $\mathcal{N}(0, 0.5)$. The prior mean of 0 is a natural benchmark and the variance of 0.5 implies that the prior is sufficiently loose such that the 90 % interval of the distribution is given by [-1.16, +1.16].

- **AR(2) parameters of factors,** ϕ_1^m, ϕ_2^m**:** The different factors in our model define the cyclical components c_{it}^k of all variables which have to be stationary by assumption. This implies that we have to ensure stationarity of the factors. Furthermore, we make the assumption that the different factors, the macro-financial factor, the macro factor, and the financial factor, differ in the persistence of their respective process, where we define the persistence as the sum of the AR-parameters of a given process. Precisely, use an informative prior on the sum $(\phi_1^{MF} + \phi_2^{MF}) \sim \mathcal{N}(0.85, 0.015^2)$ for the macro-financial factors, $(\phi_1^M + \phi_2^M) \sim \mathcal{N}(0.75, 0.015^2)$ for the macro factors and $(\phi_1^F + \phi_2^F) \sim \mathcal{N}(0.95, 0.015^2)$ for the financial factors. On the difference in the persistence of the factors we take guidance from existing studies reporting that financial cycles are considerably longer than business cycles and that business and financial cycles tend to be correlated at a medium-term cycle

[6] For example, the researcher might have prior information from theoretical models or previous empirical results on the size and/or sign of a relationship between variables and hence a model coefficient. By means of Bayesian Estimation the researcher can then put a higher probability on those regions of the distribution that support this prior information.

[7] As such, the scale parameter is expressed as a fraction of the sample size T and can be interpreted as the number of "fictitious" observations used to to construct the prior belief σ_0^2.

length (Drehmann, Borio, and Tsatsaronis, 2012; Rünstler and Vlekke, 2018; Winter, Koopman, and Hindrayanto, 2022). The prior on the first lag of each factor is much less informative. Here we set $\phi_1^{MF} \sim \mathcal{N}(1.3, 0.5)$ for the first lag of the macro-financial factors, $\phi_1^{M} \sim \mathcal{N}(1.15, 0.5)$ for the macro factors and $\phi_1^{F} \sim \mathcal{N}(1.7, 0.5)$ for the financial factors.

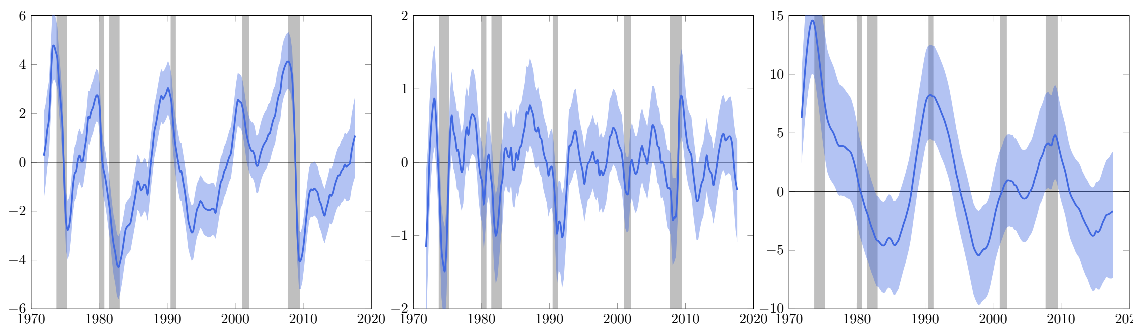
- **Slope and lag of the Phillips curve, γ_i, δ_i:** The slope of the Phillips curve is often found to be very small and statistically insignificant (e.g. see Roberts, 2006; Mishkin, 2007). We set a prior of $\gamma_i \sim \mathcal{N}(0.5, 1)$. For the coefficient on the first lag of inflation we set $\delta_i \sim \mathcal{N}(0, 1)$.

3.3 Empirical results

3.3.1 Estimates of common factors in G7 countries

Figure 3.1 displays the posterior median estimates for the common international factors along with their 90 % probability coverage intervals.[8] The turning points of the factors coincide with the NBER recession periods for the US economy which may be indicative for the world economy.

Figure 3.1: Estimated common factors (median) and 90 % coverage interval.



(a) common macro-financial **(b) common macro** **(c) common financial**

Notes: Grey shaded areas indicate NBER recessions.

First, we note that the macro-financial factor in panel (a) tends to peak prior to the beginning of a recession, while the peak of the financial factor shown in panel (c) is observed somewhat later. Second, the macro-financial factors appears to be large and positive prior to recession periods pointing to periods of overheating during boom phases.[9] Third, the amplitude of the common macro factor is fairly

[8] Cyclical fluctuations which are not common to all G7 countries are captured by country-specific macro-financial, macro, and financial factors reported in Appendix 3.C.

[9] Recall that the common factors are identified via the common deviations of the variables from their respective trends and thus indicate measure of imbalances by construction.

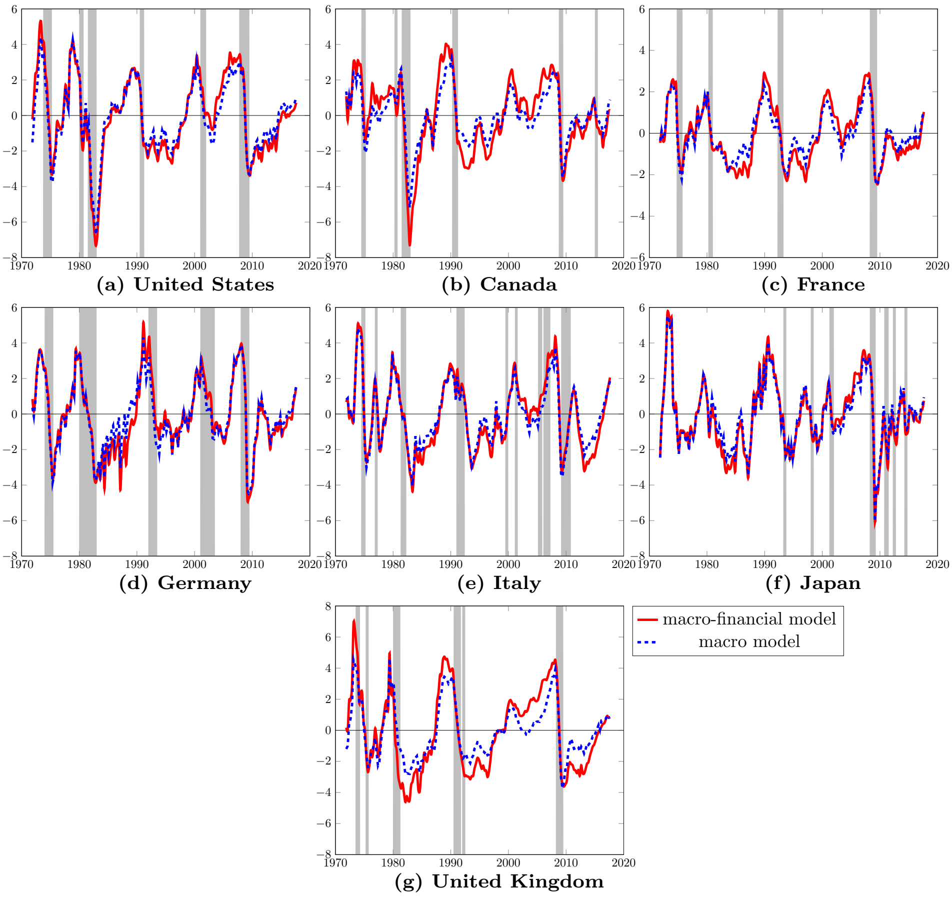
muted, though its dynamics also tends to match the NBER reference cycle. Recall that the common macro factor picks up the remaining co-movement in real activity variables across countries which is not picked up by the macro-financial factor. We interpret this as indicative evidence that the macro-financial factor picks up most of the variation in real activity gaps, a result that we discuss in greater detail in Section 3.3.2. Finally, the common financial factor shown in panel (c) picks up the remaining common fluctuations in credit and house price gaps across countries which are not synchronized with gaps in real activity variables and hence not visible in the macro-financial factor. Recall that we take guidance from the literature and assume a higher degree of persistence for the financial factor as compared to the macro factor with the macro-financial factor exhibiting a persistence in between those two. The observation that financial cycles tend to be more persistent than business cycles has emerged as a stylized fact in the literature (e.g., see Claessens, Kose, and Terrones, 2011b; Borio, 2012; Drehmann, Borio, and Tsatsaronis, 2012; Rünstler and Vlekke, 2018; Berger, Richter, and Wong, 2022). While we control the duration of the cycles with prior information on their persistence, we do not control the amplitude as we do not impose strong prior information on the size of the shock to the factors. Still, we can confirm a considerably larger amplitude of financial cycles as compared to business cycles, which is a common finding from the literature.

3.3.2 Discussing the importance of macro-financial linkages for the business cycle

We now turn to assessing the role of macro-financial linkages for the size of the output gaps in G7 countries, which we address from two perspectives. First, we ask whether the inclusion of financial variables alters the estimated output gap, a question that has received great attention in the literature (Borio, Disyatat, and Juselius, 2014, 2017). Second, we quantify the relative importance of the common and country-specific (and other) factors for explaining the variation in the output gap of a country, which is our measure of the business cycle, with the tool described in Section 3.2.2.

The (red) solid lines in Figure 3.2 show our estimates of the output gaps in the G7 countries resulting from the full model specification under consideration of real activity and financial variables. Grey shaded areas indicate country-specific recession periods. We document two key observations from Figure 3.2. First, output gap estimates largely coincide with important economic up- and downswings over the last few decades in each country, such as the two oil-crises and subsequent early-1980s recession, the recession of the early-1990s, as well as the boom and bust phase surrounding the Great Recession.

Figure 3.2: Posterior median estimates of the output gaps.



Notes: Different lines indicate estimates from (i) the full model specification including macroeconomic and financial variables (red solid) and (ii) output gap estimates implied by the trend-cycle decomposition using only macroeconomic variables. Grey-shaded areas indicate recessions which are taken from the NBER for the United States, the German Council of Economic Experts (GCEE) for Germany and the French Business Cycle Dating Committee for France. For the remaining countries a recession is defined as two consecutive quarters of negative real GDP growth.

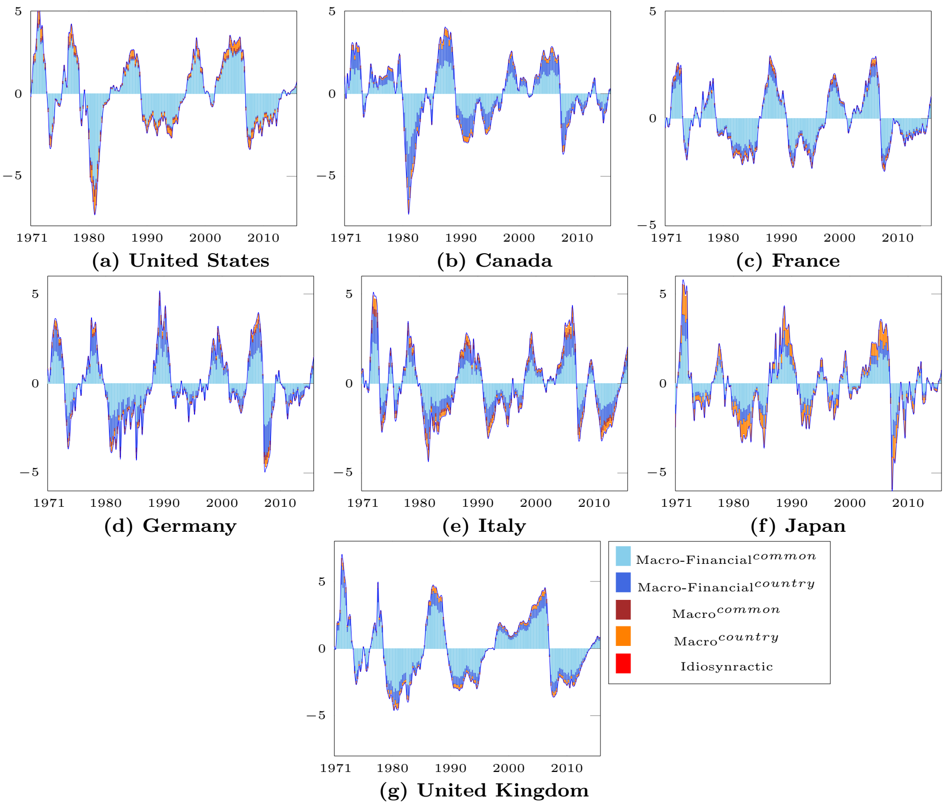
Second, output gap estimates obtained from our full set of real activity and financial variables do not differ considerably from estimates obtained with a reduced set of variables where we only consider real activity variables (blue dotted lines in Figure 3.2). In most countries and for most of the sample output gaps are almost equal regardless of the underlying set of variables. This relates to the natural question raised in the beginning of this section, asking whether the inclusion of financial variables alters the size of the output gap. This notion has been pushed by the finance-neutral output gap literature arguing that the inclusion of financial variables would be important to detect the overheating in the macroeconomy prior to the global financial crisis (e.g., see Borio, Disyatat, and Juselius, 2014, 2017). Berger, Richter, and Wong (2022) argue that this would have non-trivial consequences for both the interpretation and the measurement of the output gap, as financial variables would be required in every (multivariate) output gap estimation regardless of whether or not financial variables are of direct interest for the research question. Furthermore, this would question most of the existing approaches used for estimating the output gap like for example the production function approach widely used in policy circles. Instead, our results suggest that for most of the time the inclusion of financial variables does not matter for the measurement of the output gaps in all G7 countries. However, in some countries, predominantly the United States, Canada and the United Kingdom, there is a short period just before the Great Recession where financial variables seem to have additional information for the output gap beyond those contained in real activity variables.

Empirical results for the variance decomposition of the output gap

Furthermore, we quantify the role of macro-financial linkages as estimated by the common and country-specific macro-financial cycles for the output gap in G7 countries. As we describe in Section 3.2.2, our tool is the variance decomposition widely used in factor analysis exploiting the fact that the stationary gaps in real activity and financial variables can be decomposed into contributions from the several factors (plus an idiosyncratic part) in our model. Figure 3.3 shows this decomposition of output gap estimates. From Figure 3.3 we note that the results suggest an overwhelming contribution of the common macro-financial factor followed by a somewhat smaller contribution of the country-specific macro-financial factors to the size of the output gaps of all countries in our sample. In turn this might suggest that pure macro factors (common or country-specific) seem to play almost no role in explaining the output gap. While one might conclude from this that especially common international macro-financial linkages are of great importance for the interpretation of the output gaps in G7 countries, it is important to have a deeper look at the underlying full variance decomposition of all time series.

The full variance decomposition of the gaps of all real activity and financial variables shown in Table 3.1 provides further insights on the role of the variables for the dynamics of the factors. It reveals that the macro-financial factors (common and country-specific) are first and foremost identified via the real activity variables as the variation in the gaps of the financial variables is only to a small degree related to the macro-financial factors in most countries. For example, in the United

Figure 3.3: Variance decomposition of estimated output gaps



Notes: Variance shares for the different factors identified in the baseline model according to the common components of the factor model. International and domestic macro-financial cycle (light blue and dark blue areas) and international and domestic business cycle (brown and orange areas).

States, the common macro-financial factor explains almost 0 % of the variation in the credit gap and housing gap. These numbers are similar for most countries except for France, Japan, and the United Kingdom where variations in either the credit gap or the housing gap are related to the common macro-financial cycle. These findings suggest that the linkages between the gaps in real activity and financial variables are estimated to be rather weak. However, the variance decomposition in Table 3.1 also shows strong co-movement across countries the gaps of credit and house prices as indicated by a sizable relative importance of the common international financial factor for these variables. This confirms previous findings in the literature documenting the existence of global cycles in credit and house prices (e.g., see Hirata, Kose, Otrok, and Terrones, 2013; Jordà, Schularick, Taylor, and Ward, 2019; Potjagailo and Wolters, 2020).

Table 3.1: Variance shares of estimated gaps for all factors

Contribution of factors to cyclical variation in G7 countries (in %)								
		Common Factors			**Country Factors**			**Idiosyn-cratic**
	Gaps	MF	M	F	MF	M	F	
US	**GDP**	75.5	5.6		3.6	13.4		0.8
	Consumption	81.4	10.8		2.1	3.1		1.4
	Investment	81.3	1.1		0.1	16.2		0.3
	Credit	0.6		35.6	0.2		63.3	0.0
	House Prices	0.2		34.9	0.4		63.9	0.2
CN	**GDP**	49.8	0.4		37.9	10.0		0.3
	Consumption	50.1	2.5		38.0	0.3		7.6
	Investment	49.6	0.8		48.1	0.3		0.1
	Credit	0.3		34.0	0.2		64.1	0.5
	House Prices	6.2		38.6	24.9		13.4	0.1
FR	**GDP**	69.3	4.8		17.6	6.2		0.2
	Consumption	55.2	1.9		26.6	6.7		5.3
	Investment	79.9	4.6		9.9	0.4		2.3
	Credit	0.4		63.7	0.2		34.9	0.1
	House Prices	37.3		32.7	1.6		11.8	3.1
BD	**GDP**	49.9	5.7		33.9	8.1		0.1
	Consumption	10.5	3.8		61.3	1.3		19.3
	Investment	43.7	0.8		43.6	0.7		7.4
	Credit	0.3		54.2	0.2		44.1	0.3
	House Prices	1.5		62.8	2.0		17.6	4.5
IT	**GDP**	45.6	8.1		31.1	12.0		0.2
	Consumption	28.8	0.8		64.1	1.5		1.0
	Investment	53.5	2.2		38.0	2.9		0.3
	Credit	0.6		42.0	0.4		55.3	0.2
	House Prices	1.3		85.5	1.9		7.0	0.1
JP	**GDP**	47.3	0.4		20.6	29.3		0.3
	Consumption	11.1	0.3		76.3	6.0		4.2
	Investment	60.4	0.4		12.3	22.8		0.3
	Credit	0.6		74.4	2.5		20.7	0.3
	House Prices	45.6		19.5	2.4		22.2	0.6
UK	**GDP**	69.5	0.3		20.4	7.8		0.4
	Consumption	39.6	2.7		53.9	0.6		1.4
	Investment	84.8	0.3		3.8	0.4		8.9
	Credit	1.0		72.2	0.4		25.3	0.1
	House Prices	21.9		44.7	12.6		8.9	0.6

Notes: Variance shares measure the contribution of each factor to the cyclical variation in each variable. Median posterior shares in percent. While the factors are assumed to be orthogonal, this is not imposed in the estimation procedure. This might result in some correlation of the factors in finite sample such that variance shares do not add to 100 %. For the common factors we find a almost zero correlation of -0.06 between the macro-financial and the macro factor and a correlation of 0.46 between the macro-financial and the financial factor. US - United States, CN - Canada, FR - France, BD - Germany, IT - Italy, JP - Japan, UK - United Kingdom.

3.4 Conclusions

The global financial crisis of 2008-09 has highlighted the close links between financial markets and the real economy and has shown how financial market imbalances can spill over to the real economy. This lead to renewed interest among academics and policy institutions in understanding this connection. Recent contributions from the empirical literature suggest that: (i) there is a link between business and financial cycles (e.g. Claessens, Kose, and Terrones, 2012) particularly at medium-term frequencies (e.g. Rünstler and Vlekke, 2018; Winter, Koopman, and Hindrayanto, 2022); (ii) business and financial cycles are synchronized across countries (e.g. Rey, 2015; Kose, Otrok, and Whiteman, 2008; Hirata, Kose, Otrok, and Terrones, 2013; Winter, Koopman, and Hindrayanto, 2022); and (iii) the inclusion of financial imbalances may alter the output gap (Borio, Disyatat, and Juselius, 2017; Furlanetto, Gelain, and Taheri Sanjani, 2021).

In this paper we build on these findings and jointly estimate business and financial cycles and macro-financial linkages across the set of G7 countries. In distinction to the existing literature, our model is within the class of Unobserved Components (UC) models and starts from a trend-cycle decomposition of real activity and financial variables where we embed a hierarchical dynamic factor model to estimate common and country-specific macro-financial linkages, as well as pure business and financial factors.

Our main findings are the following. First, we document the existence of common dynamics in real activity and financial cycles across G7 countries. First, we find a high synchronization of real activity variables at the medium-term frequency and of financial variables which seem to share common dynamics at a longer duration and exhibit a higher amplitude. However, we do not find convincing evidence of a common macro-financial factor. Second, our results show that the inclusion of financial variables in the output gap estimation actually does not alter the output gap, providing further evidence to the "measurement" vs. "interpretation" discussion raised in Berger, Richter, and Wong (2022). This result is in contrast to the finance-neutral output gap literature (e.g. Borio, Disyatat, and Juselius, 2017). One implication of this finding is that financial variables are not necessarily needed for a reasonable estimation of the output gap in general. A second implication highlights the cross country similarities in business and financial cycles possibly dampening the impact of national stabilization policies.

We have taken a first step into analyzing the relevance of common macro-financial linkages across advanced economies. While we don't find strong evidence in favor of the existence of these linkages, we think this still provides an interesting starting point for future research. For example, one might choose an alternative specification for the factor model which allows for spillovers across the factors. Indeed, Ha, Kose, Otrok, and Prasad (2020) document the existence of sizable spill-over effects from international financial factors to a common business cycle in G7 countries in a dynamic factor model that allows for a VAR structure and hence for spill-over effects between the factors. This could suggest that international similarities in real activity and financial cycles are rather the result of spill-over effect than that of common shocks.

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Appendix

3.A The model in state space form

The model can be cast into state space form. Bold small letters indicate vectors and capital letters indicate matrices.
Observation equation

$$\begin{pmatrix} \mathbf{y_t} \\ \pi_\mathbf{t} \end{pmatrix} = \begin{pmatrix} I & 0 & B & 0 \\ 0 & 0 & A_g & 0 \end{pmatrix} \begin{pmatrix} \mathbf{w_t} \\ \mathbf{d_t} \\ \mathbf{f_t} \\ \mathbf{f_{t-1}} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ A_p & 1 - A_p \end{pmatrix} \begin{pmatrix} \pi_\mathbf{t-1} \\ \pi_\mathbf{t-2,4} \end{pmatrix} + \begin{pmatrix} \varepsilon^\mathbf{y}_\mathbf{t} \\ \varepsilon^\pi_\mathbf{t} \end{pmatrix}, \tag{3.9}$$

where A_g and A_p are the matrices holding the Phillips curve coefficients. Since we decompose the output gap c^{gdp}_{it} into the different factors, the Phillips curve becomes

$$\pi_{it} = \delta_i \pi_{i,t-1} + (1-\delta_i)\pi_{i,t-2,4} + \gamma_i \underbrace{\left(b^{MF}_{i,gdp} f^{MF}_t + b^M_{i,gdp} f^M_t b^{MFc}_{i,gdp} f^{MFc}_{it} + b^{Mc}_{i,gdp} f^{Mc}_{it}\right)}_{c^{gdp}_{it}} + \varepsilon^\pi_{it}$$

or in matrix notation

$$\pi_\mathbf{t} = A_p \pi_\mathbf{t-1} + (1 - A_p)\pi_\mathbf{t-2,4} + A_g \mathbf{f_t} + \varepsilon^\pi_\mathbf{t}.$$

The shocks in the observation equations are mutually uncorrelated

$$\begin{pmatrix} \varepsilon^\mathbf{y}_\mathbf{t} \\ \varepsilon^\pi_\mathbf{t} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} diag(\sigma^2_y) & 0 \\ 0 & diag(\sigma^2_\pi) \end{pmatrix} \right),$$

where σ_y^2 is of dimension N_{K*C} and σ_π^2 of dimension N_C State equation

$$\begin{pmatrix} \mathbf{w_t} \\ \mathbf{d_t} \\ \mathbf{f_t} \\ \mathbf{f_{t-1}} \end{pmatrix} = \begin{pmatrix} I & I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & \Phi_1 & \Phi_2 \\ 0 & 0 & I & 0 \end{pmatrix} \begin{pmatrix} \mathbf{w_{t-1}} \\ \mathbf{d_{t-1}} \\ \mathbf{f_{t-1}} \\ \mathbf{f_{t-2}} \end{pmatrix} + \begin{pmatrix} \eta_\mathbf{t}^\mathbf{w} \\ \eta_\mathbf{t}^\mathbf{d} \\ \eta_\mathbf{t}^\mathbf{f} \\ 0 \end{pmatrix} \tag{3.10}$$

Shock to the state equation are mutually uncorrelated as well

$$\begin{pmatrix} \eta_\mathbf{t}^\mathbf{w} \\ \eta_\mathbf{t}^\mathbf{d} \\ \eta_\mathbf{t}^\mathbf{f} \\ 0 \end{pmatrix} \sim \mathcal{N}(0, \Sigma),$$

where Σ is a diagonal matrix with σ_w^2 of dimension N_{K*C}, σ_d^2 of dimension N_{K*C} and σ_f^2 of dimension N_M on the diagonal. Note that the dimension of Σ is smaller than the number of state equations, since not all state equations contain shocks. All shock in the observation equation are uncorrelated with the shocks in the state equations.

3.B Details of the Gibbs sampler

In the following, we outline the detailed steps of the Gibbs sampler to simulate draws from the posterior distribution of parameters and latent states implied by the model in Equations (3.1)-(3.5). We denote $y_i^k = \{y_{it}^k\}_{t=1}^T$ as the vector of observed time series stacked over time, $y_i = \{y_i^k\}_{k=1}^K$ stacked over variables n, and $y = \{y_i\}_{i=1}^C$ stacked over all countries. Similarly, let w, d, and F denote the stacked vectors of unobserved trends, drifts, and dynamic factors, respectively and collect the parameters in the vector $\theta = (b, \delta, \gamma, \phi, \sigma^2)$. Collect the parameters in the vector $\theta = (b, \phi_1^m, \phi_2^m, \sigma^2)$. where $b = \{b_{i,k}^{MF}, \ldots, b_{C,K}^{MF}, b_{i,k}^{M}, \ldots, b_{C,K}^{M}, b_{i,k}^{F}, \ldots, b_{C,K}^{F}, b_{i,k}^{MFc}, \ldots, b_{C,K}^{MFc}, b_{i,k}^{Mc}, \ldots, b_{C,K}^{Mc}, b_{i,k}^{Fc}, \ldots, b_{C,K}^{Fc}\}$, $\delta = \{\delta_1, \ldots, \delta_C\}$, $\gamma = \{\gamma_1, \ldots, \gamma_C\}$, $\phi = \{\phi_1^m, \ldots, \phi_M, \phi_2^m, \ldots, \phi_2^M\}$, $\sigma^2 = \{\sigma_{y,i,k}^2, \ldots, \sigma_{y,C,K}^2, \sigma_{\pi,i,k}^2, \ldots, \sigma_{\pi,C,K}^2, \sigma_{w,i,k}^2, \ldots, \sigma_{w,C,K}^2, \sigma_{d,i,k}^2, \ldots, \sigma_{d,C,K}^2, \sigma_{f,m}^2, \ldots, \sigma_{f,M}^2\}$.

1. Sample the unobserved states (w, d, F) according to the state space model in Equations (3.9)-(3.10) from $f(w, d, F|y, \theta)$ with the Kalman filter using the simulation smoother of Durbin and Koopman (2002) implemented as explained in Jarociński (2015).

2. Once we have estimated the unobserved states, we can treat them as known and observed variables such that the observation and state equations simply collapse to a series of independent linear regressions. Hence, conditioning on the states from step 1., we estimate the constant parameters, i.e. factor loadings, AR-coefficients, Phillips curve coefficients and variance parameters, from a set of linear regressions of the form

$$y = X\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2). \tag{3.11}$$

   We use independent priors for β and σ^2 as described in Section 3.2.3. We follow the standard literature and draw σ^2 from the posterior Inverse-gamma distribution $IG \sim (c_1, C_1)$ and β from the posterior normal distribution $\mathcal{N}(a_1, V_1)$

where the posterior hyperparameters are (e.g. see Kim and Nelson, 1999)

$$c_1 = c_0 + \frac{T}{2} \tag{3.12}$$

$$C_1 = C_0 + \frac{1}{2}(y - X\beta)'(y - X\beta) \tag{3.13}$$

$$a_1 = \left(V_0^{-1} + \frac{1}{\sigma^2}X'X\right)^{-1}\left(V_0^{-1}a_0 + \frac{1}{\sigma^2}X'y\right) \tag{3.14}$$

$$V_1 = \left(V_0^{-1} + \frac{1}{\sigma^2}X'X\right)^{-1} \tag{3.15}$$

We draw the parameters from the above described posterior distribution in the following order:

2.1 Conditioning on the unobserved states, draw the factor loadings $b_{i,k}^m$ and the innovation variances of the observation equation $\sigma_{y,i,k}^2$.

2.2. Conditioning on the unobserved states, draw the innovation variances $\sigma_{w,i,k}^2$ and $\sigma_{d,i,k}^2$ for the measurement equation describing the trend level and time-varying drift components in Equations (3.3) and (3.4).

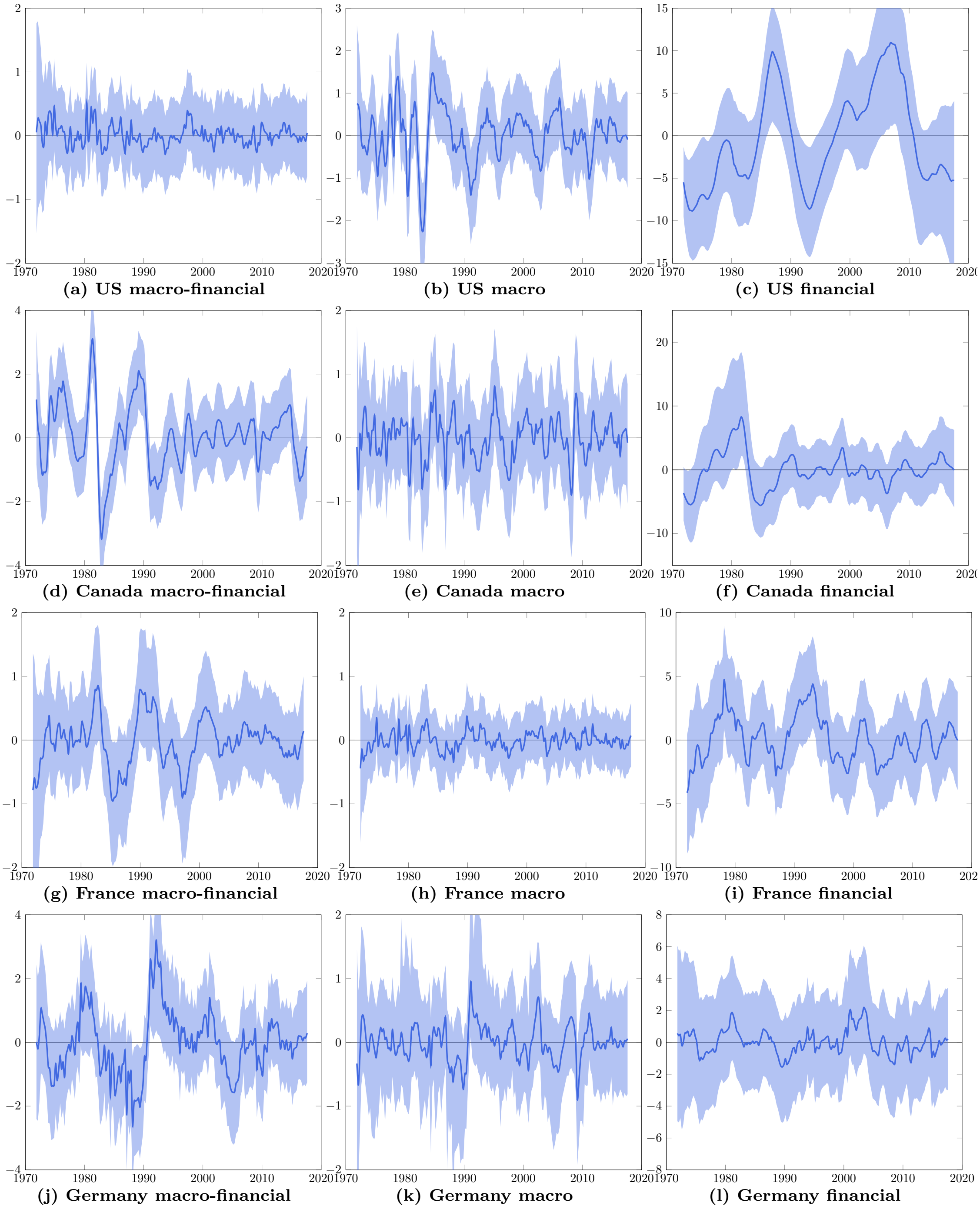
2.3 Draw the parameters of the Phillips curve δ_i and γ_i and the innovation variances $\sigma_{\pi,i}^2$.

2.4 Draw the parameters $\phi^m = (\phi_1^m, \phi_2^m)$ for the second-order autoregressive factors f_t^m as described by Equation (3.5) and the factor innovation variances $\sigma_{f,m}^2$.

3.C Additional results

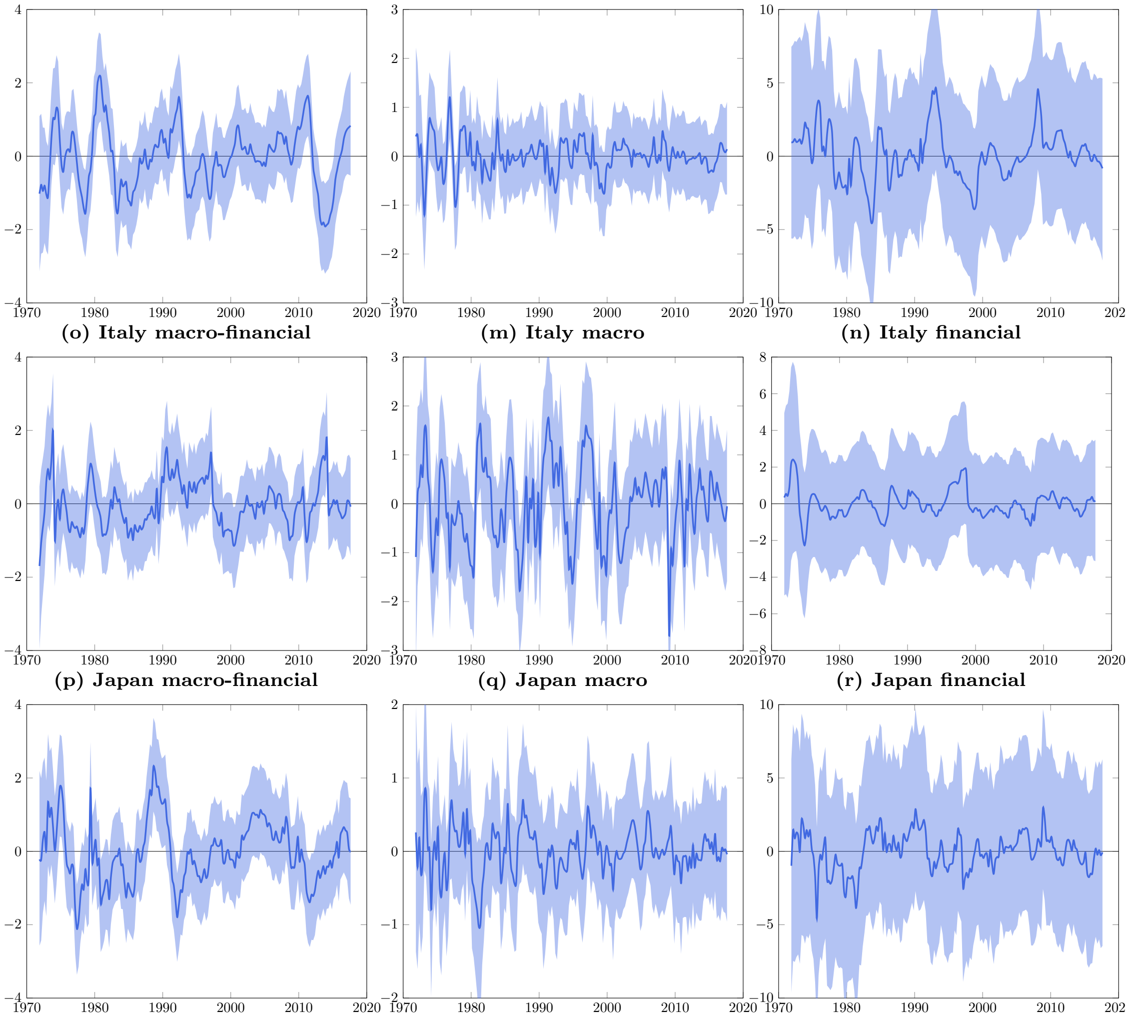
Country-specific factors

Figure 3.C.1: Posterior median and 90 % probability coverage interval of country-specific factors.



(a) US macro-financial (b) US macro (c) US financial
(d) Canada macro-financial (e) Canada macro (f) Canada financial
(g) France macro-financial (h) France macro (i) France financial
(j) Germany macro-financial (k) Germany macro (l) Germany financial

Figure 3.C.2: Posterior median and 90 % probability coverage interval of country-specific factors (cont.)



(o) Italy macro-financial **(m) Italy macro** **(n) Italy financial**

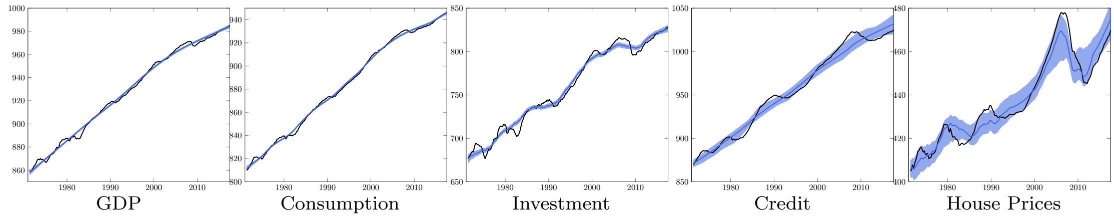
(p) Japan macro-financial **(q) Japan macro** **(r) Japan financial**

(u) UK macro-financial **(s) UK macro** **(t) UK financial**

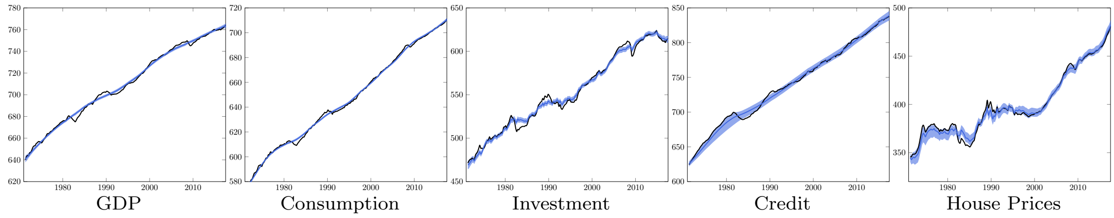
Trends

Figure 3.C.3: Estimated posterior median of the trends and 90 % coverage interval and data

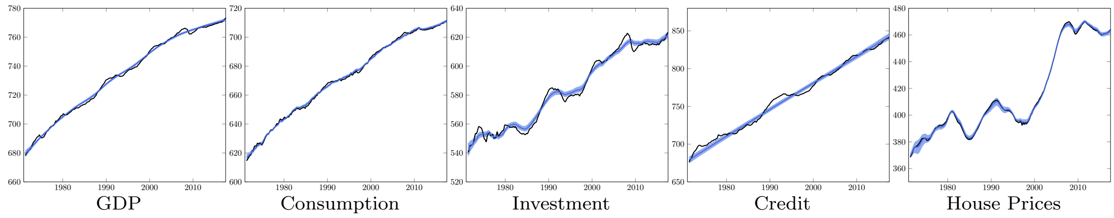
(a) United States



(b) Canada



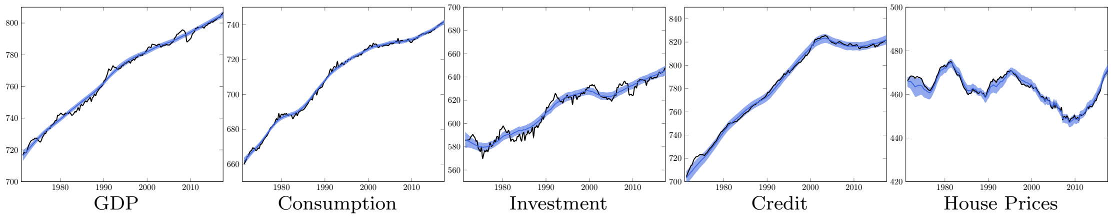
(c) France



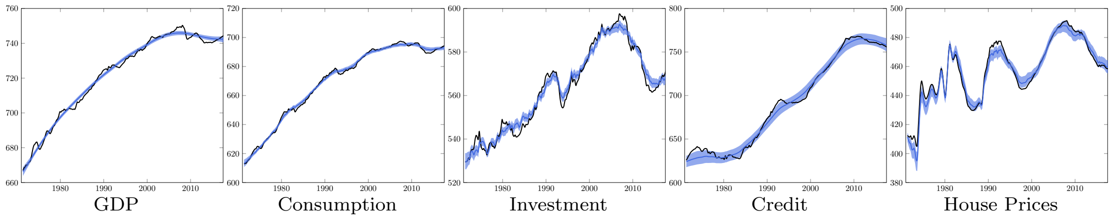
Notes: Original data expressed in log of the series multiplied by 100.

Figure 3.C.4: Estimated posterior median of the trends and 90 % coverage interval and data (ctd.)

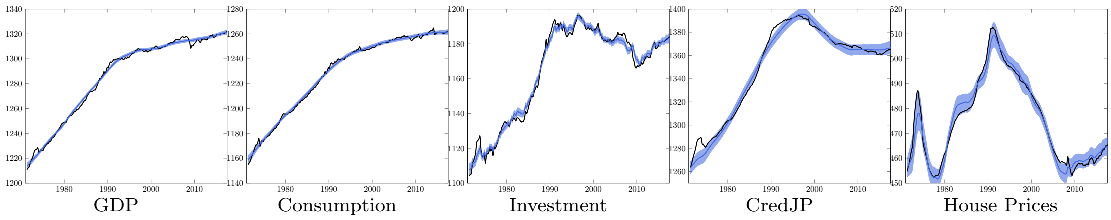
(a) Germany



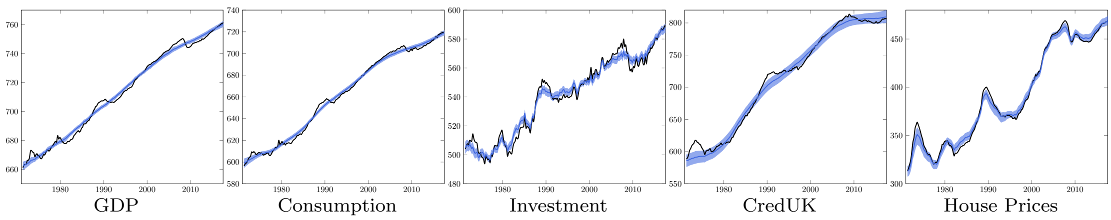
(b) Italy



(c) Japan



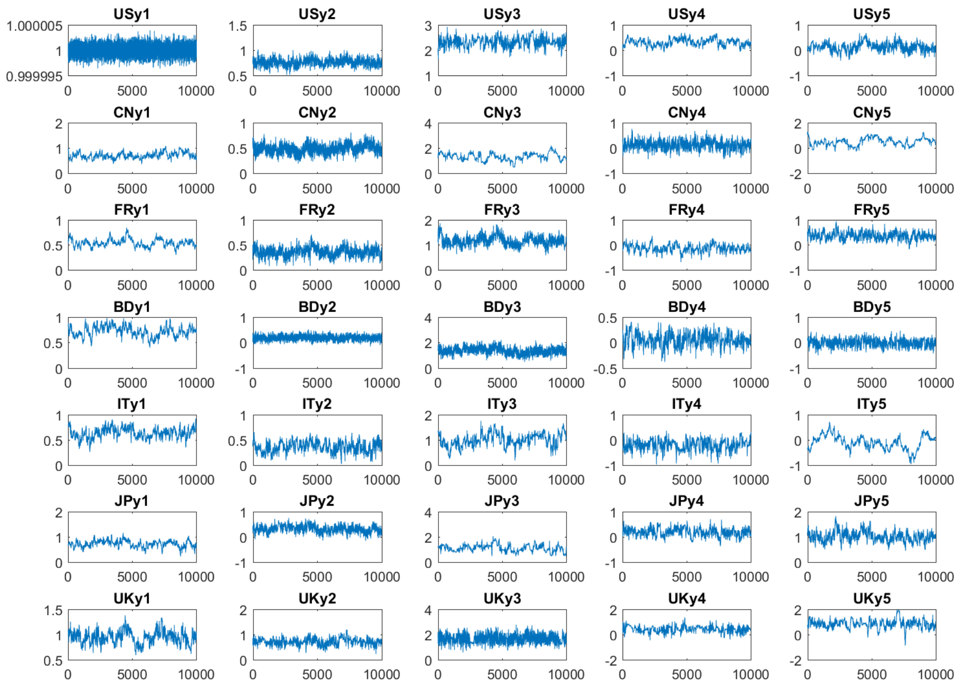
(d) United Kingdom



Notes: Original data expressed in log of the series multiplied by 100.

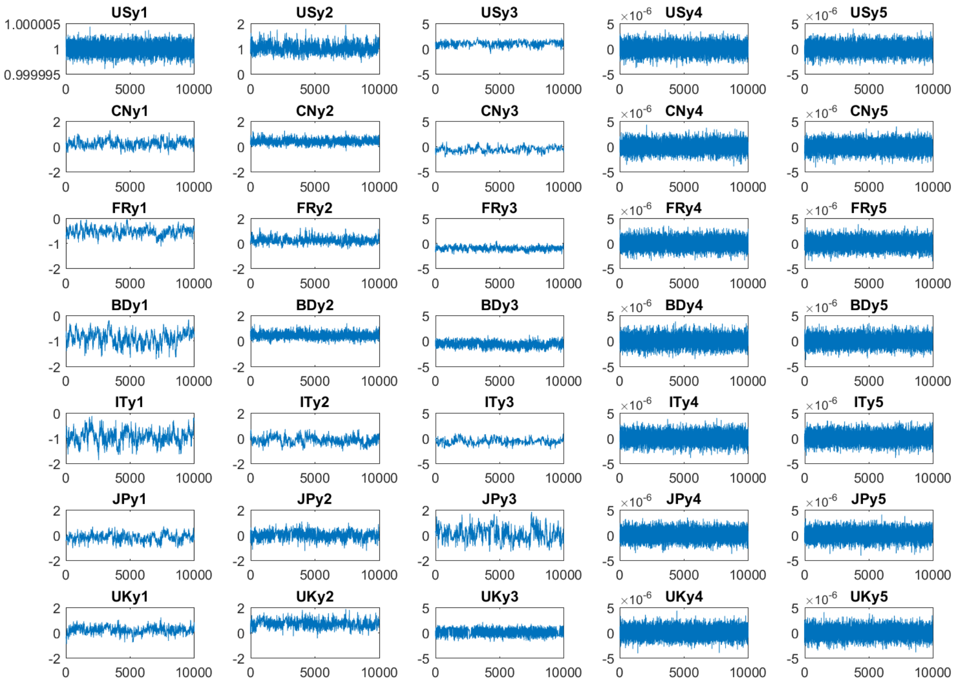
3.D Convergence of the Gibbs sampler

Figure 3.D.1: Trace plots for MCMC draws for the factor loadings on the *common macro-financial factor*



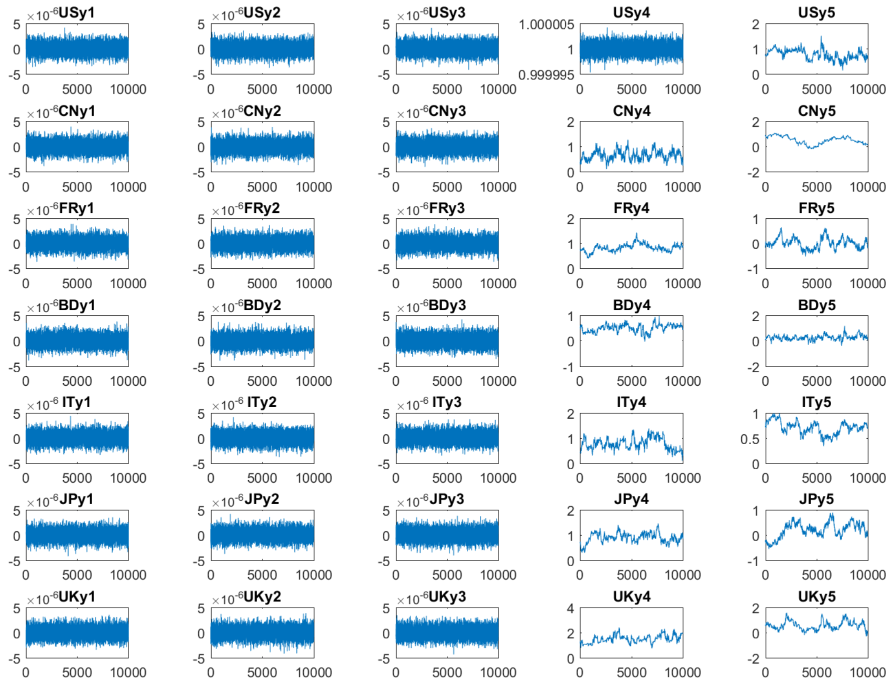
For the retained 10.000 MCMC draws from the posterior. $y1$ - GDP, $y2$ - consumption, $y3$ - investment, $y4$ - credit, $y5$ - house prices. $b_{US}^{gdp} = 1$ for identification.

Figure 3.D.2: Trace plots for MCMC draws for the factor loadings on the *common macro factor*



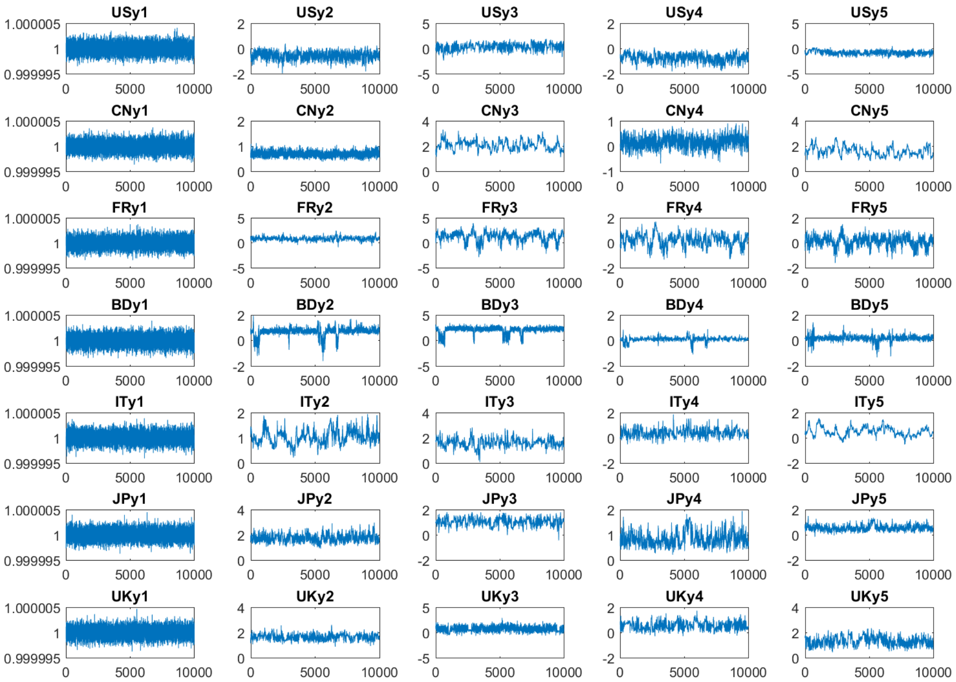
For the retained 10.000 MCMC draws from the posterior. $y1$ - GDP, $y2$ - consumption, $y3$ - investment, $y4$ - credit, $y5$ - house prices. $b_{US}^{gdp} = 1$ for identification. Financial variables do not load on the macro factor and are set to 0.

Figure 3.D.3: Trace plots for MCMC draws for the factor loadings on the *common financial factor*



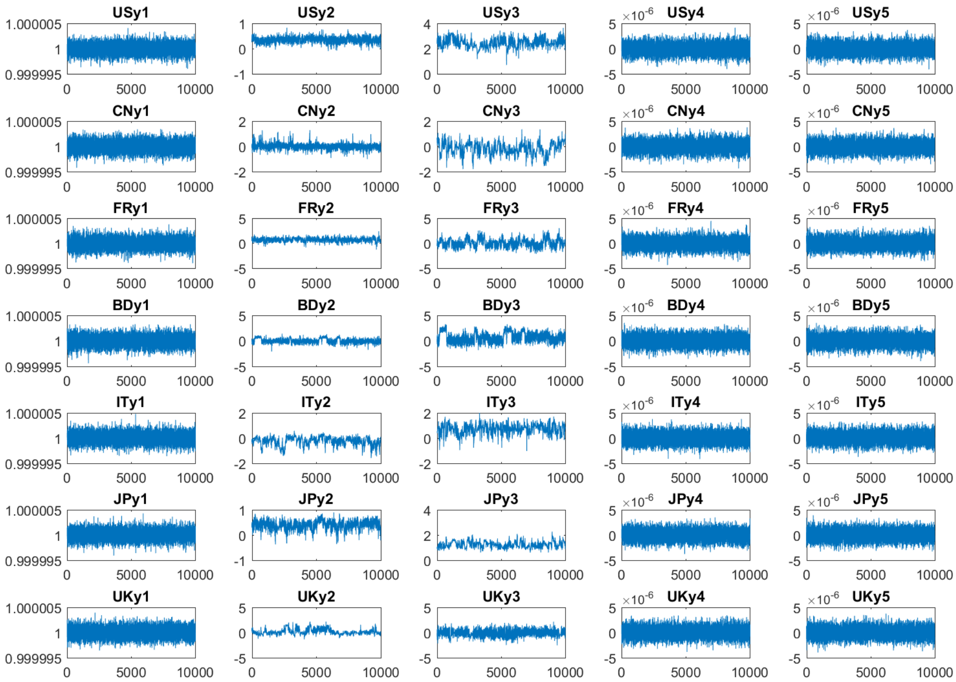
For the retained 10.000 MCMC draws from the posterior. $y1$ - GDP, $y2$ - consumption, $y3$ - investment, $y4$ - credit, $y5$ - house prices. $b_{US}^{credit} = 1$ for identification. Real activity variables do not load on the financial factor and are set to 0.

Figure 3.D.4: Trace plots for MCMC draws for the factor loadings on the *country-specific macro-financial factor*



For the retained 10.000 MCMC draws from the posterior. $y1$ - GDP, $y2$ - consumption, $y3$ - investment, $y4$ - credit, $y5$ - house prices. $b_i^{gdp} = 1$ for identification.

Figure 3.D.5: Trace plots for MCMC draws for the factor loadings on the *country-specific macro factor*



For the retained 10.000 MCMC draws from the posterior. $y1$ - GDP, $y2$ - consumption, $y3$ - investment, $y4$ - credit, $y5$ - house prices. $b_i^{gdp} = 1$ for identification. Financial variables do not load on the macro factor and are set to 0.

Figure 3.D.6: Trace plots for MCMC draws for the factor loadings on the *country-specific financial factor*

For the retained 10.000 MCMC draws from the posterior. $y1$ - GDP, $y2$ - consumption, $y3$ - investment, $y4$ - credit, $y5$ - house prices. $b_i^{credit} = 1$ for identification. Real activity variables do not load on the financial factor and are set to 0.

4 A Unified Approach for Jointly Estimating the Business and Financial Cycle, and the Role of Financial Factors

with Tino Berger and Benjamin Wong
published in the
Journal of Economic Dynamics and Control

Abstract

We jointly estimate the U.S. business and financial cycle through a unified empirical approach which also simultaneously quantifies the role of financial factors. Our approach uses the Beveridge-Nelson decomposition within a medium-scale Bayesian Vector Autoregression. First, we show, both in reduced form and when we identify a structural financial shock, that variation in financial factors had a larger role post-2000 and a more modest role pre-2000. Our results suggest that the financial sector did play a role in overheating the business cycle pre-Great Recession. Second, while an identified financial shock can generate a negative correlation between the lagged credit cycle and the contemporaneous output gap, the *unconditional* correlation between the credit cycle and the output gap is still positive. The latter at least suggests that one should be careful in associating an increase in the financial cycle to bust in the business cycle.

JEL Classification: C18, E51, E32
Keywords: Business Cycle, Financial Cycle, Financial shocks

Journal of Economic Dynamics & Control 136 (2022) 104315

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

A unified approach for jointly estimating the business and financial cycle, and the role of financial factors ☆

Tino Berger [a], Julia Richter [a,b], Benjamin Wong [c,*]

[a] Georg-August University Göttingen, Germany
[b] German Council of Economic Experts, Germany
[c] Monash University, Australia

ARTICLE INFO

Article history:
Received 30 October 2021
Revised 17 January 2022
Accepted 21 January 2022
Available online 24 January 2022

JEL classification:
C18
E51
E32

Keywords:
Business cycle
Financial cycle
Financial shocks

ABSTRACT

We jointly estimate the U.S. business and financial cycle through a unified empirical approach which also simultaneously quantifies the role of financial factors. Our approach uses the Beveridge-Nelson decomposition within a medium-scale Bayesian Vector Autoregression. First, we show, both in reduced form and when we identify a structural financial shock, that variation in financial factors had a larger role post-2000 and a more modest role pre-2000. Our results suggest that the financial sector did play a role in overheating the business cycle pre-Great Recession. Second, while an identified financial shock can generate a negative correlation between the lagged credit cycle and the contemporaneous output gap, the *unconditional* correlation between the credit cycle and the output gap is still positive. The latter at least suggests that one should be careful in associating an increase in the financial cycle to bust in the business cycle.

1. Introduction

The financial crisis of 2008-09 emphasized how developments in the financial market can spill over into the real economy, highlighting the importance to model and understanding the role of the financial sector and how the financial sector of the economy interacts with the macroeconomy (see Adrian and Shin, 2010, for a review). Within the policy sphere, it is important to understand the business and financial cycle because each is respectively used to understand imbalances in the real economy and financial sector.

The key contribution of our paper is to jointly model the business and financial cycle within a unified empirical approach. Our approach goes beyond just estimating both the business and financial cycle within a common empirical framework.

☆ This paper reflects the personal views of the authors and not necessarily those of the German Council of Economic Experts. A previous version of this paper was circulated titled "Financial Factors and the Business Cycle". We thank an anonymous referee, Francesco Furlanetto, Lutz Kilian, Mariano Kulish, James Morley, Anh D.M. Nguyen, Adrian Pagan, Hélène Rey, and James Yetman, and participants of seminars and conferences for comments and helpful discussions. We also thank Abby Nguyen for providing excellent research assistance. This research was supported by the Australia-Germany Joint Research Co-operation Scheme initiative of Universities Australia and the German Academic Exchange Service (DAAD). Benjamin Wong also acknowledges funding support from the Australian Research Council (DP190100202 and DE200100693). All remaining errors are ours.

* Corresponding author.
E-mail addresses: tino.berger@wiwi.uni-goettingen.de (T. Berger), julia.richter@svr-wirtschaft.de (J. Richter), benjamin.wong@monash.edu (B. Wong).

https://doi.org/10.1016/j.jedc.2022.104315
0165-1889/

Because we allow many variables to simultaneously evolve endogenously within a medium-sized VAR, we are also able to account for how much of the variation in the business and financial cycle can be attributed to financial variables and/or financial shocks. From a broad perspective, ours is a unified approach, to the extent that we can jointly estimate the business and financial cycle as well as account for SVAR work which seeks to identify financial shocks, all within a single framework. Henceforth, we take the (relatively) uncontroversial characterization of the output gap, or the cyclical component of real GDP, as the business cycle, and both the housing and credit cycle, or the cyclical component of house prices and credit, as the financial cycle.

Our key results are as follows. First, it appears that the role of financial factors played for both the output gap and financial cycles were much smaller pre-2000s, its role appears to have been much larger after the 2000s. In particular, our analysis suggests that loose financial conditions did overheat the real economy in the 2000s pre-Great Recession. From our more reduced form analysis, we find that a reasonable share of the positive output gap in the 2000s can be attributed to the excess bond premium, a credit spread constructed by Gilchrist and Zakrajsĕk (2012) to measure credit conditions through capturing the risk-bearing capacity of the financial sector. Our identification exercise also reveals that our identified financial shock added somewhere between 2 to 4% to the output gap in the 2000s. Second, while an identified financial shock can generate a negative correlation between the lagged credit cycle, the *unconditional* correlation between the obtained output gap and credit cycle is positive. Our finding suggests that one should be careful in associating an increase in the financial cycle to bust in the business cycle. Indeed, our work would suggest that the *average* credit boom is not likely associated with a bust in the business cycle.

Our focus on modeling and quantifying the relationship of the business and financial cycle with financial factors is deliberate for at least two reasons. First, policy is often framed through the cyclical component of real activity and financial variables, which are the business and financial cycle respectively. For example, the output gap, or cyclical component of real GDP, is commonly used in policy settings, such as central banks, as being a summary measure of the business cycle, as well as capacity pressures. Similarly, macroprudential policy is also often framed in terms of the cyclical component of financial variables.[1] In such settings, the cyclical component of financial variables is taken to be a signal of financial imbalances and risk (e.g. see Drehmann and Yetman, 2021). Our focus on the cyclical components is thus natural as this is precisely how macroeconomic stabilization and macroprudential policy are formulated. Second, we note that our approach is not unusual given broad segments of the extant literature. For example, an existing strand of the literature shares a similar focus of aiming to understand how financial factors shape the output gap, likely due to the reasons we outlined.[2] We also note that the practice of taking the cyclical component of house prices and credit as the financial cycle is not unusual relative to extant work (e.g., see Aikman et al., 2015; Borio et al., 2017; Rünstler and Vlekke, 2018).

Briefly, our empirical approach builds off Morley and Wong (2020) and involves estimating a medium scale Bayesian Vector Autoregression (BVAR) containing both U.S. macroeconomic and financial variables, and subsequently applying the Beveridge-Nelson (BN) (1981) decomposition to obtain both the output gap and measures of the financial cycle. We emphasize that our approach is unified and internally consistent to the extent that the output gap and financial cycle are obtained from the *same* time series model, namely our BVAR. We stress this is a non-trivial distinction relative to extant methods that first separately obtain the output gap and financial cycle before conducting subsequent analysis (e.g. Aikman et al., 2015; Albuquerque et al., 2015; Claessens et al., 2012), as it is well known how such analysis may be distorted by how one first obtains these cycles (e.g., see Canova, 1998) within the context of the business cycle facts. Moreover, a key aspect of our empirical approach is that, because the output gap and financial cycle are obtained from the same BVAR, interpretation of the output gap and financial cycle are possible through standard VAR objects such as the forecast errors or identified structural shocks. It is the latter feature which will enable us to quantify the role of financial shocks for the output gap by appealing to the broader structural VAR literature (see Caldara et al., 2016; Furlanetto et al., 2019; Gilchrist and Zakrajsĕk, 2012).

We contrast our empirical approach to Borio et al. (2017), Rünstler and Vlekke (2018) and de Winter et al. (2021), which we regard as the closest in spirit to our work with regards to how one might model the relationship of financial factors to the output gap or jointly modeling the business and financial cycle. Borio et al. (2017) use the Hodrick-Prescott (HP) filter as a starting point, and subsequently use credit growth as an exogenous variable after casting the HP filter into state-space form. As it is well known, the HP filter may induce spurious cycles (see Cogley and Nason, 1995; Hamilton, 2018). In contrast, our approach, because it is based upon an explicitly specified time series, cannot, by construction, produce spurious cycles. Moreover, our approach does not treat credit as an exogenous variable in determining the output gap but instead allows real GDP growth, credit growth, and various macroeconomic and financial variables to evolve endogenously. This point is important because to the extent that decisions about granting or seeking credit are a function of how one views the macroeconomy, credit should be an endogenous variable. Work such as Rünstler and Vlekke (2018) and de Winter et al. (2021) use Unobserved Components (UC) models to decompose real GDP, credit, and house prices into trend and cyclical components, and characterize the relationship between the subsequently extracted cyclical components. While UC models arguably are immune to spurious cycles, and thus at least from that perspective can be viewed as an improvement on the approach by Borio et al. (2017), our approach has the advantage of linking variation from the business and financial cycles

[1] For example, macroprudential regulatory frameworks such as Basel III, treat the cyclical component of the credit-to-GDP ratio as the financial cycle.

[2] For example, see Aikman et al. (2015), Borio et al. (2017), Cagliarini and Price (2017), Rünstler and Vlekke (2018), Furlanetto et al. (2021), Constantinescu and Nguyen (2021), de Winter et al. (2021) etc.

through the VAR forecast errors and/or identified financial shocks. It should also be noted, given we use the Beveridge-Nelson (BN) decomposition from a BVAR to obtain the output gap and the financial cycle, the trend and cycle from a BN decomposition and UC models are conceptually linked and identical through the reduced form of the UC model (see Morley et al., 2003). In this regard, our empirical approach is thus conceptually akin to the UC model, except that the use of a BVAR enables us to explicitly identify the role of financial shocks, an option that is unavailable to standard UC models.

Finally, we note that part of our work also relates to broader work on how financial factors alter the output gap, albeit through applying a very different set of tools. In this vein, more structural models such as Furlanetto et al. (2021) re-define the output gap within a DSGE environment where financial frictions are a source of inefficiencies, and thus the output gap also represents inefficiencies stemming from variation in financial frictions. The aforementioned work by Borio et al. (2017) embed financial sector information in conjunction with the Hodrick-Prescott filter to estimate output gaps that are “finance-neutral”. Relative to the more fully structural approach by Furlanetto et al. (2021), our approach has less structure, though we can still conduct a structural identification to quantify the role of the identified financial shock in driving the output gap. Relative to the “finance-neutral” approach, our empirical approach is more flexible and broad-based as we incorporate information from not only financial but also other macroeconomic variables.

The remainder of this paper is organized as follows. Section 2 introduces the empirical framework. Section 3 presents our estimates of the financial and business cycle. Section 4 investigates the role of financial factors in driving both the business and financial cycle. Section 5 considers some robustness issues. Section 6 concludes.

2. Empirical framework

We construct trend and cycle using the Beveridge and Nelson (1981) (BN) decomposition, who define the trend of a time series as its long-horizon conditional expectation minus any future deterministic drift. For a time series $\{y_t\}$ which has a trend that follows a random walk process with a constant drift μ, the BN trend at time t, τ_t, is

$$\tau_t = \lim_{j\to\infty} \mathbb{E}_t\left[y_{t+j} - j\cdot\mu\right]. \tag{1}$$

The cycle of the series at time t, c_t, is then defined as

$$c_t = y_t - \tau_t. \tag{2}$$

The evaluation of the conditional expectation in Equation (1) requires specifying a suitable empirical model. We build on Morley and Wong (2020) by using a medium-sized 23 variable BVAR as our empirical model. Based on the estimates of the empirical model, we then obtain trends and cycles of the various variables within the BVAR. For the business cycle, we take this as the cyclical component of real GDP. Consistent with the labeling in the wider literature and policy circles, we interchangeably refer to the business cycle as the output gap.

Guided by the broader literature, we take the cyclical component of house prices and credit as estimates of the financial cycles, noting our choice of variables to consider for the financial cycle is also consistent with the UC model by Rünstler and Vlekke (2018). While there is less agreement about the variable of interest when measuring the financial cycle, there appears to be an emerging consensus that the cyclical component of house prices and credit embed much of the longer frequency movement that one seeks to isolate when estimating a financial cycle (e.g., see Borio et al., 2014; Galati et al., 2016).[3]

2.1. Decomposition into trends and cycles

Suppose we are interested in detrending K time series, where we denote each of these time series as $y_{i,t}$ where $i \in \{1, 2, \dots, K\}$. Let $\mathbf{x_t}$ be a vector of n variables where $\Delta y_{i,t} \subset \mathbf{x_t}$.[4] We assume that $\mathbf{x_t}$ has a VAR(p) representation with the following companion form:

$$(\mathbf{X_t} - \boldsymbol{\mu}) = \mathbf{F}(\mathbf{X_{t-1}} - \boldsymbol{\mu}) + \mathbf{H}\boldsymbol{e_t}, \tag{3}$$

where $\mathbf{X_t} = \{\mathbf{x}'_\mathbf{t}, \mathbf{x}'_\mathbf{t-1}, \dots, \mathbf{x}'_\mathbf{t-p}\}'$, $\boldsymbol{\mu}$ is the vector of n unconditional means of $\mathbf{x_t}$, $\mathbf{F}$ is the companion matrix with eigenvalues that all are inside the unit circle, $\mathbf{H}$ maps the VAR forecast errors to the companion form, and $\boldsymbol{e_t}$ is a vector of serially uncorrelated forecast errors with covariance matrix $\boldsymbol{\Sigma}$. Denoting $\tau_{i,t}$ and $c_{i,t}$ as respectively the BN trend and cycle of the series $y_{i,t}$,

$$y_{i,t} = \tau_{i,t} + c_{i,t}. \tag{4}$$

[3] Drehmann et al. (2012) argue that the cyclical component of house prices and credit are suitable variables to measure the financial cycle given stock prices appear to have cyclical characteristics that do not accord with what one thinks of a financial cycle. The subsequent adoption by wider work to consider both credit and house price also suggests that their view has been influential in this emerging consensus. Nonetheless, for completeness, we present results for the stock market cycle in Section C of the online appendix.

[4] $\mathbf{x_t}$ can contain variables that are differenced or in levels. The mix of I(1) and I(0) variables does not matter as long as together, $\mathbf{x_t}$ implies a stationary VAR. We only require the variables which we are interested in detrending to be differenced, as we require variables to be I(1) in the levels to apply the BN decomposition.

Let $\mathbf{s_q}$ be a selector row vector with 1 at its q^{th} element, and zero otherwise. Further, let $\Delta y_{i,t}$ be in the k^{th} position of $\mathbf{x_t}$. Applying the definition of the BN decomposition, the cycle, $c_{i,t}$, can be calculated as (see Morley, 2002)

$$c_{i,t} = -\mathbf{s_k F}(\mathbf{I}-\mathbf{F})^{-\mathbf{1}}(\mathbf{X_t}-\boldsymbol{\mu}). \tag{5}$$

Morley and Wong (2020) show that we can further decompose the obtained BN trends and cycles as a function of either the VAR forecast errors or structural shocks. Let $c_{ij,t}$ represent the share of the forecast error of the j^{th} variable in $\mathbf{x_t}$ on the cycle $c_{i,t}$. Similarly, let $\Delta y_{i,t}$ once again occupy the k^{th} position in $\mathbf{x_t}$. Morley and Wong (2020) show that we can write $c_{ij,t}$[5] as

$$c_{ij,t} = -\sum_{l=0}^{t-1}\mathbf{s_k F^{l+1}}(\mathbf{I}-\mathbf{F})^{-\mathbf{1}}\mathbf{Hs_j's_j e}_{t-l}. \tag{6}$$

Equation (6) decomposes the K cycles which we obtain through our VAR into shares of forecast errors of all the n variables contained in $\mathbf{x_t}$. We refer to Equation (6) as the informational decomposition, as it associates fluctuations in the cycles with the information contained within the other variables. At the same time, note that

$$c_{i,t} = \sum_{j=1}^{n} c_{ij,t}, \tag{7}$$

which implies that the obtained cycle from our VAR fully decomposes into the forecast errors of all the n variables contained in $\mathbf{x_t}$. Within our empirical framework, $c_{i,t}$ will represent objects of interest such as the output gap, which will be our measure of the business cycle, and the cyclical component of housing prices and credit, which represents our measure of the financial cycle. Accordingly, we will use the expression in Equation (6) to understand the role of financial variables in driving the output gap by associating fluctuations in the output gap with the forecast errors of the financial variables such as credit, house prices, stock prices, credit spreads, etc.

The decomposition in Equation (6), while informative, does not attach any causal interpretation. Attaching a causal interpretation will require identifying structural shocks. Let $\boldsymbol{\epsilon}_t$ represent a $n \times 1$ vector of orthogonal structural shocks, with the variance normalized to unity, or $\mathbb{E}\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t' = \mathbf{I}$. The structural VAR literature shows that identifying a structural shock requires specifying a mapping

$$\boldsymbol{e}_t = \mathbf{A}\boldsymbol{\epsilon}_\mathbf{t}, \ \text{where } \mathbf{AA}' = \boldsymbol{\Sigma}. \tag{8}$$

Let $c^S_{ij,t}$ be the share of the j^{th} structural shock on $c_{i,t}$. Using the mapping defined by Equation (8), we can substitute in Equation (6) to obtain

$$c^S_{ij,t} = -\sum_{l=0}^{t-1}\mathbf{s_k F^{l+1}}(\mathbf{I}-\mathbf{F})^{-\mathbf{1}}\mathbf{HAs_j's_j}\boldsymbol{\epsilon}_{t-l}. \tag{9}$$

Equation (9) now allows us to interpret the business and financial cycle as a function of orthogonalized shocks, and so allows for a structural or causal interpretation. For our structural analysis, we will identify a financial shock with guidance from the wider empirical literature to understand how financial shocks drive both the business and financial cycle.

We briefly reiterate two points raised in the introduction to remind the reader of our modeling choice. First, our concept of trend and cycle is equivalent to Unobserved Components models as shown by Morley et al. (2003). However, as demonstrated by Morley and Wong (2020), and also Berger et al. (2020) in a nowcasting setting, the key advantage of using a BVAR is that we can directly link fluctuations in the cycles to variation of different variables within the BVAR, thus allowing us to build a richer picture of which financial variables are linked to fluctuations in the output gap. Moreover, Morley and Wong (2020) and Kamber and Wong (2020) show that standard identification tools from the SVAR literature can be easily brought into the empirical framework, a step which will be crucial for considering causality. Second, our empirical approach is immune to spurious cycles, in the Cogley and Nason (1995) and Hamilton (2018) sense, relative to using approaches such as a Hodrick-Prescott or bandpass filter (see Murray, 2003, on spurious cycles in the bandpass case).[6]

2.2. Estimation and data

We estimate a 23 variable BVAR of U.S. macroeconomic and financial variables. The set of variables in our BVAR are real GDP, the CPI, employment, real private consumption, industrial production, capacity utilization, the unemployment rate,

[5] Morley and Wong (2020) also derive analogous expressions for the trends, but as our focus is on the business and financial cycles, we omit discussion about the trends.

[6] A key point emphasized by both Cogley and Nason (1995) and Hamilton (2018) is that if the underlying data generating process was a random walk, the Hodrick-Prescott filter will attribute cycles that are spurious since the underlying time series has no forecastability, and the cycles are thus meaningless or spurious. Since our specification nests a random walk for any differenced variable, our approach will consistently estimate the random walk process for these variables/equations, and so our approach will not fall afoul with the issue of spurious cycles.

housing starts, the producer price index for all commodities, hours worked, nonfarm real output per hour, personal income, real gross domestic investment, the fed funds rate, the 10-year government bond yield, real M1, real M2, total credit to non-financial institutions, the S&P 500 index, real energy prices, the VIX index, real house prices, and the excess bond premium introduced by Gilchrist and Zakrajsěk (2012). Most of the data is sourced from the FRED database over the sample period 1973Q1-2020Q1. Data for the excess bond premium is taken from Gilchrist and Zakrajsěk (2012) and its subsequent updates by the Board of Governors.[7] Most of the variables are standard, motivated in part by the specification of Banbura et al. (2010) and Morley and Wong (2020). We provide details of the precise data source, description, and transformation in Section A of the online appendix.

We briefly note that our choice to work with a 23 variable BVAR is because we require a variable set that spans all the relevant information for both the business and financial cycles. More precisely, Morley and Wong (2020) show that a condition of estimating the true BN cycle is the inclusion of all the relevant forecasting information for the variables from which we are obtaining the BN cycle. At the same time, because we are making inference on the effect of a structural financial shock as part of our analysis, Forni and Gambetti (2014) show that one should include all the information that spans the SVAR shocks. The choice of the 23 variable medium-sized BVAR, as opposed to a more standard smaller six to eight variable VAR, should act as a sufficient guard against omitting relevant information.[8]

Given the rest of the variables are standard, we only comment on the excess bond premium, which was introduced by Gilchrist and Zakrajsěk (2012). The excess bond premium is a credit spread that measures the risk-bearing capacity of financial intermediaries. Faust et al. (2013) show that the inclusion of credit spreads can help with the prediction of real economic activity. This suggests from at least the perspective of both Morley and Wong (2020) and Forni and Gambetti (2014), the inclusion of the excess bond premium, as a credit spread, is necessary as this is relevant information for aiding with the estimation of the output gap, as well as the identification of structural financial shocks. We also note that variation in the excess bond premium also plays a key role in the literature on identifying structural financial shocks (e.g. Caldara et al., 2016; Gilchrist et al., 2009), and so its inclusion within our context would also aid in the identification of structural financial shocks.

Some variables exhibit a break in the mean, implying $\boldsymbol{\mu}$ in Equation (3) has to be adjusted. As shown by Morley and Wong (2020), these breaks in the mean can compromise the BN decomposition, as stationarity requires a variable to be mean-reverting. We thus proceed as follows. We first apply conventional transformations to the variables. To adjust for possible breaks in means, we slightly vary the treatment for the variables for which we are deriving a business or financial cycle, and the other variables.

Drift Adjustment - Business and Financial Cycle Variables For variables that we use to make inferences on the business and financial cycle, a break in the mean implies a break in the drift since these variables are differenced before estimation. Given that the definition of the BN decomposition from Equation (1) depends on the drift, Kamber et al. (2018) show that a break in the drift can play a crucial role in obtaining reliable measures of trend and cycle. We therefore tested the variables associated with the respective financial and business cycles to ensure that the assumption of a constant drift cannot be rejected by a standard Bai and Perron (2003) test.[9] These variables under consideration are real GDP for the business cycle, and credit and house prices for the financial cycles. We found a break in the drift for credit in 2008Q1. This is not entirely surprising as the financial crisis of 2008/09 resulted in not only a stall in credit during the recession, but also a continued flattening of the drift due to financial regulation post-2008 in the aftermath of the crisis, notably resulting from initiatives such as the Basel Accords (notably Basel III). We therefore adjusted for a break in the drift of credit in 2008Q1.

Mean Adjustment - Other Variables For the other variables, our concern is mainly to guard against possible breaks in the mean in compromising our inference of the business and financial cycle. In particular, if there is a break in the mean in the other variables, this may imply excessive persistence instead of a quicker revision to the new (post-break) mean, and this can impart excessive persistence to our estimate of the business and financial cycle.[10] While Morley and Wong (2020) opted to difference variables if there was some evidence of a break in the mean, such an approach might be overly conservative in throwing out useful information in the level. For example, capacity utilization is a variable that exhibits a break in the mean. However, the level of capacity utilization provides a lot of information about the state of the business cycle. By differencing such a variable, we throw out a lot of useful information in the level. Kamber and Wong (2020) thus opted to adjust for breaks in the mean if there was compelling evidence to suggest so, an approach that we adapt to our setting. More precisely, we first test for a difference in the mean between the first and second half of the sample using a two-sample *t*-test, similar to Morley and Wong (2020). If the test rejects the null hypothesis of equal means at the 10% significance level, we follow the procedure by Kamber and Wong (2020) and use a sup-F statistic (see Andrews, 1993) to locate a break in the mean at

[7] See https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/recession-risk-and-the-excess-bond-premium-20160408.html.

[8] Preliminary analysis suggests that a 15 variable BVAR may be informationally sufficient for the output gap, though it is a bit more mixed whether the 15 variable suffices for the financial cycles. Given our Bayesian shrinkage does not impose a large cost of including the additional 8 variables, we work with the 23 variable BVAR.

[9] We tested for the break in the drift by allowing for heteroskedasticity and autocorrelation consistent (i.e. Newey and West, 1987) (HAC) standard errors.

[10] The idea that excessive persistence can result from a break in the mean is not new and has been explored and shown by Perron (1990), amongst other contributions.

an unknown breakpoint and use this unknown breakpoint to adjust for a break in the mean.[11] Details on the breaks are provided in Section A of the online appendix.

The estimation of the BVAR is standard. We utilize the natural-conjugate Normal-Wishart prior which draws on elements of the Minnesota Prior (e.g., see Litterman, 1986; Robertson and Tallman, 1999). Consider the VAR(p) for the vector of variables $\mathbf{x_t}$ which are demeaned before estimation:[12]

$$\mathbf{x_t} = \mathbf{\Phi_1 x_{t-1}} + \cdots + \mathbf{\Phi}_p \mathbf{x_{t-p}} + \boldsymbol{e_t}$$
$$= \begin{bmatrix} \phi_1^{11} & \dots & \phi_1^{1n} & \phi_2^{11} & \dots & \phi_2^{1n} & \dots & \dots & \phi_p^{1n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\ \phi_1^{n1} & \dots & \phi_1^{nn} & \phi_2^{n1} & \dots & \phi_2^{nn} & \dots & \dots & \phi_p^{nn} \end{bmatrix} \begin{bmatrix} \mathbf{x_{t-1}} \\ x_{\mathbf{t-2}} \\ \vdots \\ x_{\mathbf{t-p}} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ \vdots \\ e_{n,t} \end{bmatrix}, \tag{10}$$

where $\mathbb{E}(\boldsymbol{e_t e_t'}) = \mathbf{\Sigma}$ and $\mathbb{E}(\boldsymbol{e_t e_{t-i}'}) = \mathbf{0}\ \forall i > 0$. We then apply shrinkage to the VAR slope coefficients using a Minnesota-type prior specification for the prior means and prior variances as follows:

$$\mathbb{E}[\phi_i^{jk}] = 0 \tag{11}$$

$$Var\left[\phi_i^{jk}\right] = \begin{cases} \frac{\lambda^2}{i^2}, & \text{if } j = k \\ \frac{\lambda^2}{i^2}\frac{\sigma_j^2}{\sigma_k^2}, & otherwise, \end{cases} \tag{12}$$

where the degree of shrinkage is governed by the hyperparameter λ, with $\lambda \to 0$ shrinking to the assumption that the variables in the VAR are independent white noise processes or, equivalently for all of the differenced variables in the VAR, independent random walk processes in levels.

We obtain σ_l^2 by taking the residual variances after fitting an AR(4) on the l^{th} variable using least squares, which is a common practice (e.g., Banbura et al., 2010; Koop, 2013). The term $1/i^2$ governs the basic structure of the Minnesota Prior to down-weight more distant lags and the factor σ_j^2/σ_k^2 adjusts for the different scale of the data.

We follow Morley and Wong (2020) and choose λ by minimizing the one-step-ahead out-of-sample forecast error of output growth. The natural conjugate Normal-Inverse-Wishart prior implies posterior moments that can be calculated either analytically or through the use of dummy observations. We will use dummy observations to estimate the BVAR (e.g., Banbura et al., 2010; Del Negro and Schorfheide, 2011; Woźniak, 2016). For brevity, we relegate these details to Section B of the online appendix.

3. Estimates of business and financial cycles

Fig. 1 presents our measure of the U.S. business cycle, the estimated U.S. output gap, together with our measure of the U.S. financial cycle, the estimated U.S. housing and credit cycle, alongside their associated 90% credible interval. Our point estimate is based on the BVAR posterior mode (i.e. we take the posterior mode of the BVAR parameters and thereafter construct the cycles by applying the BN decomposition to those BVAR parameters). The estimated output gap lines up with the NBER reference cycles, with turning points coinciding with NBER-dated recessions. We also note that our estimated output gap appears to be large and positive just before the Great Recession, lining up with accounts that the real economy was overheating in the 2000s (e.g., see Borio et al., 2017; Taylor and Wieland, 2016). Turning to the estimates of the financial cycle, namely estimated the housing and credit cycle, our estimates are consistent with the general narratives. In particular, whether one looks at the credit or house price cycle, our estimates imply a boom of the financial cycle in the 2000s and a bust during the Great Recession.

Recall that our estimates of the business and financial cycles only rely on an underlying BVAR and the definition of the long-horizon forecast to define the trend and cycle. Because our estimates of the business and financial cycle do not rely on an *a priori* view of the length of financial and business cycles, we can reassess the view on the relative duration of the business and financial cycle through the lens of our model. As Cagliarini and Price (2017) point out, a widely held view that the financial cycle has a much longer duration than the business cycle may be partly driven by assumptions on which frequencies to isolate, potentially obscuring the distinction between assumptions and conclusions.[13] Fig. 2 presents

[11] We tested for a break at the midpoint as a first pass as we wanted to also strike a balance against adjusting for too many breaks. If one cannot find a break in the mean using the midpoint of the sample, then we view any possible breaks in the mean as probably not sufficiently large to warrant attention. Only if we find a statistically significant difference in the mean between the first and the second half of the sample do we use the sup-F statistic to be more precise about the dating of the break.

[12] If we find a break in the mean, we adjust the $\mathbf{x_t}$ vector before estimation. This approach will be equivalent to placing a flat prior on the mean and makes the estimation of the VAR and BN decomposition straightforward. As our estimation procedure optimizes on the degree of shrinkage, the analytical properties from using the natural-conjugate prior, as opposed to Monte Carlo sampling, is a key ingredient in making our estimation procedure feasible. As noted by Morley and Wong (2020), one could model the break explicitly, though this will result in a more involved estimation procedure as we lose the analytical properties of the natural-conjugate prior and potentially makes estimation less feasible.

[13] For example, users of the bandpass filter take frequencies of $1\frac{1}{2}$ to 8 years as coinciding with the business cycle (e.g., see Baxter and King, 1999; Christiano and Fitzgerald, 2003). For the financial cycle, extant work such as Drehmann et al. (2012) and Aikman et al. (2015) choose 8 to 20 or 30 years as frequencies to isolate for characterizing the financial cycle.

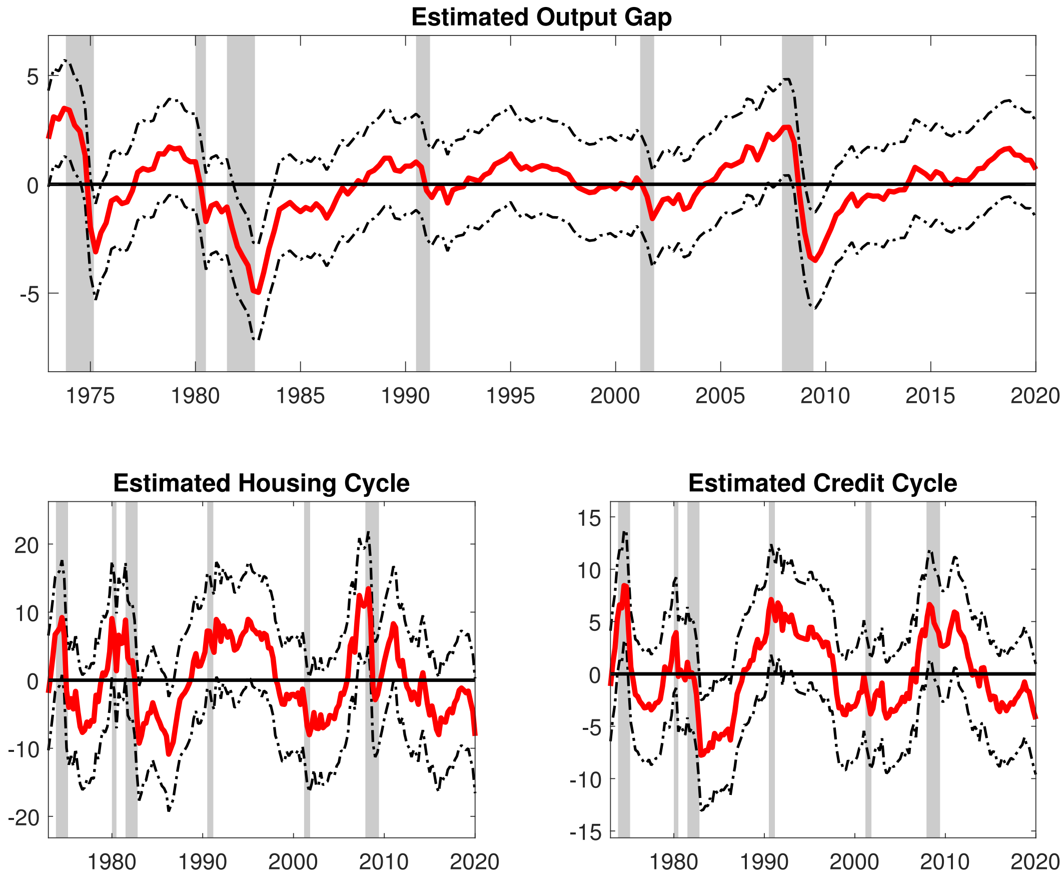


Fig. 1. Estimated cycles from the BVAR. Units are in percent deviation from trend. Grey shaded areas indicate NBER recessions. 90% credible interval calculated as per Kamber et al. (2018) .

the estimated spectral density of the estimated output gap, housing cycle, and credit cycle and its associated 90% credible interval.[14] We highlight the frequencies between $1\frac{1}{2}$ to 8 years, 8 to 10 years, and, 10 to 20 years. Recall that $1\frac{1}{2}$ to 8 years correspond with the frequencies regularly isolated by a bandpass filter as being consistent with "business cycle frequencies" (e.g., see Baxter and King, 1999; Christiano and Fitzgerald, 2003). Our point estimate for the spectral density is similarly based on the posterior mode as per the point estimate in Fig. 1.

We find that our estimated output gap is the only cycle that features a non-trivial degree of fluctuations between $1\frac{1}{2}$ to 8 years. That is, we find very little of the variation of either the housing or credit cycle is within the frequencies associated with $1\frac{1}{2}$ to 8 years. Instead, it appears that much of the variation of the housing and credit cycle occurs at the 10 to 20 year frequency, with both featuring a dominant peak of the spectral densities within the 10 to 20 year window. More precisely, the dominant peak in the spectral density of the housing and credit cycle occurs at frequencies coinciding with 16 and 19 years respectively, very similar to extant estimates (e.g. Aikman et al., 2015; Rünstler and Vlekke, 2018). We note that from the posterior distribution, the dominant peak of the spectral density in the financial cycle appears fairly precisely estimated. While the output gap does feature fluctuation between the traditional business cycle frequencies of $1\frac{1}{2}$ to 8 years, we also find a non-trivial degree of fluctuation outside the traditional business cycle frequencies. Indeed, while we note that the traditional frequencies associated with the business cycle are $1\frac{1}{2}$ to 8 years and noting the caveat that the broader literature uses different methods which may compromise comparability, Comin and Gertler (2006) emphasize non-trivial business cycle frequencies in the 2 to 50 year window, while Rünstler and Vlekke (2018) also find the dominant cycle to be just outside the 8 years range.[15]

Overall, we find mixed evidence of whether the financial cycle to be substantially longer than the business cycle. A key reason for our finding is that while the peaks of the spectral density for both the housing and credit cycle appear to be

[14] In estimating the spectral density, we follow Schüler (2020) and use a Parzen window of $12\sqrt{T}+1$ to smooth the periodogram.

[15] Our estimated credit cycle is 0.24 correlated with a credit cycle obtained via a HP filter with a smoothing parameter of 400,000 and 0.17 with the Rünstler and Vlekke (2018) model. Our estimated credit cycle also peaks around the same time as these alternative measures. Interestingly, when we allowed for more variability on the smoothness of the trend in the alternative measures, the correlation to the HP filter and Rünstler and Vlekke (2018) model both rise to 0.29, which echoes some previous work. For example, both Beltran et al. (2021) and Drehmann and Yetman (2021) show properties of the credit cycle in these alternative measures can change with how one calibrates the smoothness of the trend. We note that our approach imposes no such restrictions on the smoothness of the trend since it is entirely predicated on the forecastability of variables in the BVAR. Nonetheless, it is useful to note that our estimated credit gap is positively, albeit weakly, correlated with these alternative measures.

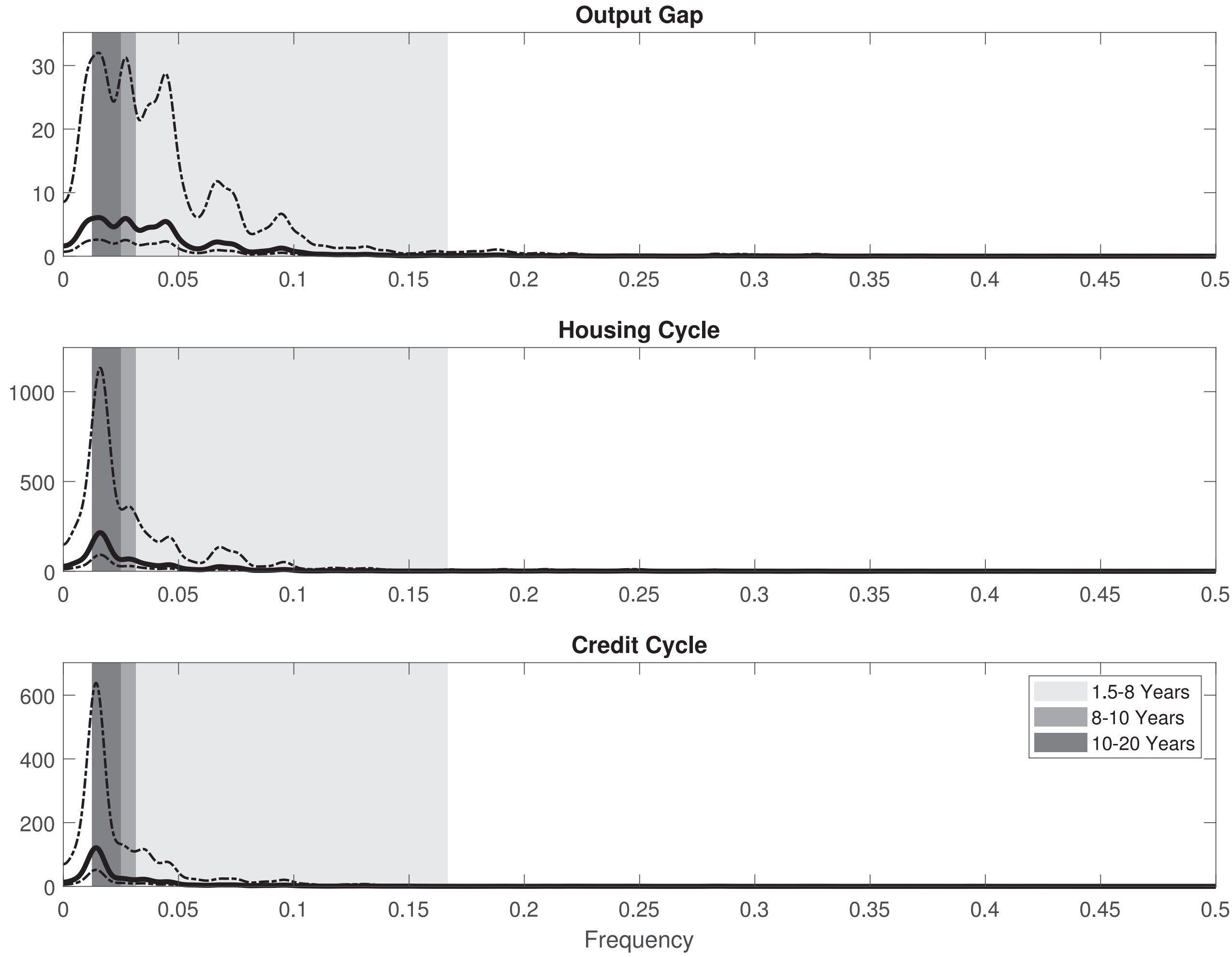


Fig. 2. Estimated spectral density of the estimated cycles with 90% credible interval. The frequencies associated with $1\frac{1}{2}$ to 8 years, 8 to 10 years, and 10 to 20 years are highlighted.

very sharply identified within the 10 to 20 year window, the peak of the spectral density for the output gap is fraught with a large degree of uncertainty. For example, while the posterior mean difference of the implied dominant frequency of the business cycle is 10 quarters shorter than that of the credit cycle, our estimated posterior probability that the dominant frequency of the financial cycle implies a longer cycle than that implied by the dominant frequency of the business cycle is 60%, which while larger than a 50-50 probability, does on balance constitutes mixed and perhaps weak evidence.[16] We also note, once again with the caveat of being in a different model setting, Kulish and Pagan (2021) tested the Rünstler and Vlekke (2018) model and are unable to reject the null hypothesis that the financial cycle in their model is longer in duration relative to the business cycle, a similar conclusion also arrived by Cagliarini and Price (2017). Through constructing the posterior distribution of the estimated spectral density, our results would suggest that imprecision involved in estimating the dominant frequency of the business cycle may reconcile the mixed evidence in the wider literature.

4. The role of financial factors in driving the business and financial cycles

We now turn to the role of financial factors in driving the business and financial cycle. We address this question mainly with two tools that we introduced in Section 2; the informational decomposition and structural analysis where we explicitly identify a structural financial shock through guidance from the broader literature.

4.1. Informational decomposition of the output gap

Figs. 3 and 4 present the informational decomposition for the estimated output gap and financial cycles calculated using Equation (6). The contributions are calculated from the forecast errors of five financial variables in our BVAR system; credit, the excess bond premium, stock prices, the VIX, and house prices. Fig. 4 reports the individual shares of the forecast errors

[16] Note that we can make probability statements as these quantities are obtained via a Bayesian posterior distribution.

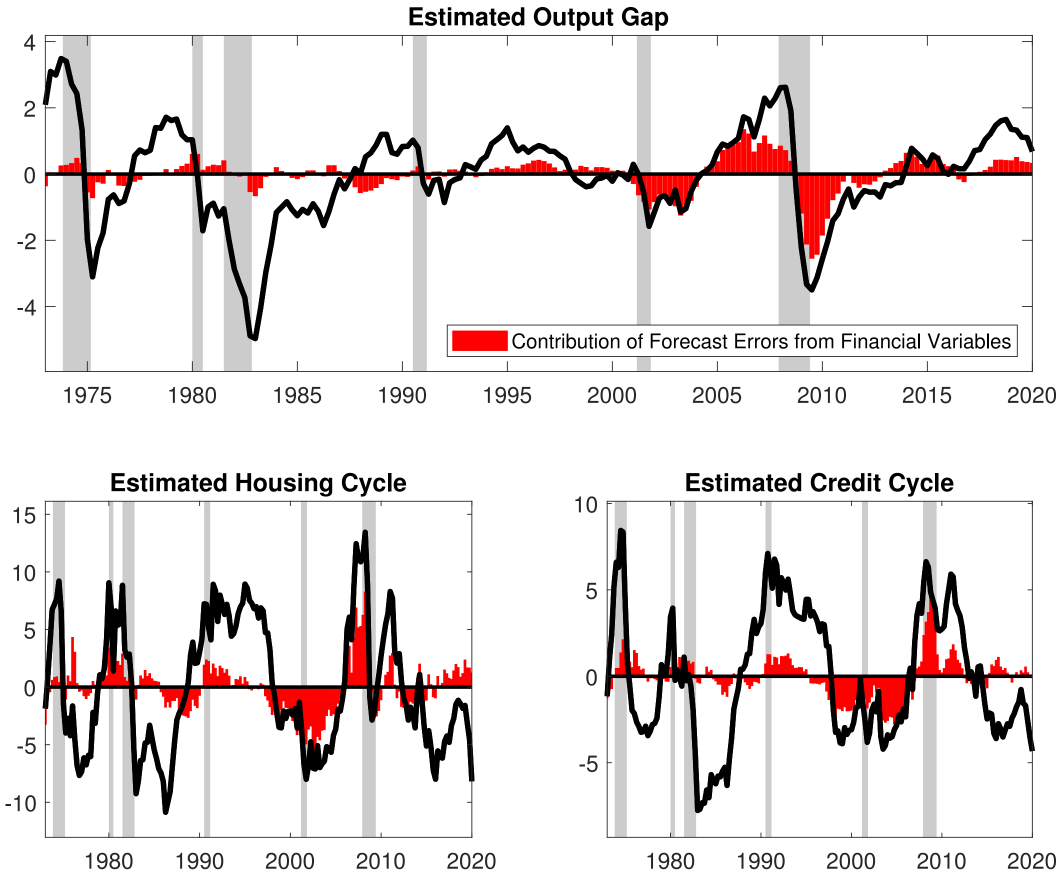


Fig. 3. Informational decomposition of the estimated cycles. Solid line denotes the estimated cycle. Cycles are measured in percent deviation from the trend. Grey shaded areas indicate NBER recessions. The bars represent the total contribution of the contribution from the BVAR forecast errors from five financial variables (credit, the excess bond premium, the S&P 500, the VIX index, and the house price) The individual contributions are presented in Fig. 4.

of the five chosen financial variables, while Fig. 3 sums up these contributions. We emphasize that the informational decomposition is not causal, so any conclusions about causal mechanisms from the information decomposition should only be viewed as suggestive. In particular, the information contained within the forecast errors of financial variables could originate from shocks outside the financial sector and/or forecast errors that have little or a negligible role do not necessarily indicate their respective variables have no role.[17]

We document two general key observations from Fig. 3. First, the role of financial variables seems to have been important during the 2000s, but its impact is rather negligible before the 2000s, and especially so before the mid 1990s. It is a more open question whether, towards the end of the sample, the role of the financial variables associated with the output gap has returned to the more negligible role pre-2000. Second, financial variables have been particularly important during times where one would *a priori* attach a role for financial factors as having been important for the business cycle. For example, we find an important role for financial variables on the output gap in periods of financial stress, such as the burst of the dot-com bubble and the outbreak of the financial crisis as well as during the build-up of large financial imbalances as seen during the 2000s.

Turning to the individual financial variables in the bottom panel of Fig. 4, we find that of all the financial variables, the forecast errors from the excess bond premium and house prices contribute sizeably to both the output gap and financial cycles. As described previously, the excess bond premium reflects the risk-bearing capacity of financial intermediaries, and thus can be seen as a measure of excess credit (see Gilchrist and Zakrajsěk, 2012). That we find a prominent role for the information contained in the excess bond premium despite the inclusion of several other financial variables suggests that the link of how financial factors affected the output gap in the 2000s is likely linked to excess credit. Our evidence is consistent with an interpretation that excess credit contributed substantially to the overheating of the U.S. economy before the financial crisis. House prices have also been shown to play an important role in providing information about the output

[17] The latter point is worth elaborating on with a stylized example. Suppose variable A Granger causes variable B, and variable B Granger causes variable C, but variable A does not Granger causes variable C. Clearly in this case, variable B matters for the estimation of the BN cycle of variable C (see Evans and Reichlin, 1994). However, the forecast errors of variable A will matter for the informational decomposition of the cycle of variable C through variable B. Therefore, even if the forecast errors of variable B do not show up in the informational decomposition of the cycle of variable C, variable B is still important, because, without the role of variable B, the forecast errors of variable A would never show up in the informational decomposition of the cycle of variable C.

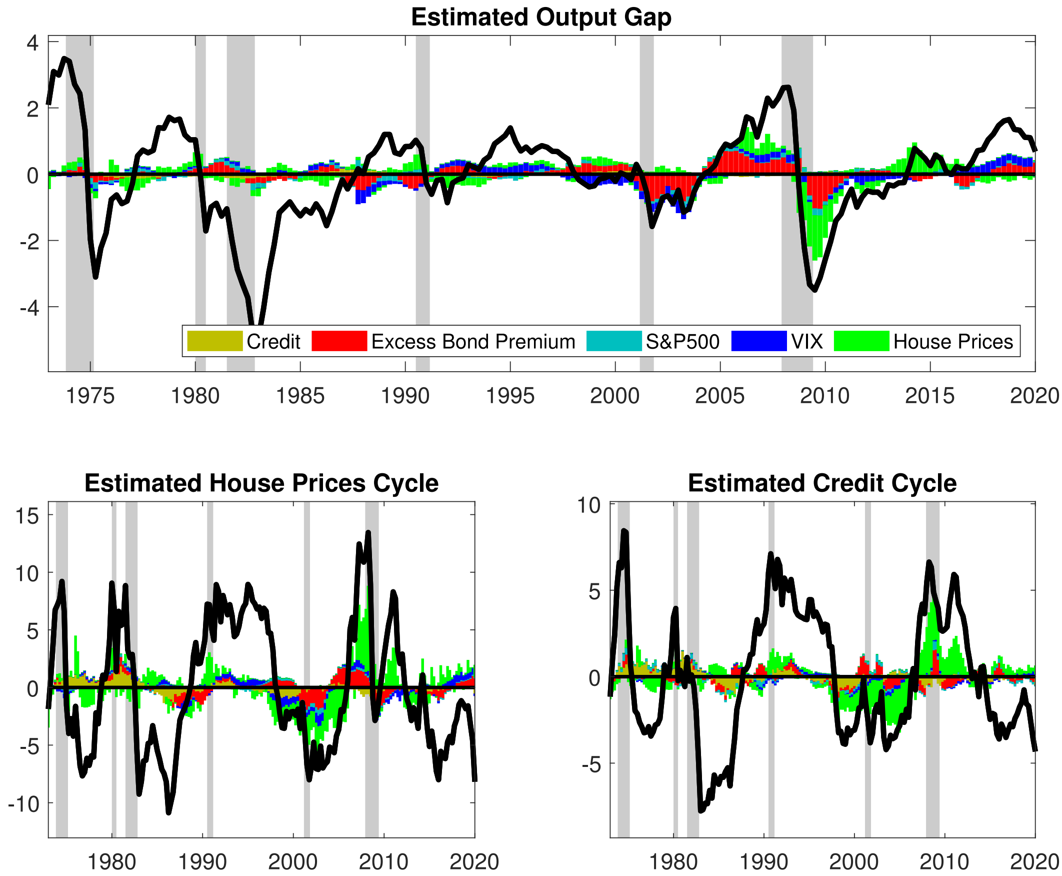


Fig. 4. Informational decomposition of the estimated cycles. Solid line denotes the estimated cycle. Cycles are measured in percent deviation from the trend. Grey shaded areas indicate NBER recessions. The bars represent the individual contribution from the BVAR forecast errors from five financial variables (credit, the excess bond premium, the S&P 500, the VIX index, and the house price).

gap, which is consistent with Leamer's (2007) observation that "housing is the business cycle". In particular, house prices contribute to the positive output gap in the 2000s, and also explain a large share of the negative output gap in the period during and just after the 2008/09 recession. The latter is a finding that is perhaps less surprising given it is well known that the housing bust played a big role in the 2008/09 recession.

While we once again stress that the interpretation from the informational decomposition is not causal, it represents a useful starting point. That the forecast errors of house prices and the excess bond premium contain information for both the output gap and measures of the financial cycle suggest that they would have probably played a role in linking and understanding the business and financial cycle during the 2000s.

A natural question is whether the presence of financial variables for output gap estimation helps with measurement, as opposed to its inclusion purely based upon for purposes restricted to interpretation. The finance-neutral output gap literature uses financial variables as forcing variables when estimating the output gap, and so the inclusion of financial variables is for both interpretation, as well as the measurement of the output gap (e.g., see Borio, 2014). The distinction might, at first sight, appear trivial, but is actually important, because if one requires financial variables for measurement as opposed to just interpretation, then arguable, *all* output gap estimation, or at least multivariate approaches, must necessarily include financial variables routinely whether or not financial variables are of direct relevance for the question of interest. Our modeling approach is well suited to provide some perspective to the issue of "interpretation" vis-a-vis "measurement". In particular, in our approach, estimating the output gap requires *all* relevant multivariate information for output growth to be included (see Evans and Reichlin, 1994; Morley and Wong, 2020). Put differently, if financial variables do not contain any information above and beyond the output gap, then one would obtain a similar output gap even without the financial variables. In other words, if one obtains the same output gap without the financial variables, then financial variables are only needed to interpret the output gap but play no role in the *measurement* of the output gap. Fig. 5 plots our benchmark output gap obtained with 23 variables against an output gap estimated with 18 variables where we excluded the five financial variables. For most of the sample, it appears that one would obtain the same output gap, except for a period just before the Great Recession, where one would estimate a larger output gap if we included the financial variables. In other words, while we find that one would often not require financial variables for measurement of the output gap, we find that one would need financial variables just before the Great Recession for measurement of the output gap. In particular, our results suggest that it is precisely in the period just before the Great Recession that financial variables provide information beyond that contained

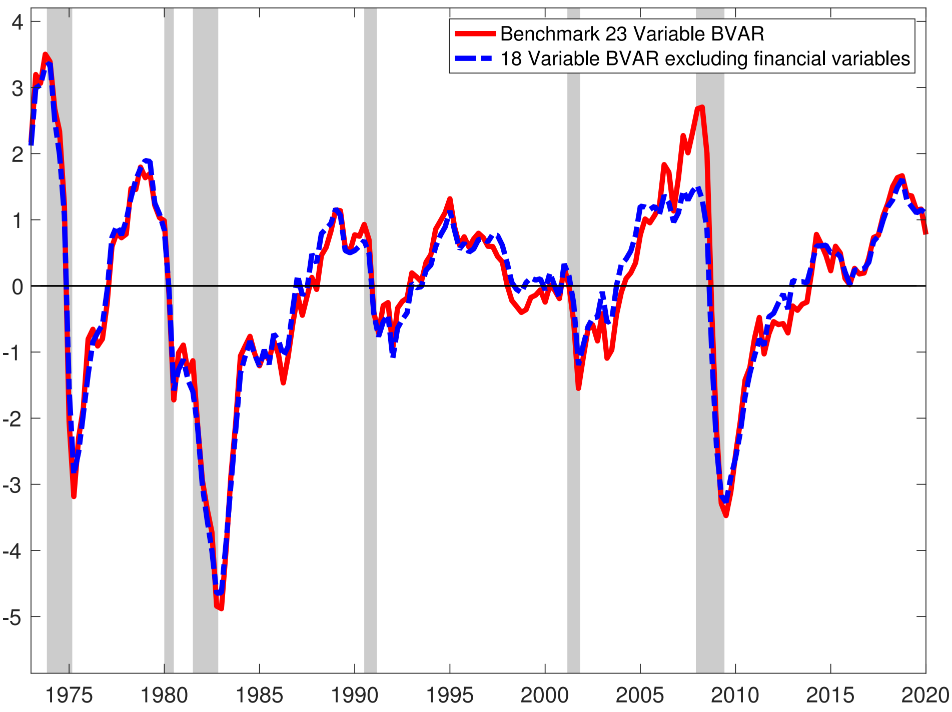


Fig. 5. Estimated output gaps with and without financial variables. Units are in percent deviation from trend.

in information such as the unemployment rate to estimate the output gap. Our result nuances a key consensus on using multivariate information to estimate the output gap. In particular, a broad consensus has concurred that the unemployment rate may be all the multivariate information one needs to estimate the output gap, at least for the U.S. (see Barbarino et al., 2020; Morley and Wong, 2020; González-Astudillo and Roberts, 2021), suggesting that one may not need financial variables for the measurement of the output gap. While our results would largely agree with this consensus, we also show that one may, at times such as the one just before the Great Recession, require information embedded in financial variables to help with the measurement of the output gap.

4.2. The role of identified financial shocks

As stressed in the previous subsection, while useful, the informational decomposition cannot attribute causality. While the informational decomposition only requires fitting a standard BVAR on a set of financial and macroeconomic variables, quantifying causal effects requires explicit identifying assumptions.

While we are more agnostic as to the precise definition of a financial shock, a broad element of what we seek to isolate is the exogenous variation of credit availability emanating from the financial sector. Our approach is thus to draw guidance from three existing identification schemes to identify financial shocks so that our conclusions are less sensitive to any particular identification scheme. The three identification schemes we will employ are a Cholesky decomposition, a penalty function approach that we take guidance from Caldara et al. (2016), and a sign restriction approach inspired by Furlanetto et al. (2019) combined with a narrative restriction approach inspired by Antolín-Díaz and Rubio-Ramírez (2018). The Cholesky and penalty function identification rely on exploiting variation in the excess bond premium for identification. Recall the excess bond premium is an indicator of the risk-bearing capacity of financial intermediaries, so the identified financial shock in these settings is conceptually closer to exogenous variation in the financial sector's ability to provide credit. This is also consistent with the loosening and tightening of the credit constraint, a mechanism that is very much at the heart of the financial friction/financial accelerator literature (e.g. Bernanke et al., 1999; Bernanke and Gertler, 1989). The sign restriction approach by Furlanetto et al. (2019) on the other hand, define and identify a financial shock as a boom in investment and stock prices. We design a set of sign restrictions, consistent with Furlanetto et al. (2019), which we further refine by specifying a narrative restriction where the financial shock is the overwhelming driver of the increase in the excess bond premium between 2008Q3 to 2008Q4. This type of restriction is akin to what Antolín-Díaz and Rubio-Ramírez (2018) refer to as Type B restrictions, and the event we have in mind is the collapse of Lehman in September 2008 and credit freezing in 2008Q4. The identification of a financial shock amounts to finding a column of the **A** matrix. We provide further discussion of the implementation of the identification schemes as well as present associated impulse response functions in sections D and E of the online appendix.

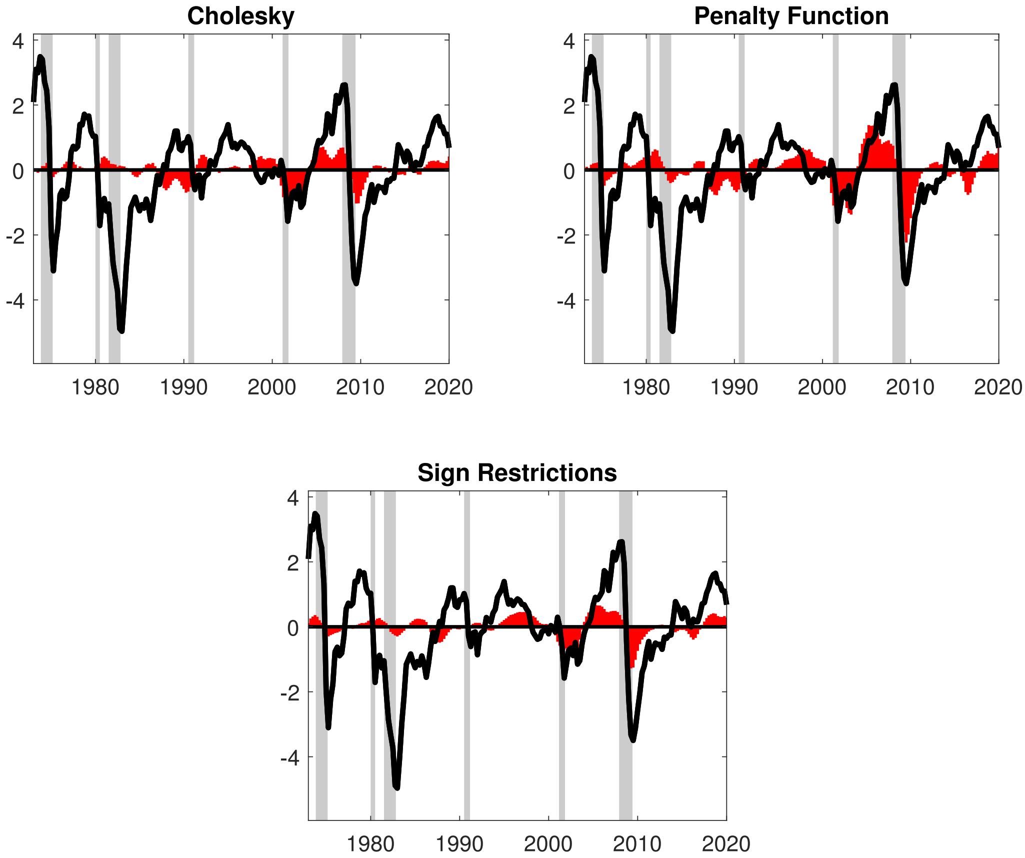


Fig. 6. Contribution of the financial shock to the estimated output gap. The solid line represents the estimated output gap. The output gap is measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The title refers to the different identification schemes. The bars represent the contribution of financial shocks to the estimated output gap. The contribution from the sign restriction approach is averaged across draws that satisfy the sign and narrative restrictions.

The Role of Financial Shocks in Driving the Output Gap

Fig. 6 presents the contribution of the financial shock to the output gap for all three different identification schemes. These shares are calculated conditional on the posterior mode of the BVAR parameters in and equivalent to reporting Equation (9) across different **A**'s.[18] While the share of financial shocks on the output gap differs between the three identification strategies, we highlight two key similarities across the three different strategies. First, the share of financial shocks tends to be much smaller pre-2000s, but appears to be much larger since the 2000s. Second, financial shocks appear to contribute positively to the output gap in the 2000s before the Great Recession, and then played a large role in the negative output gap during the Great Recession. We also note that financial shocks also played a sizable negative role in the 2000/01 recession, which was associated with the bust of the dot-com bubble.

To more precisely quantify how much financial shocks contributed to the overheating of the U.S. output gap in the 2000s, Fig. 7 presents our estimate of how much financial shocks contributed to the U.S. output gap between 2002Q1 and 2005Q4 along with the associated credible sets and credible intervals. We choose this time period as 2002Q1 marked the first quarter after the 2000-01 recession. We choose 2005Q4 as the end of 2005 was the height of the asset bubble. To construct these credible sets and intervals, for each draw of the posterior distribution, we construct the implied output gap sequence of identified financial shocks, then calculate the role of financial shocks on the output gap for the time period in question.[19] Because the financial shock is an identified (orthogonal) structural shock, the interpretation from Fig. 7 would be our estimated counterfactual reduction in the output gap from 2002Q1 to 2005Q4 in the absence of the identified financial shock. The bounds of the 68% credible interval are taken from the 16th and 84th quantiles of the posterior distribution. Because the

[18] For the sign restriction results, we averaged over the 1000 rotations which satisfy the sign and narrative restrictions conditional on the posterior mode parameters. Our approach to averaging across the admissible rotations is similar to Forbes et al. (2018), who averaged across the different solutions when calculating their historical decomposition. We do this as the average contribution from all the shocks, identified or unidentified, across all the retained solutions sums up to the output gap.

[19] Note that this would entail subtracting the contribution of financial shocks on the output gap in 2002Q1 from the contribution of financial shocks on the output gap in 2005Q4 for each draw of the posterior distribution. For the Cholesky and penalty function identification, this effectively requires us to just take a draw from the reduced form and then construct all these associated quantities. For the sign and narrative restrictions, we have to construct membership of the posterior distribution by allowing for satisfying both the sign and narrative restriction as described by Antolín-Díaz and Rubio-Ramírez (2018), then construct the associated quantities for each draw of the posterior distribution.

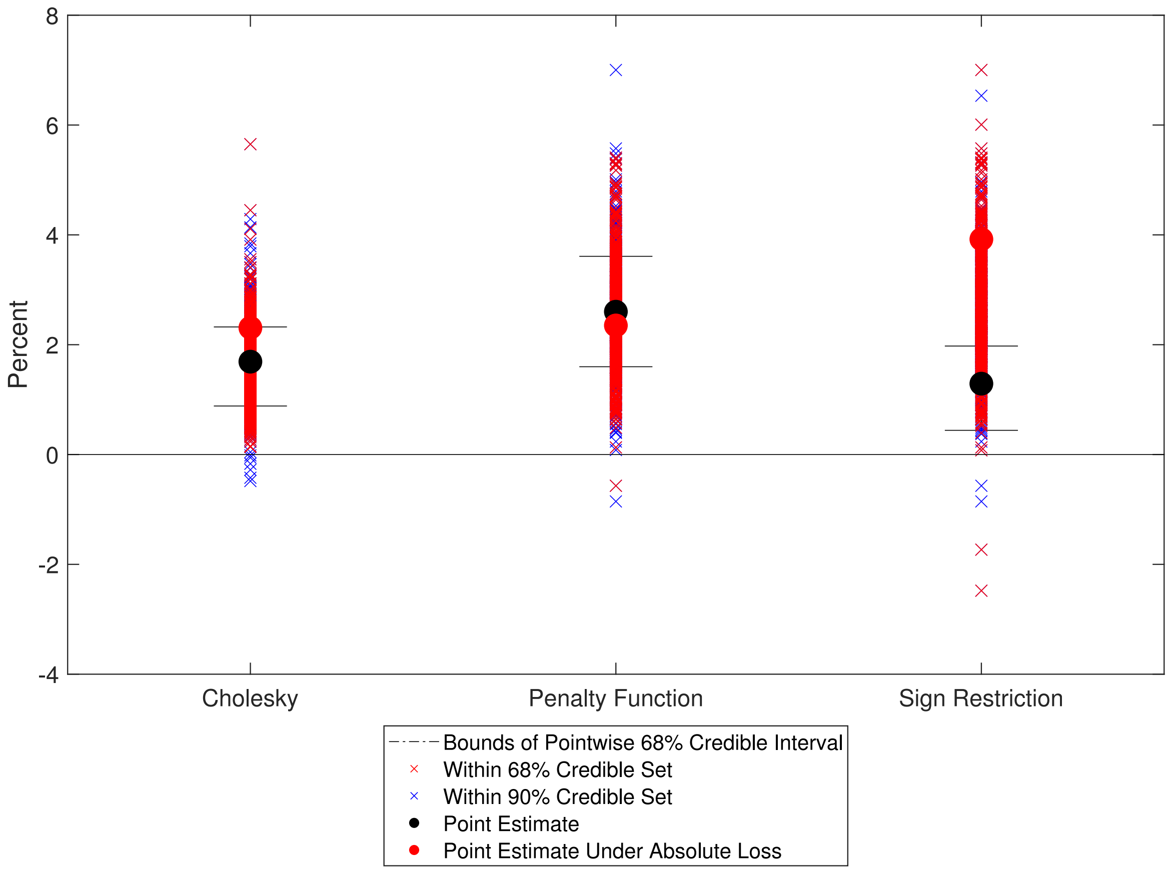


Fig. 7. Contribution of the identified financial shock to the estimated output gap (in percent) for the period 2002Q1-2005Q4 under the three identification schemes. The solid lines represent the pointwise bound of the 68% credible interval. The x represent membership in either 68% or 90% credible set obtained under absolute loss function described by Inoue and Kilian (2021). The point estimates for both Cholesky and penalty function identification are obtained conditional on the mode of the VAR posterior distribution.

quantiles may obscure information about the dynamics as the role of financial shocks is derived from a path rather than a point on a distribution (see Inoue and Kilian, 2021, for the analogous argument from the perspective of an impulse response function), we also present the associated credible sets calculated via the absolute loss function as described by Inoue and Kilian (2021).[20]

We take the posterior mode as our point estimate for both the Cholesky and penalty function identification, and for the sign restriction, the mean across 1000 rotations which satisfy the sign and narrative restriction but conditional on the posterior mode of the reduced form, to just retain comparability to Fig. 6. We also consider an optimal point estimate under absolute loss, for the posterior draw which evaluates the minimum loss. All the point estimates, under our preferred approach conditioning on the posterior mode and under absolute loss, imply the identified financial shocks added somewhere between 2 to 4% to the output gap. In other words, in a counterfactual without the identified financial shock, the increase in the output gap between 2002Q1 to 2005Q4 would have been 2 to 4 percentage points lower, which is reasonably large, considering the historical magnitude of the estimated output gap in Fig. 1. Given the lower bound of the 68% credible set is greater than zero under all three identifications, it implies that at least 84% of the posterior draws estimate a role of where identified financial shocks led to an increase in the output gap between 2002Q1 to 2005Q4. Turning to the credible sets, we first focus on the posterior draws within 68% credible set. Apart from 1 draw for the penalty function, and 2 draws for the sign restrictions, all elements of the credible set estimate a role for the financial shocks leading to an increase in the output gap. Note that once one moves to the credible set setting, the estimates implied by these sets are not continuous, in the sense that we are just reporting elements associated with draws from the posterior distribution which one evaluates a smaller loss from the associated loss function. It is noteworthy while there is a greater dispersion relative to the bounds of the credible interval, almost all elements of the credible set across all three identification schemes are still bunched up

[20] It is a more unresolved issue whether using impulse response function, as Inoue and Kilian (2021) do, is the most appropriate approach to evaluate the loss function given impulse response functions are not the focus of our analysis. We choose to evaluate the loss function based on the impulse response function to a financial shock to mostly maintain comparability with the description found in Inoue and Kilian (2021), as well as the credible sets we present in section E of the online appendix. Note that our approach would be tantamount to treating the impulse response function as the primary object of interest from the BVARs, which one may argue is not necessarily true in our setting, but an appropriate compromise given the issue is still not entirely resolved. We thank Lutz Kilian for the many discussions on this issue with us.

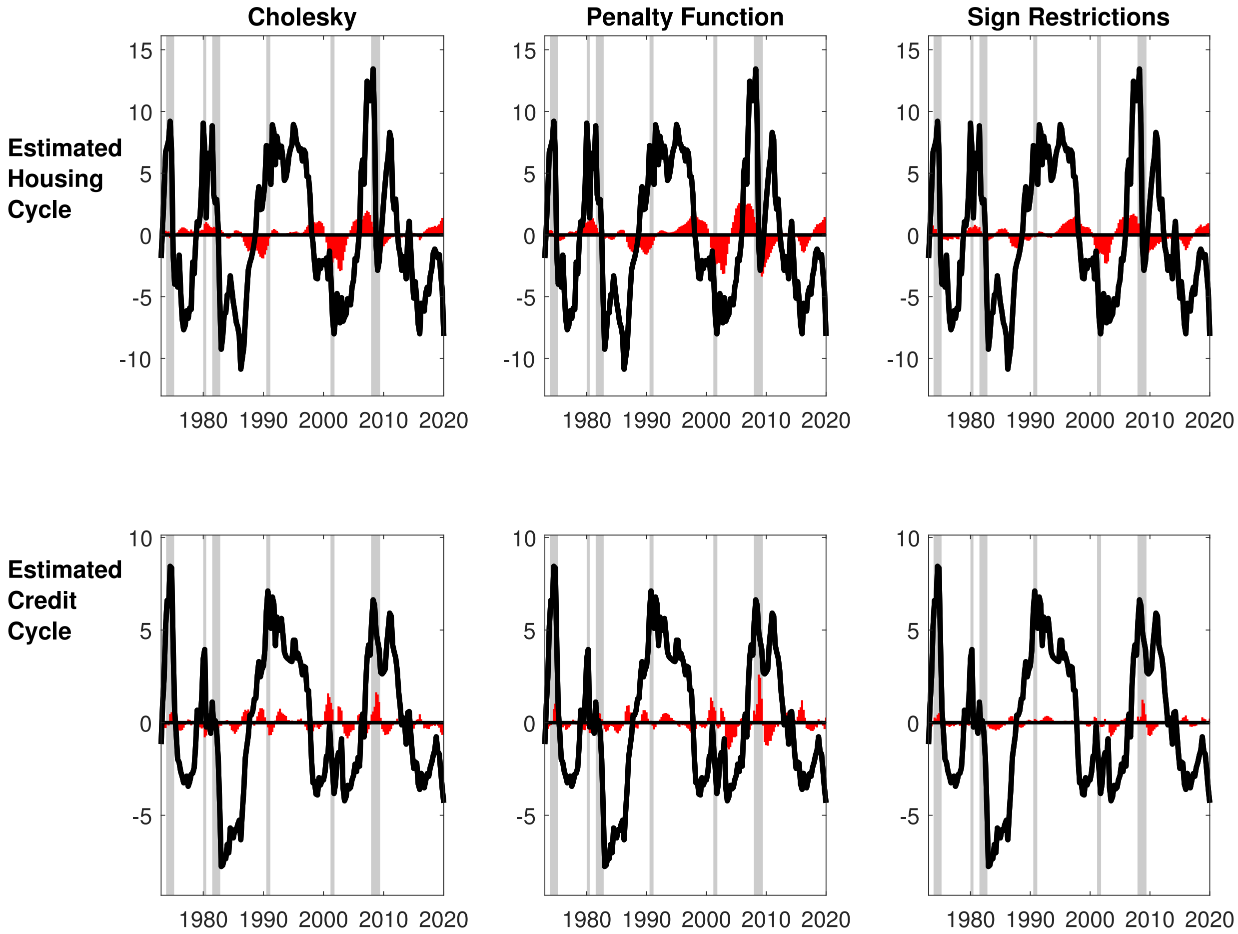


Fig. 8. Contribution of the financial shock to the estimated financial cycles. The solid line represents the estimated housing cycle (top panels) and estimated credit cycle (bottom panels). The cycles are measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The headers refer to the different identification schemes. The bars represent the contribution of financial shocks to the estimated financial cycle. The contribution from the sign restriction approach is averaged across draws that satisfy the sign and narrative restrictions.

between our 2 to 4% estimate. Finally, we show that even if we considered a 90% credible set, our conclusion is almost identical to using a 68% credible set.

Therefore, based on the overall evidence presented, our results point to a prominent role of financial shocks in contributing sizably to a large and positive output gap before the 2008/09 recession. Our interpretation is consistent with the notion that loose credit conditions originating from financial shocks in the 2000s likely fueled a boom in the business cycle which later led to the bust. While there is some uncertainty around the estimates of how much financial shocks matter, our estimates suggest financial shocks led to between a 2 to 4% increase in the estimated output gap with the credible interval and credible sets suggesting a very high probability that financial shocks led to some degree of overheating of the business cycle between 2002Q1 to 2005Q4. It is reassuring that even without a consensus on how to identify financial shocks, three different identification strategies provide a consistent account of how financial shocks drive the business cycle.

To round out our analysis, we also quantify the role of the estimated financial shocks on the financial cycle. Fig. 8 present these results. In general, the role of financial shocks across all three identification schemes is fairly similar. However, the role of financial shocks on both the house price cycles and credit cycles appears to be a bit different. We note that the role of financial shocks with the estimated house price cycles appears more like the role of financial shocks with the output gap. While the role of financial shocks in the credit cycle, at least with the Cholesky and penalty function identification, appears more muted, we still find a role for financial shocks in driving the credit cycle in the 2000s. Nonetheless, we note that relative to raw fluctuations of the estimated financial cycle, the role of identified financial shocks when accounting for the variation in the financial cycle is still much smaller than the role that the identified financial shock has in accounting for variation in the output gap.

4.3. Discussion

The results of how financial shocks affect the business cycle are consistent with the more reduced form informational decomposition. In particular, the forecast errors of the financial variables contributed more since the 2000s and played a large role in the overheating of the business cycle, a result which is also consistent with the role of the structural identified financial shock.

From our results, it would appear that the role of the financial variables is much larger than that of financial shocks. For example, if we zoom in on the output gap during the 2000s, the role of the identified financial shocks as shown in Fig. 6 is about half that of the role of financial variables, as shown by Figs. 3 and 4, depending on the precise identification scheme used. While we again stress that the informational decomposition is in reduced form, and so the role of these forecast errors should not be interpreted as causal, we briefly reconcile the differences we observe between the informational decomposition in Figs. 3 and 4 with the structural decomposition in Figs. 6 and 8 during the 2000s boom, given a key narrative is that financial factors appear to play a role in overheating the real economy, as indicated in our decomposition of the output gap and financial cycles.

To begin, it should not be entirely surprising that the role associated with the forecast errors of the financial variables is larger than that ascribed to financial shocks. After all, the forecast errors reflect variation from all the identified and unidentified shocks. Given we are only identifying one shock, one would expect the role of the financial shock to be much smaller than that reflected by the forecast errors of the financial variables since we expect shocks from the real economy, which we do not identify in our exercise, should also drive a non-trivial proportion of this variation in the forecast errors of the financial variables. From Fig. 4, during the period from 2000 to 2008, the key financial variables whose forecast errors are driving the output gap and the financial cycles are the excess bond premium and house prices. At first glance, the role of the financial shocks driving the output gap in the 2000s is approximately the same as being ascribed to the excess bond premium.[21] Therefore, it would appear during the 2000s boom, the forecast error of house prices is approximately the difference between the role attributed to forecast errors of the financial variables and the role attributed to financial shocks. Note that the preceding statement does not necessarily mean house prices did not have a role in the 2000s boom. Our analysis almost certainly suggested that house prices had a role given the excess bond premium in the informational decomposition and financial shocks had a non-trivial role in the house price cycle in the 2000s. Because the excess bond premium (and financial shocks) had a non-trivial role in the house price cycle, it is almost certainly true that whatever the role the financial shocks had on the output gap in the 2000s, it had a similar role in the housing cycle.

A key insight from comparing both the informational decomposition and the structural decomposition is that it reveals that one needs to largely explain the house price forecast errors within the model to provide a fuller account of the business and financial cycle in the 2000s. Put differently, while some of the current SVAR approaches to identifying financial shocks which we explore in our structural analysis go a long way in understanding the business and financial cycle in the 2000s, one would need to find a set of, or a single, shocks which can explain the forecast errors of house prices to fully reconcile the business and financial cycle in the 2000s.

We also relate our work to contributions in the wider literature to construct both "finance-neutral" output gaps (e.g. Borio et al., 2017), or considering the output gap as the difference between actual output and a counterfactual in the absence of financial frictions (e.g. Furlanetto et al., 2021). While our work has a flavor of both, we discuss more broadly the differences and similarities to this body of work. When considering "finance-neutral" output gaps, Borio et al. (2017) state that traditional output gap estimates are inflation-centric, and thus they consider information from financial variables to estimate the transitory component of real GDP. Within our framework, our output gap has no notion of being inflation or finance-centric. Instead, following on from the discussion by Evans and Reichlin (1994) and Morley and Wong (2020), when conducting a multivariate BN decomposition, any variable that is relevant for forecasting output growth is relevant for the output gap. While our analysis in Fig. 5 suggests the inclusion of financial variables can sometimes be important for the measurement of the output gap, especially just before the financial crisis, we caution that this alternative output gap is not "inflation"-centric in any sense, and Fig. 5 could at best be described as the "non-financial" output gap. Moreover, this "non-financial" output gap also accounts for the fact that financial and macro variables are correlated, and so omitting financial information would merely shift some of the role played by the financial variables to macroeconomic variables, as all that matters for the output gap in our framework is information from the various variables. Despite these conceptual differences and the obvious caveats, our account with the "non-financial" output gap would correspond with what has been found in the "finance-neutral" output gap literature (e.g., Borio et al., 2017) just before the Great Recession, though allowing for macro and financial variables to be correlated suggests that the "finance-neutral" output gap probably over-estimates the role of finance in the 2000s since they treat the financial variable (i.e. credit) to be exogenous. We also stress that while our results are consistent with regards to the view that the 2000s coincides with the perspective of the finance-neutral work, we do find a very small contribution of financial variables to the output gap pre-2000s, which suggests that any distinction of our output gap and a non-financial output gap pre-2000s is probably less relevant, at least in our setting.

The more structural approach taken by Furlanetto et al. (2021) views the output gap as reflecting inefficiencies arising from frictions, in the tradition of New-Keynesian DSGE models. Trend output is the counterfactual level of output in the absence of these frictions and the output gap is the difference between actual and the counterfactual output. Conceptually, the frictions in their setup are propagation mechanisms and relevant for *all* shocks. A direct comparison relative to the more structural approach of Furlanetto et al. (2021) is naturally challenging, as a fully-specified DSGE model requires one to be

[21] We confirm this when we looked at the role of financial shocks on the role of the excess bond premium forecast errors in the informational decomposition. While there were slight differences across the identification schemes, financial shocks accounted for most of the share of the role of the forecast errors of the excess bond premium in the informational decomposition.

explicit about the different frictions in the model. Even so, we note that a key result in their paper is that the inclusion of financial frictions implies a more positive output gap in the 2000s and before the Great Recession, consistent with our key result that the financial sector played an important role in overheating the business cycle pre-Great Recession.

4.4. Does the financial cycle lead the business cycle or vice versa?

So far, the analysis has been focused on estimating the business and financial cycle, as well as quantifying how important financial factors have been in driving the U.S. business cycle. In this section, we focus on the links between the financial and business cycle. In particular, an active body of work is interested in characterizing features on the comovement between the financial and business cycle to understand the links between them (e.g. Aikman et al., 2015; Claessens et al., 2012; Oman, 2019; Rünstler and Vlekke, 2018; de Winter et al., 2021).

As cross-correlations have traditionally played an important role in understanding the links between the cyclical components of different macroeconomic variables, we now adapt our empirical framework to understand cross-correlations. In particular, we are interested in shedding light on issues such as whether the financial cycle leads the business cycle or vice versa. From Equations (3) and (5), we know from Morley (2002) that $\mathbf{F}(\mathbf{I}-\mathbf{F})^{-\mathbf{1}}(\mathbf{X_t}-\boldsymbol{\mu})$ contains the estimated BN cycles. Following Kamber et al. (2018), the following can be used to calculate the variances of the estimated BN cycles

$$\boldsymbol{\Psi} = \mathbf{F}(\mathbf{I}-\mathbf{F})^{-\mathbf{1}}\boldsymbol{\Omega}[(\mathbf{I}-\mathbf{F})^{-\mathbf{1}}]'\mathbf{F}', \tag{13}$$

where $\boldsymbol{\Omega}$ is the variance of $\mathbf{X_t}$ and $vec(\boldsymbol{\Omega}) = [\mathbf{I}-\mathbf{F}\otimes\mathbf{F}]^{-\mathbf{1}}vec(\boldsymbol{Q})$, where

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{\Sigma} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{0} & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix}. \tag{14}$$

It follows that elements of $\boldsymbol{\Psi}$ will contain the cross-covariance between any pair of $c_{i,t}$ and $c_{j,t-m}$ where $i, j \in \{1, 2, \dots, K\}$ and $m \in \{0, 1, 2, \dots\}$.[22] It is then straightforward to normalize $\boldsymbol{\Psi}$ into a correlation matrix to obtain the cross-correlation of $c_{i,t}$ and $c_{j,t-m}$, where $\Delta y_{i,t}$ and $\Delta y_{j,t}$ are respectively in the k^{th} and l^{th} position in $\mathbf{x_t}$, and

$$corr(c_{i,t}, c_{j,t-m}) = \mathbf{s_k}\boldsymbol{\psi}\mathbf{s}'_{\mathbf{nm+l}}, \tag{15}$$

where $\boldsymbol{\psi}$ is the correlation matrix associated with $\boldsymbol{\Psi}$. More precisely, Equation (15) can be used to quantify objects such as the correlation of the output gap with the credit cycle four quarters ago and vice versa, providing a richer framework to understand the interaction between the financial and business cycle. $\boldsymbol{\psi}$, though, only contains the unconditional cross-correlations between measures of the business and financial cycle. It is straightforward to modify this cross-correlation conditional on a financial shock. Let $\boldsymbol{\alpha}$ be the column of the matrix $\mathbf{A}$ which identifies the financial shock in our exercise. If we modify Equation (14) such that

$$\tilde{\boldsymbol{Q}} = \begin{bmatrix} \boldsymbol{\alpha\alpha'} & \mathbf{0} & \dots \\ 0 & 0 & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix} \tag{16}$$

and substitute $\tilde{\boldsymbol{Q}}$ for $\boldsymbol{Q}$ at every step of the calculation of $\boldsymbol{\Psi}$, we can now obtain the cross-correlations of the business and financial cycle *conditional* on a financial shock. Unconditional correlations are the outcome of various shocks, and within our framework, the financial and business cycle are just outcomes of the various, identified and unidentified, shocks. The characterization of conditional cross-correlations adds a further dimension to the analysis. In particular, while the unconditional cross-correlations are important to characterize, these may have little to do with financial shocks. Unconditional cross-correlations, like our informational decomposition exercise, also do not allow us to make causal statements. Characterizing conditional cross-correlation allows our framework to make a causal link to how financial shocks can drive particular lead-lag relationships between the business and financial cycle.

Table 1 presents unconditional correlations, as well as the unconditional and conditional 4-quarter cross-correlations between our estimates of the output gap, credit cycle, and house price cycle, which we take as measures of the business and financial cycle. We also present the contemporaneous correlations between the different estimated cycles. We first focus on the top panel, which presents the unconditional cross-correlations. All entries are positive, which suggests that unconditionally, we expect booms in the financial cycle to be followed by booms in the business cycle and vice versa. While it should not be surprising that booms in the financial cycle lead to booms in the business cycle, unconditionally, this provides very little rationale for any form of regulation or macroprudential regulation to restrain credit or even house

[22] If one fitted a VAR(p) and cast it into the form implied by Equation (3), we can obtain cross-covariances up to $p-1$. To calculate the cross-covariances for cycles where $m \geq p$, one will still estimate the same VAR(p), but subsequently just augment the state vector $(\mathbf{X_t}-\boldsymbol{\mu})$ in Equation (3) with longer lags, as well as input appropriate entries in $\mathbf{F}$ to calculate $\boldsymbol{\Psi}$.

Table 1
Unconditional and conditional cross-correlations.

	Output Gap(t)	House Price Cycle(t)	Credit Cycle(t)
Unconditional Cross-Correlations			
	Output Gap(t)	**House Price Cycle(t)**	**Credit Cycle(t)**
Output Gap(t)	1		
House Price Cycle(t)	0.39	1	
Credit Cycle(t)	0.32	0.85	1
	Output Gap(t)	**House Price Cycle(t)**	**Credit Cycle(t)**
Output Gap(t-4)	0.36	0.37	0.57
House Price Cycle(t-4)	0.17	0.90	0.82
Credit Cycle(t-4)	0.01	0.74	0.91
Conditional Cross-Correlations			
Cholesky			
	Output Gap(t)	**House Price Cycle(t)**	**Credit Cycle(t)**
Output Gap(t-4)	0.31	0.20	0.50
House Price Cycle(t-4)	0.74	0.90	−0.35
Credit Cycle(t-4)	−0.69	−0.89	0.78
Penalty Function			
	Output Gap(t)	**House Price Cycle(t)**	**Credit Cycle(t)**
Output Gap(t-4)	0.04	−0.02	0.81
House Price Cycle(t-4)	0.53	0.61	0.29
Credit Cycle(t-4)	−0.72	−0.80	0.74
Sign Restrictions, percentage of negative correlations			
	Output Gap(t)	**House Price Cycle(t)**	**Credit Cycle(t)**
Output Gap(t-4)	0	0	10.6
House Price Cycle(t-4)	0	0	13.8
Credit Cycle(t-4)	82.3	69.3	7.1

prices. A boom in the credit cycle is followed by a boom in the house price cycle and vice versa, which is consistent with the reinforcing dynamics of credit and housing booms, as documented by Jordà et al. (2015).

However, the picture changes somewhat once we condition these correlations on a financial shock, as per Equation (16). We first condition on a financial shock identified through our Cholesky and penalty function identification since these identification techniques provide a unique solution to the identification of the financial shock. For both the Cholesky and penalty function identification, we observe that once we condition on a financial shock, the credit cycle lagged 4 quarters is now strongly negatively correlated with the output gap and the house price cycle. Because sign restrictions do not point identify the financial shock, but instead produce a set of admissible solutions (see Fry and Pagan, 2011), to check for whether our sign restriction identification produces conditional correlations in line with our other two identification strategies, we count the proportion of conditional correlations from the various sign restriction solutions which are negative, and thus switch sign from the unconditional correlation.[23] This is presented in the bottom panel of Table 1. We observe a sign switch in the majority of our sign-restricted solutions for the conditional correlation of the lagged credit cycle on the house price cycle, and more importantly, for the output gap. Therefore, we conclude that a majority of our sign-restricted identified solutions are in line with the sign switch that we document for the Cholesky and penalty function approach.

Fig. 9 provides some intuition on why we observe a sign switch conditional on a financial shock. Presented are the impulse response function of real GDP and credit to a one standard deviation financial shock identified using the Cholesky decomposition, though the precise identification matters less given all three identification schemes show similar patterns. The impulse response functions of the level of real GDP and credit are based on cumulating the impulse response functions of real GDP growth and credit growth since both variables enter the BVAR in first differences. The definition of trend in the BN decomposition is the forecast of the long-horizon forecast. Given that, by definition, the impulse response function is the response to only a financial shock being introduced into the system at time 0, the trend becomes where the level of real GDP and credit settle in the long-run. This is denoted by the dotted line in Fig. 9. The difference between the impulse response function and the long horizon forecast, denoted by the dotted line, thus becomes the output gap and credit cycle which we obtain via the BN decomposition.

The dynamics of real GDP are such that while it falls quickly in response to the financial shock, there is a hump-shaped response where the level of real GDP starts to recover 4 to 5 quarters after the financial shock. This also means that the eventual fall in real GDP relative to before the financial shock is more marginal as the level eventually largely recovers. Given the level of the impulse response function of real GDP is below this long-horizon level, a negative output gap opens up for up to 10 quarters after the financial shock. On the other hand, the dynamics of credit are quite different. In response to a financial shock, credit falls slowly towards its long-horizon forecast. Because credit is above its long horizon level for up

[23] Note that the unconditional correlation is the same across all the sign restricted solutions as this quantity is derived from the same reduced form.

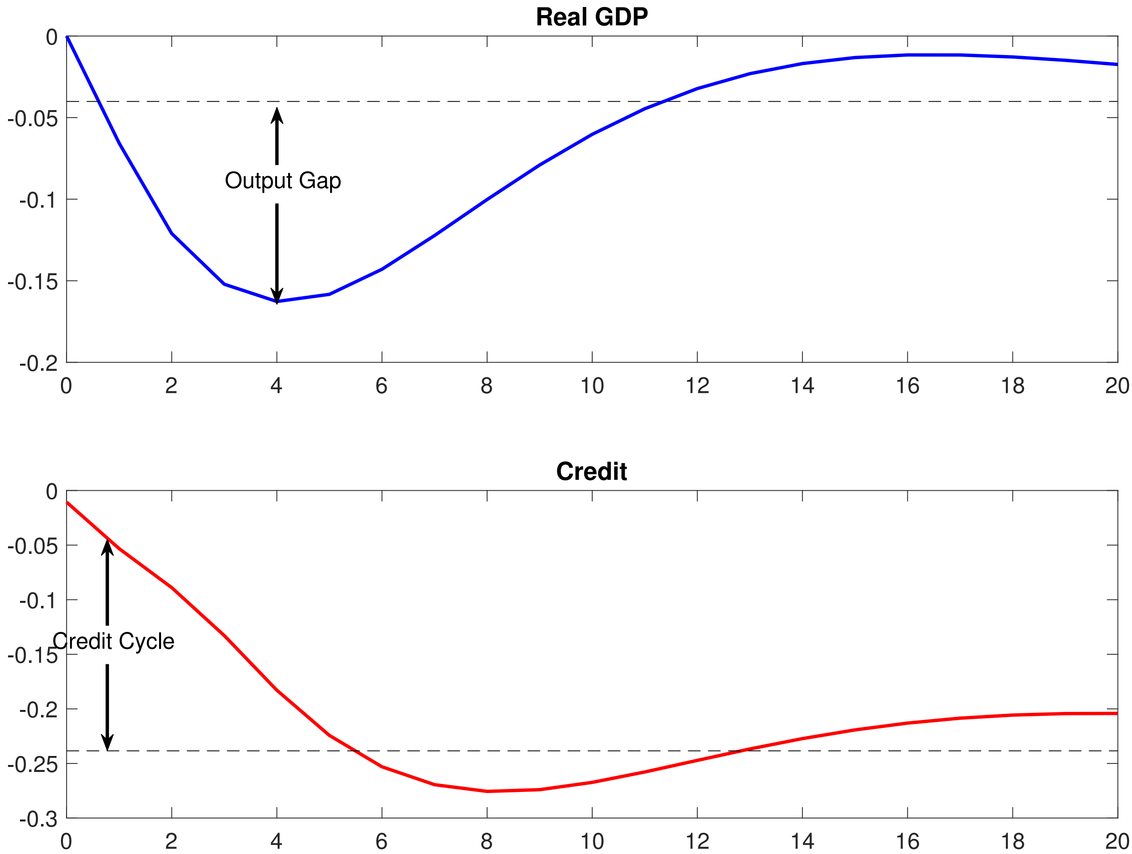


Fig. 9. Impulse response function to a one standard deviation financial shock identified using Cholesky Decomposition.

to 6 quarters after the shock, a positive credit cycle opens up initially as the level of credit adjusts towards its long-horizon level. Fig. 9 also clarifies the source of the negative correlation of the output gap and credit cycle conditional on a financial shock. Because the level of credit adjusts slowly, but the long-horizon level falls by more, the credit cycle and the output gap thus become negatively correlated conditional on the financial shock. The impulse response function should make clear that credit and real GDP are still positively correlated conditional on a financial shock since they both move in the same direction in response to the shock, it is only the conditional correlation of their cycles that become negatively correlated.

A key takeout of the analysis in this section is that unconditionally, the financial and business cycle are positively correlated (i.e. they comove). This would appear to contradict a key narrative that excess credit leads to a systemic event that results in a bust in the business cycle. While we can generate a positive credit cycle leading to a business cycle bust when conditioning on a financial shock identified by the broader structural VAR literature, the dynamics implied by Fig. 9 also do not entirely fit with this narrative. In particular, if a financial shock identified by the broader literature can generate dynamics consistent with this narrative, a financial shock that leads to a narrowing of credit spreads should lead to credit being above its longer-run level, with a boom in real GDP above in longer run level before both of the levels of real GDP and credit reversing below its long-run level, with *all* these dynamics generated endogenously in response to the identified shock. In particular, one should also observe a sign switch at particular horizons in the impulse response function, which we do not. We, therefore, make two points. First, while the narrative of a boom in the credit cycle leading to a boom-bust cycle in the business cycle is plausible, it is unlikely that this is due to a financial shock identified by the broader VAR literature, which is more akin to a credit spread type shock. Second, and perhaps practically, the fact that a financial shock can lead to an opening up of a positive credit cycle also questions whether one should necessarily use the credit cycle as an indicator of future systemic crises. In particular, for the sort of financial shock identified by the broader literature, we are already in the midst of a systemic crisis, and the positive credit cycle opens up because the systemic event lowers the long-run level of credit, thus opening up a positive credit cycle. While it is fair to note that we use a different method to extant work to estimating the credit cycle, our measure of the credit cycle peaks precisely at the same time as measures such as using the HP filter or UC model. In these models, because the level of credit falls slowly after the shock, this also leads to a credit cycle opening up when the shock occurs.[24] While this does not say anything about a positive credit cycle being an indicator

[24] One possibility is that because credit is a stock, the credit cycle is based on new credit accorded in each period. The new credit plays the role of investment in the capital accumulation equation, which suggests one may need to consider a flow measure to construct indicators that do not peak at the time of a systemic crisis. We thank the anonymous referee for pointing out this possibility.

during the build-up phase, we think it is fair to say that the *peak* of the credit cycle may be a manifestation of the systemic event.[25]

Summing up, a key conclusion is that *unconditionally*, the credit cycle and output gap are strongly positively correlated, with the contemporaneous correlation much stronger (0.32) relative to the correlation between the lagged credit cycle and the contemporaneous output gap (0.01). While an identified financial shock can switch this correlation between the lagged credit cycle and the output gap, the dynamics are also such that it is unlikely that the sort of financial shock identified by the SVAR literature is the type of shock that leads to the narrative that endogenously generates the boom-bust cycle in the credit cycle. Perhaps more generally, our results would suggest a more nuanced view of how the business cycle interacts with the financial cycle, or more specifically the credit cycle. Based on our analysis, the average boom in the business cycle will be associated with a boom in the financial cycle and vice versa. More broadly, our results would at least suggest that macroprudential policy targeted at crude measures of credit cycles, may be too blunt of an instrument since one should not *a priori* expect all positive deviations of the financial cycle relative to trend to be associated with business cycle busts.

5. Robustness

We briefly discuss some of the following robustness issues, though relegate the presentation of these results to the online appendix.

Shifts in mean We explore two possibilities for a shift in the mean, or $\boldsymbol{\mu}$ in Equation (5), as this may affect the estimation of the cycle. First, we explore the possibility of a sharp break in the drift of real GDP as this has a first-order implication for the measurement of the business cycle. When we set up our baseline specification, we could not reject the possibility of a break in the drift of real GDP with a Bai and Perron (2003) test. However, this result is sensitive to how we adjusted for the standard errors when testing for the break. An alternative specification dates a break in 2006Q2, which is consistent with wider work (e.g. Berger et al., 2016; Eo and Morley, 2022; Kamber et al., 2018) dating a slowdown in GDP growth just before the Great Recession.[26] We note that the inherent uncertainty of whether, and if so when, a break in the drift in U.S. real GDP has occurred is not entirely surprising given the mixed evidence on the issue (Check and Piger, 2020). We allowed for a break in the drift of U.S. GDP in 2006Q2, and present these results in Section F of the online appendix, but note that our main results are robust. Second, the breaks may not be discrete. We, therefore, allowed for the possibility of a smooth change in the mean of all variables. Taking guidance from Stock and Watson (2012), we demeaned all variables before estimation by using a biweight kernel with a bandwidth of 100 quarters before estimation. We also present these results in Section F of the online appendix, but note that our results are also robust to this choice of demeaning.

Informational Sufficiency We checked if our model is informational sufficient. Taking guidance from Forni and Gambetti (2014), we constructed a factor by extracting the first principal component from FRED-QD, and tested whether this extracted factor Granger cause any of the 23 BVAR equations in an out-of-sample forecasting exercise. Using the procedure described by Clark and West (2006) to test for predictability in nested models, we did not find evidence that the extracted factor from the FRED-QD dataset Granger causes any of our VAR variables in an out-of-sample forecasting exercise, suggesting our 23 variable BVAR system is informational sufficient.

Disentangling Uncertainty from Financial Shocks It is known that it is challenging to disentangle the role of uncertainty shocks from financial shocks. Similar to Caldara et al. (2016) and Furlanetto et al. (2019), we also attempted to disentangle the role of uncertainty shocks from financial shocks. In the penalty function identification, this is a similar exercise to Caldara et al. (2016) when they reverse the order of identifying uncertainty shocks first before financial shocks.[27] In the sign restriction setting, we identify an uncertainty shock using the same sign restriction as the financial shock, except that for the uncertainty shock, the ratio of the increase in the VIX relative to the excess bond premium is larger than the financial shock. This is effectively the same exercise to Furlanetto et al. (2019) who attempt this disentanglement by imposing a sign restriction on the ratio of the VIX to excess bond premium. We present these results in Section G of the online appendix, but just briefly comment on the results. In the penalty function setting, it is not entirely surprising that the results are sensitive to reversing the order since Caldara et al. (2016) already document sensitivity when using the VIX to identify uncertainty shocks. Nonetheless, the *sum* of the effect of financial and uncertainty shocks appear to be quite similar either when one first identifies the financial shock then uncertainty shock, or vice versa. Given the sum of the shares is quite insensitive, it suggests that if one was inclined to take guidance from the penalty function identification while pinning down the role of financial or uncertainty shocks might be tricky, the general conclusions hold if we are prepared to group the two shocks. In the sign restriction setting, identifying a second uncertainty shock does not affect our main conclusion. The role of the

[25] This point is an important distinction to highlight because given parts of the literature judge the credit cycle *solely* on the basis of whether it can predict future systemic events (e.g. Drehmann and Yetman, 2021; Hartwig et al., 2021), it is possible that a positive credit cycle *before* the peak is able to correctly predict the crisis. For example, a widely used dating for systemic crisis used by the evaluation literature only has two crises in our sample (see Laeven and Valencia, 2018), so we cannot judge whether this is indeed the case. This would appear to be an important issue to revisit for future work if one could obtain a large cross-country sample of credit cycles estimated using the BN decomposition.

[26] To be precise, our baseline specification for the Bai and Perron (2003) test allows for heteroskedasticity and autocorrelation consistent (HAC) standard errors, which cannot date a break with the usual degree of statistical significance. If we do not allow for HAC standard errors, we will date a break in 2006Q2.

[27] Note that because we only identify a single shock in our penalty function exercise, namely the financial shock, the role of financial shocks will be identical to a setting where one first identifies the financial shocks, then uncertainty shock using the penalty function identification.

identified financial shock on the output gap is almost identical between our baseline results identifying a single financial shock, or the alternative of jointly identifying both uncertainty and financial shocks.

Choice of Particular Financial and Housing Indicators We also explored using loans, rather than credit, and house prices from the OECD and Federal Housing Finance Agency, rather than the BIS in our baseline analysis. Note that some of these alternative data sources may cause mismatches with our baseline sample. These results are also presented in Section H of the online appendix. Our main results are also robust to the change in the choice of the particular financial and housing indicators we use for the empirical analysis.

6. Conclusion

Building off a standard BVAR in conjunction with the Beveridge-Nelson decomposition, we jointly estimate the U.S. business and financial cycle within a unified approach which also allows us to interpret the estimated business and financial cycles through the lens of the forecast errors or structural shocks. First, we find that the role of financial factors in driving the business cycle appears to be much larger since the 2000s. We find this result regardless of whether in the more reduced form informational decomposition, or when we identify a structural financial shock. In particular, we find evidence that the financial sector did overheat the business cycle in the 2000s before the Great Recession, with our structural analysis pointing towards financial shocks adding as much as between 2 to 4% to the output gap during the 2000s. We also uncover evidence of a more nuanced relationship between the credit cycle and the output gap. In particular, we show that the unconditional correlation between the credit cycle and the output gap is often positive, though does turn negative when conditioned on a financial shock. One implication of our findings is that macroprudential policy may need to distinguish between the underlying causes of the credit cycle rather than relying on simple rules of thumb that prescribe unconditionally curbing all positive fluctuations in the credit cycle.

Our framework provides several interesting avenues for future work given the ability to interpret multiple cycles and also link these fluctuations to identified shocks. In particular, while we have restricted our analysis to the U.S., one could extend our framework to understanding financial and business cycles across multiple economies. In particular, extending work such as Miranda-Agrippino and Rey (2020) to jointly model financial cycles across multiple economies and also teasing out whether financial cycles comove across multiple economies, and if so what causes such comovement, would be an interesting avenue to pursue.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jedc.2022.104315.

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Online Appendix to

A Unified Approach for Jointly Estimating the Business and Financial Cycle, and the Role of Financial Factors *

Tino Berger[1], Julia Richter[1,2], and Benjamin Wong[3]

[1]Georg-August University Göttingen
[2]German Council of Economic Experts
[3]Monash University, Australia

January 15, 2022

*This paper reflects the personal views of the authors and not necessarily those of the German Council of Economic Experts. Email: Berger: tino.berger@wiwi.uni-goettingen.de Richter: julia.richter@svr-wirtschaft.de Wong: benjamin.wong@monash.edu

A Data

The data is mostly sourced from the Federal Reserve Economic Data (FRED), with a few series from other sources. “Adjust” refers to any data transformations: “ln” indicates natural logarithms, “Δ” indicates that the variable has been differenced, and 'break' indicates that the series has been adjusted for a break in the mean. Like Kamber and Wong (2020), we date the break using a sup-F statistic.

Series	**FRED Mnemonic or Source**	**Adjust**
Real Gross Domestic Product	GDPC1	ln, Δ
Real Personal Consumption Expenditures	PCECC96	ln, Δ
Real Gross Private Domestic Investment	GPDIC1	ln, Δ
Real Personal Income	PI	ln, Δ, break in 1984Q4
Industrial Production Index	INDPRO	ln, Δ
Capacity Utilization (Manufacturing)	CAPUTLB00004SQ	break in 2001Q1
All Employees: Total Nonfarm Payrolls	PAYEMS	ln, Δ, break in 2000Q2
Civilian Unemployment Rate	UNRATE	break in 1987Q2
Nonfarm Business Sector: Hours of All Persons	HOANBS	ln, Δ, break in 2000Q2
Nonfarm Business Sector: Real Output Per Hour of All Persons	OPHNFB	ln, Δ
Housing Starts: Total: New Privately Owned Housing Units Started	HOUST	ln, Δ
Real Residential Property Prices for U.S. (BIS)	QUSR628BIS	ln, Δ
Consumer Price Index for All Urban Consumers: All Items	CPIAUCSL	ln, Δ, break in 1982Q1
Producer Price Index for All Commodities	PPIACO	ln, Δ, break in 1981Q3
Effective Federal Funds Rate	FEDFUNDS	break in 1991Q3
10-Year Treasury Constant Maturity Rate	GS10	break in 1997Q4
Real M1 Money Stock	M1SL	ln, Δ, break
Real M2 Money Stock	M2SL	ln, Δ
Real Credit	Total Credit to Private Non-Financial Sector, Adjusted for Breaks (CRDQUSAPABIS) deflated by GDP Deflator (GDPDEF)	ln, Δ, break in 2008Q1
Excess Bond Premium	Gilchrist and Zakrajšek (2012), updated by Boston Fed	
S&P 500 Index	Yahoo Finance	ln, Δ
Real Energy Prices	Pinksheet (World Bank) deflated by CPI	ln, Δ
CBOE Volatility Index	VXOCLS and backcasted through Caggiano, Castelnuovo, and Groshenny (2014)	

B Bayesian Estimation and Dummy Observations

We estimate the medium-sized 23 variable BVAR by utilizing the natural-conjugate Normal-Wishart prior which draws on elements of the Minnesota prior (see e.g. Litterman, 1986).

In order to estimate the BVAR, we cast Eq. (10) in the main text into a system of multivariate regressions of the form (see e.g. Robertson and Tallman, 1999; Banbura, Giannone, and Reichlin, 2010)

$$Y = X\beta + u, \tag{B.1}$$

where $Y = [Y_1, \ldots, Y_T]'$, $X = [X_1, \ldots, X_T]'$ with $X_t = [Y'_{t-1}, \ldots, Y'_{t-p}]$ and $u = [u_1, \ldots, u_T]'$. The Normal-Wishart prior distribution then takes the form

$$vec(\beta)|\Sigma \sim \mathcal{N}(vec(\beta_0), \Sigma \otimes \Omega_0) \quad \text{and} \quad \Sigma \sim \mathcal{IW}(S_0, a_0), \tag{B.2}$$

where we set the prior parameters $\beta_0, \Omega_0, S_0,$ and a_0 such that they are consistent with the structure given by Eqs. (11) and (12) in the main text and the expectation of Σ being $diag(\sigma_1^2, \ldots, \sigma_n^2)$. The prior in Eq. (B.2) can be implemented by means of dummy observations (see e.g. Del Negro and Schorfheide, 2011; Woźniak, 2016):

$$Y_d = \begin{pmatrix} 0_{np,n} \\ diag(\sigma_1 \ldots \sigma_n) \end{pmatrix}, \quad X_d = \begin{pmatrix} J_p \otimes diag(\sigma_1 \ldots \sigma_n)/\lambda \\ 0_{np,n} \end{pmatrix}, \tag{B.3}$$

where Y_d and X_d are the dummy observations chosen according to Eqs. (11) and (12) in the main text, $J_p = diag(1, \ldots, p)$, $S_0 = (Y_d - X_d B_0)'(Y_d - X_d B_0)$, $B_0 = (X_d' X_d)^{-1} X_d Y_d$, $\Omega_0 = (X_d' X_d)^{-1}$, and $a_0 = T_d - np$, where T_d is the number of rows for both Y_d and X_d. The first block of the dummy observations imposes the prior belief on the VAR slope coefficients and the second block contains the prior for the covariance matrix.

Consider the regression in Eq. (B.1) augmented with the dummy observations:

$$Y^* = X^*\beta + u^*, \tag{B.4}$$

where $Y^* = [Y', Y_d']', X^* = [X', X_d']'$ and $u^* = [u', u_d']'$. Estimating the BVAR then simply amounts to conducting least squares regression of Y^* on X^*. The posterior distribution than has the form

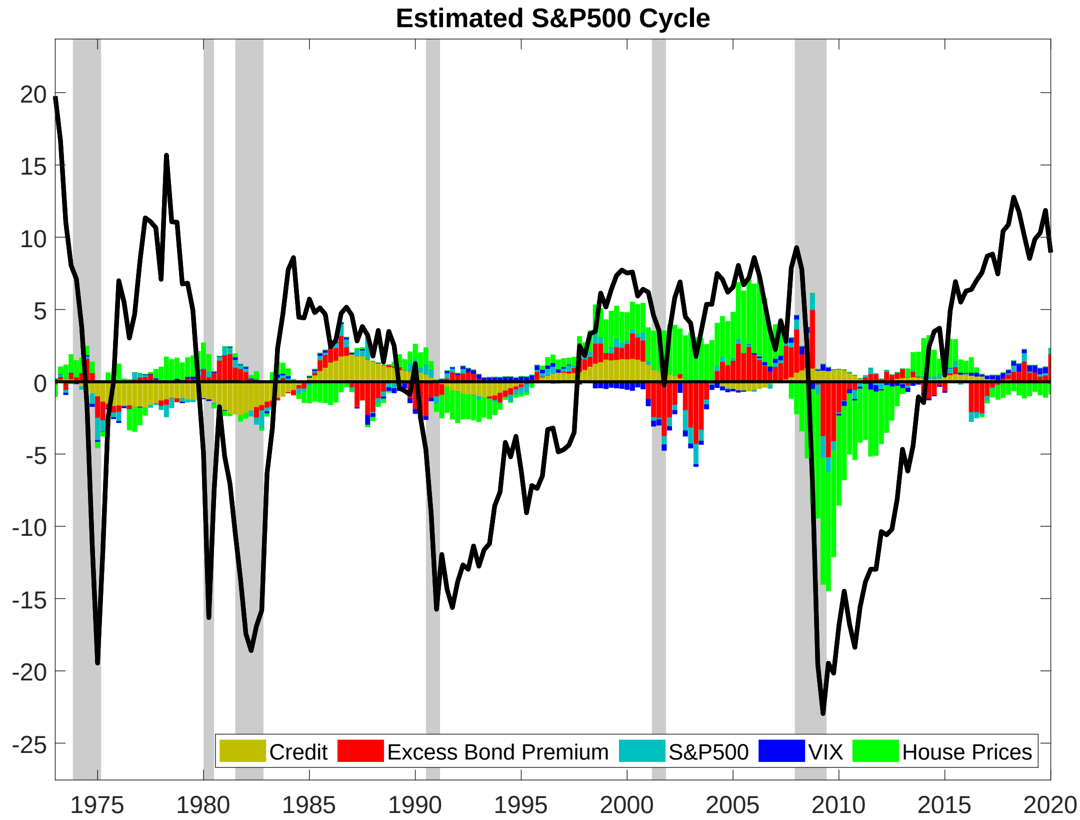
$$vec(\beta)|\Sigma, Y \sim \mathcal{N}(vec(\tilde{\beta}), \Sigma \otimes (X^{*\prime}X^*)^{-1}) \tag{B.5}$$

$$\Sigma|Y \sim \mathcal{IW}(\tilde{\Sigma}, T_d + T - np + 2), \tag{B.6}$$

where $\tilde{\beta} = (X^{*\prime}X^*)^{-1}X^{*\prime}Y^*$ and $\tilde{\Sigma} = (Y^* - X^*\tilde{\beta})'(Y^* - X^*\tilde{\beta})$.

C Decomposition of the Stock Price Cycle

Figure C.1: Informational decomposition of the estimated stock price cycle.



Notes: Solid line plots the estimated stock price cycle in % deviations from the trend. Grey shaded areas indicate NBER recessions. The bars represent the contributions of each of the BVAR forecast errors from our five financial variables (credit, the excess bond premium, the stock price index, the VIX, and house prices) for the stock price cycle.

Figure C.1 presents the informational decomposition of the estimated stock prices cycle calculated using Equation (6) in the main paper. The contributions are calculated from the forecast errors of the five financial variables in our BVAR system: real credit, the excess bond premium, stock prices, the VIX, and house prices.

We note that the stock market cycle has characteristics that look very much like the output gap. Similar to the output gap, amongst the financial variables, we find that the forecast errors of the excess bond premium and of house prices contribute much to the stock price cycle. We present Figure C.1 for completeness as we had mentioned in the main text, there is no consensus on the definition of the financial cycle. A reason why the financial cycle literature has not often considered the stock market cycle is because the stock market features a high degree of high-frequency volatility. In fact, as we show, the stock market cycle appears more like the output gap than the credit and housing price cycle which we had estimated in the main text.

D Details of Identification of Financial Shocks

Cholesky Decomposition We follow Gilchrist and Zakrajšek (2012) by utilizing (orthogonal) variation in the excess bond premium to identify financial shocks. Mechanically, implementation amounts to ordering the excess bond premium after slow-moving variables such as GDP, investment, etc., and before fast-moving variables, which are often the financial market variables such as stock prices. This assumes that slow-moving variables do not react contemporaneously to the financial shock and shocks in the fast-moving block. At the same time, shocks in the fast-moving block do not have a contemporaneous effect on the excess bond premium. Specifying a slow-moving and fast-moving block is a reasonably common strategy for using the Cholesky decomposition within a system that features both financial and macroeconomic variables (e.g., see Christiano, Eichenbaum, and Evans, 1999; Bernanke, Boivin, and Eliasz, 2005). As Gilchrist and Zakrajšek (2012) point out, the identified shock is a shock to the excess bond premium which is orthogonal to other shocks in the economy. We will interpret this shock as a structural financial shock within this setting. While we are aware of possible misgivings against the zero restrictions implied by the Cholesky decomposition, we add that this is a standard identification strategy used in the wider literature (e.g., Gilchrist, Yankov, and Zakrajšek, 2009; Walentin, 2014), which at least provides a first pass at identifying a financial shock before moving on to other identification strategies.

Penalty Function Drawing inspiration from Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016), we consider a penalty function approach in order to identify financial shocks. This entails using a penalty function to identify the financial shock by solving for the shock to maximize the variance of the excess bond premium over the first 4 quarters.[1] Like the Cholesky decomposition, the penalty function approach also relies on orthogonalized variation in the excess bond premium to identify financial shocks. The penalty function approach though, relaxes many of the zero restrictions one utilizes in the Cholesky decomposition, which one may view as being more tenable.

Sign Restrictions We also consider sign restrictions to identify the financial shock. The identification of the financial shock closely mimics Furlanetto, Ravazzolo, and Sarferaz (2019). Furlanetto, Ravazzolo, and Sarferaz (2019) derive their sign restrictions by characterizing a financial shock as a shock which induces an investment and stock market boom/slump. Guided by Furlanetto, Ravazzolo, and Sarferaz (2019), Table 1 summarizes the sign restrictions to identify a financial shock. The signs are normalized where a positive financial shock leads to investment and stock market slumps. The identification strategy also imposes investment to fall

[1] We note that our objective differs from Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016), who use the penalty function approach to distinguish financial shocks from uncertainty shocks. Unlike them, we do not attempt to identify an uncertainty shock. In their approach, the choice of whether to first identify the financial or uncertainty shock may matter. Therefore, strictly speaking, our approach is only identical to Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016) in the setting where a financial shock is identified *before* the uncertainty shock. We will return to the issue if one wanted to also identify an uncertainty shock in the Section G on the online appendix.

Real GDP	-	Housing starts	-	M1	NA
CPI Inflation	-	PPI Inflation	NA	M2	NA
Federal Funds Rate	-	Hours Worked	NA	Credit	-
Employment	-	Personal Income	-	S&P 500	-
Consumption	-	10-year rate	NA	Real Energy Prices	NA
Industrial Production	-	Productivity	NA	VIX	+
Capacity Utilization	-	Investment	-	Property prices	-
Unemployment	+	Excess Bond Premium	+	Investment/GDP ratio	-

Table 1: The table describes the sign restriction on each variable in order to identify the financial shock. NA indicates that the response of the variable to a financial shock is left unrestricted. The sign restriction is restricted to only hold upon impact.

more than GDP in response to a positive financial shock as an investment boom/slump forms part of their identification strategy.[2] All the sign restrictions hold only upon impact. While we do not identify more than a single financial shock, guided by Antolín-Díaz and Rubio-Ramírez (2018) we impose a narrative in addition to the sign restrictions. In smaller systems, identifying more shocks, as Furlanetto, Ravazzolo, and Sarferaz (2019) do, can yield sharper inference. Because the size of our system makes identifying more shocks a more challenging endeavor, we use the narrative sign restriction, as opposed to identifying more shocks, as a means of introducing additional information to help with the identification of the financial shock. The events we have in mind are the collapse of Lehman in September 2008 and credit freezing in 2008Q4. We therefore implement a narrative sign restriction that the sign of the financial shock is positive in both 2008Q3 and 2008Q4. In addition, the financial shock is the overwhelming driver of the increase in the excess bond premium between 2008Q3 to 2008Q4.

[2]Note that as Furlanetto, Ravazzolo, and Sarferaz (2019) normalize the financial shock to induce an investment boom, so positive financial shocks in their identification causes the investment to GDP ratio to rise. However, since we normalize a positive financial shock to induce an investment slump, to make the sign of the financial shock consistent with our other two identification strategies, investment will fall more than GDP in response to a positive financial shock, so the investment to GDP ratio falls.

E Impulse Response Functions

Figure E.2 presents the impulse response functions to a one standard deviation structural financial shock for all three identification strategies. All the impulse response functions are conditional on the posterior mode of the BVAR estimates and we report the impulse response functions of the sign restrictions using the Fry and Pagan (2011) median target approach.[3] On the estimated impulse response functions, we draw attention to two key points. First, while the sign restrictions do impose the responses of particular variables to financial shocks as per Table 1, the responses of all variables to a financial shock identified using all three identification strategies have the same sign. The effect of a financial shock is therefore qualitatively similar across all three identification strategies, and the difference is largely confined to the extent of the magnitude of the responses. Therefore, our results should provide at least some confidence that all three identification strategies are providing reasonable estimates of the effect of financial shocks. Second, while we restrict the sign of prices and GDP to fall in the sign restrictions, we obtain similar results with the other two identification strategies where the sign is left unrestricted. Therefore, there is at least consistent evidence that the identified financial shock in all three settings is an aggregate demand shock, or at least one where the effect on the aggregate demand side of the economy dominates.

To get a sense of the sampling uncertainty, Figures E.3 to E.8 present the posterior distribution of the impulse response functions (IRF) to a one standard deviation financial shock.

Figures E.3 and E.4 present the IRF constructed using the posterior mode of the BVAR parameters (or equivalently the posterior mean or median within our class of priors) and the equal tailed 68% pointwise credible interval. Figure E.5 presents the posterior median, together with equal tailed 68% credible interval from the sign and narrative restrictions identification.[4]

As is well known, pointwise quantiles may obscure information about the dynamics and also across different structural models (see Fry and Pagan, 2011; Inoue and Kilian, 2020). An alternative approach is to report membership of a credible set under a suitably specified loss function. We therefore used the procedure described by Inoue and Kilian (2020) under absolute loss to construct credible sets and the optimal estimator for the IRFs. The absolute loss function is defined over the first 21 quarters of the response to a one standard deviation financial shock. Figures E.6 to E.8 report the 68% credible sets as well as the optimal estimator under absolute loss.

For both the Cholesky decomposition and penalty function approach, the posterior distribution of the impulse response functions is constructed by taking 1000 draws from the

[3]There is a known issue of representativeness of the impulse response function as sign restrictions only identify a set and do not provide a unique solution (see Fry and Pagan, 2011). We report the Fry and Pagan (2011) median target approach here from 1000 admissible solutions conditional on the posterior mode parameters. This is just for illustrative purposes as we wish to just compare the sign of the responses of all three identification schemes.

[4]We present the posterior median for the sign restriction case as the posterior mode from the reduced form does not necessarily imply a unique structural model. While we are aware of known misgivings, this is common practice (see, e.g. Antolín-Díaz and Rubio-Ramírez, 2018).

posterior distribution of the reduced form, as per Eqs. (B.5) and (B.6) and subsequently applying the identification strategy to the reduced form. The posterior distribution for the sign restrictions with the narrative is constructed using the algorithm by Antolín-Díaz and Rubio-Ramírez (2018), where we first take a draw from our reduced form posterior distribution and multiplied a Cholesky factorization of the draw from the posterior distribution of the covariance matrix by a randomly drawn orthonormal matrix. If the draw satisfies the sign and narrative restriction, we keep the draw, otherwise we discard it. We iterate the algorithm until we find 1000 draws that satisfy the sign and narrative restrictions from this first stage. Thereafter, we construct the resampled importance weights, as described by Antolín-Díaz and Rubio-Ramírez (2018), and use the importance weights to reweight the 1000 draws from the first stage to construct the posterior distribution of impulse response functions.

All three identification procedures imply qualitatively very similar responses to a financial shock. Overall, the responses identified with the Cholesky decomposition are less pronounced as compared to those identified with the penalty function and sign restrictions. For the penalty function approach, this result is not surprising, because the objective function that we are using to identify the financial shock maximizes responses of the excess bond premium. We also note that the credible sets under absolute loss do imply more estimation uncertainty relative to the intervals constructed using pointwise quantiles, which is fairly common.

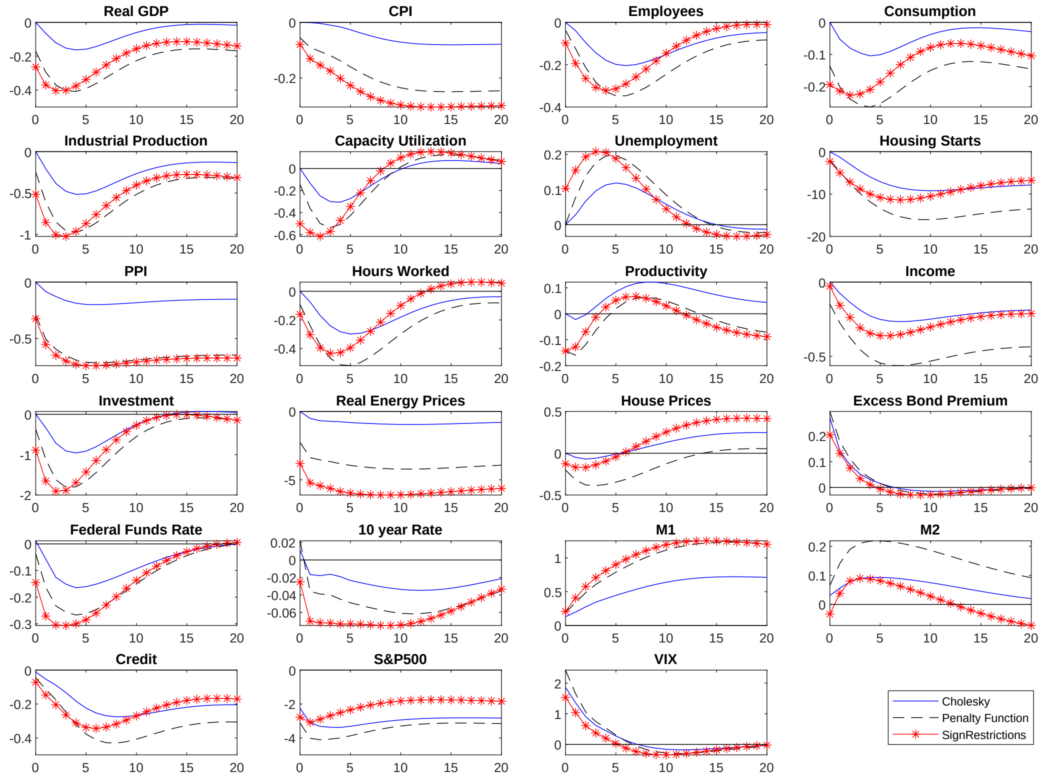


Figure E.2: Estimated impulse response function to a one standard deviation financial shock. All impulse response functions are calculated from the posterior mode of the estimated BVAR. The sign restriction impulse response function is chosen from a representative model using the Fry-Pagan median target from 1000 rotations of the posterior mode parameters which satisfy the sign and narrative restrictions. The x-axis is in terms of quarters after the shock. Capacity utilization, unemployment, Federal funds rate, and 10 years rate are in terms of percentage point deviation. VIX and excess bond premium are in their natural units. All other variables are in terms of percentage change.

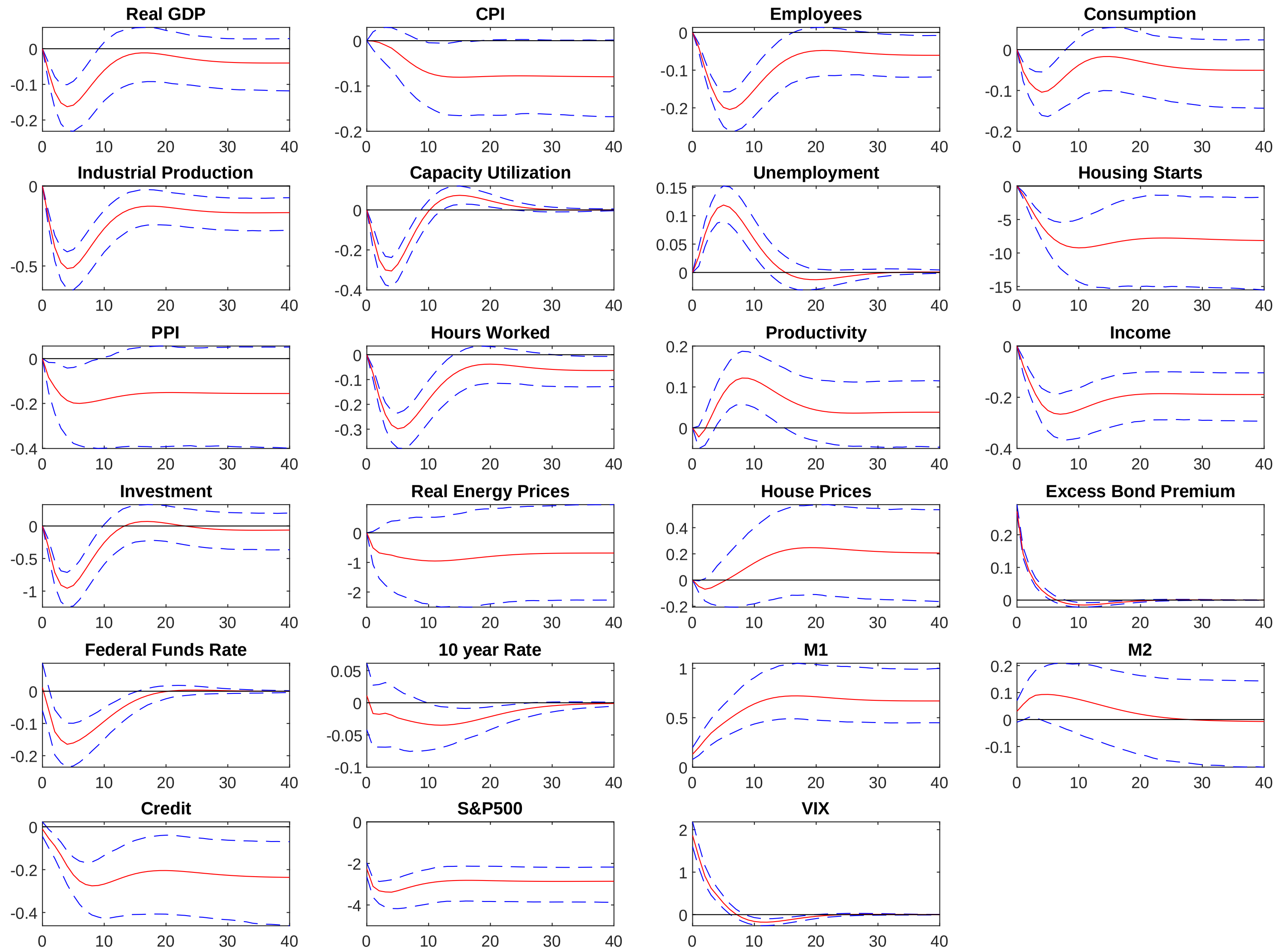


Figure E.3: Impulse response functions to a one standard deviation financial shock (Cholesky identification). Posterior mode with 68% pointwise credible interval. The x-axis is in terms of quarters after the shock. Capacity utilization, unemployment, Federal funds rate, and 10 years rate are in terms of percentage point deviation. VIX and excess bond premium are in their natural units. All other variables are in terms of percentage change.

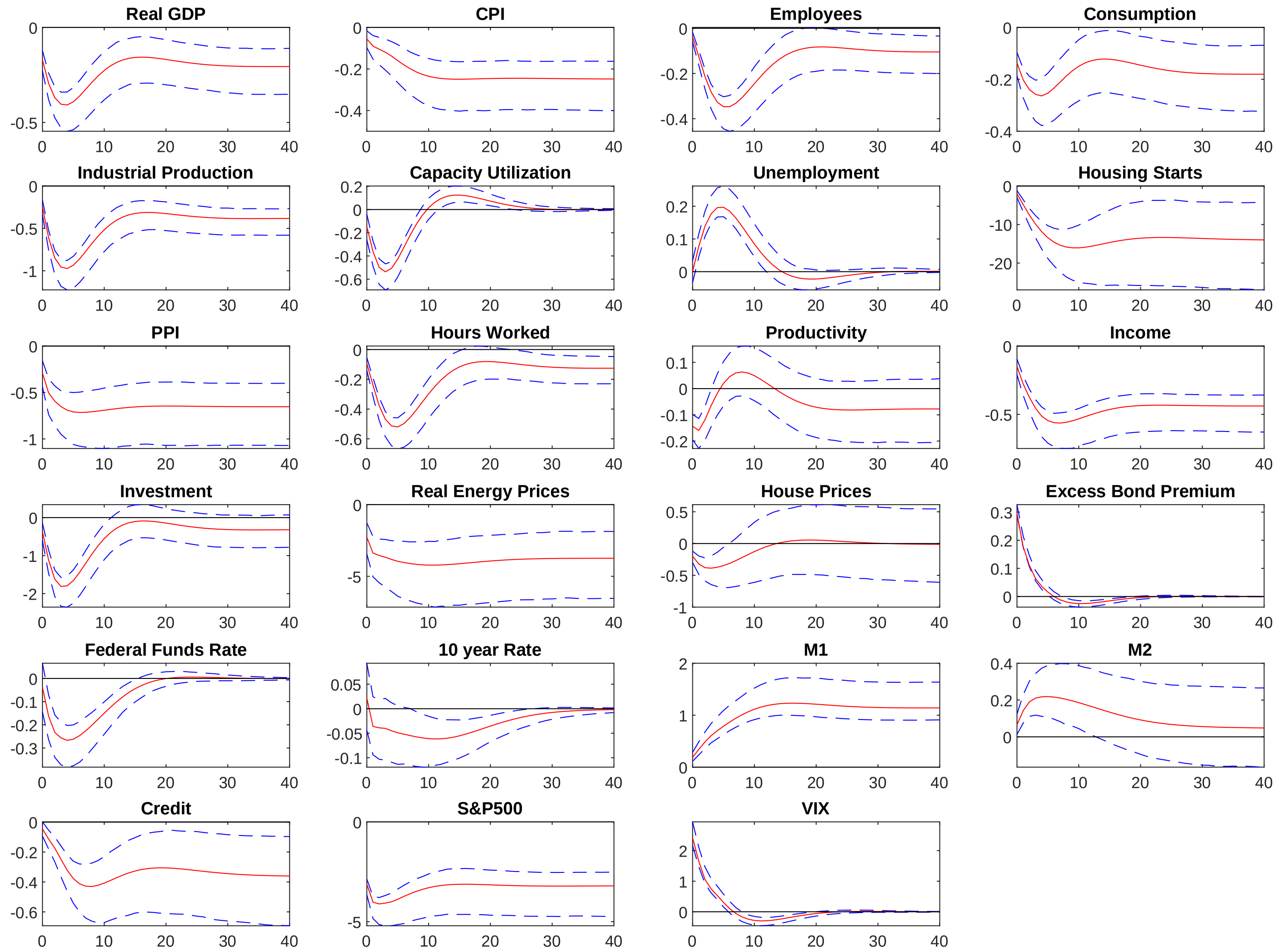


Figure E.4: Impulse response functions to a one standard deviation financial shock (penalty function identification). Posterior mode with 68% pointwise credible interval. The x-axis is in terms of quarters after the shock. Capacity utilization, unemployment, Federal funds rate, and 10 years rate are in terms of percentage point deviation. VIX and excess bond premium are in their natural units. All other variables are in terms of percentage change.

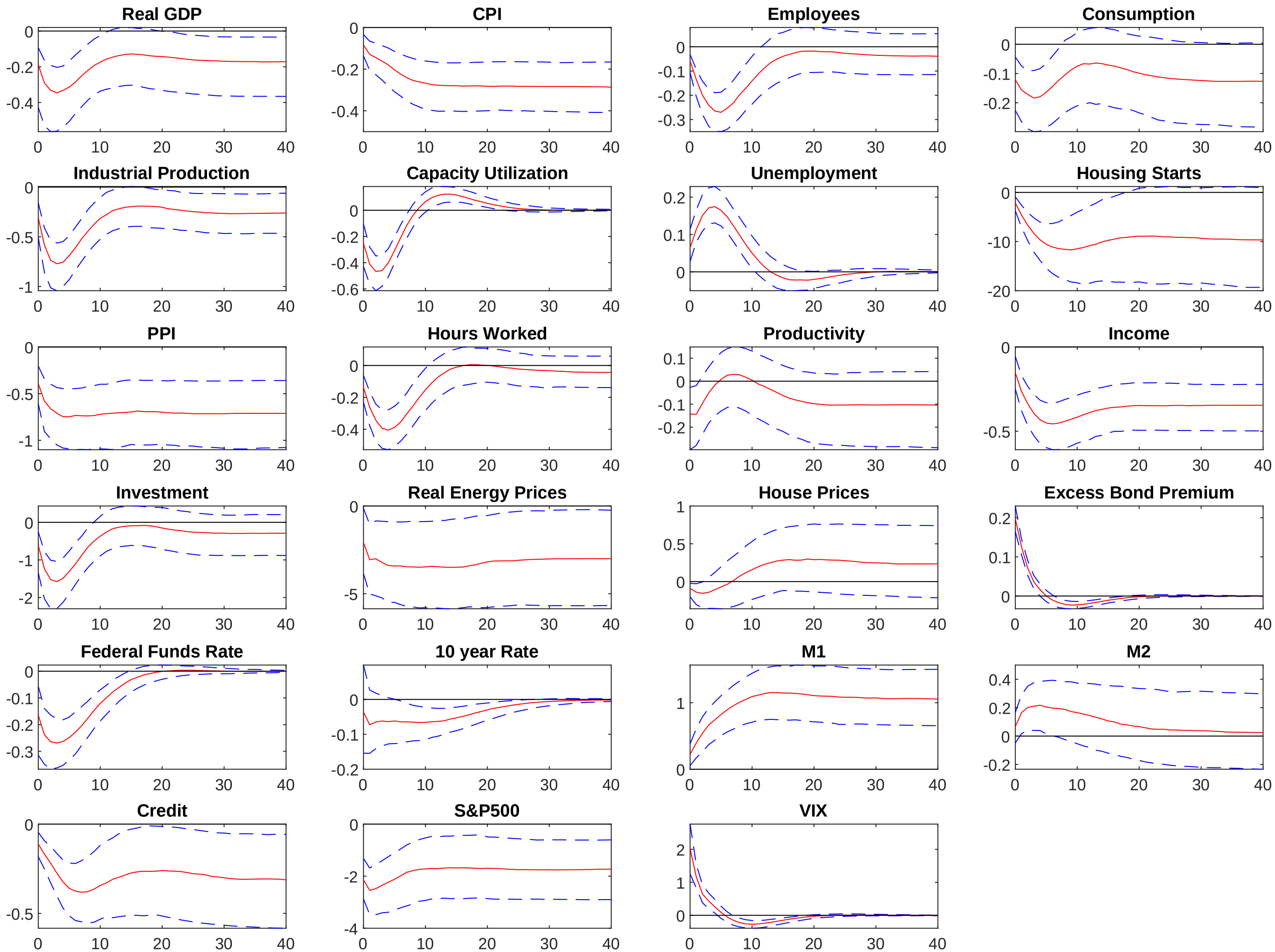


Figure E.5: Impulse response functions to a one standard deviation financial shock (sign restrictions). Posterior median with 68% pointwise credible interval. Posterior distribution are obtained as described in Antolín-Díaz and Rubio-Ramírez (2018). The x-axis is in terms of quarters after the shock. Capacity utilization, unemployment, Federal funds rate, and 10 years rate are in terms of percentage point deviation. VIX and excess bond premium are in their natural units.

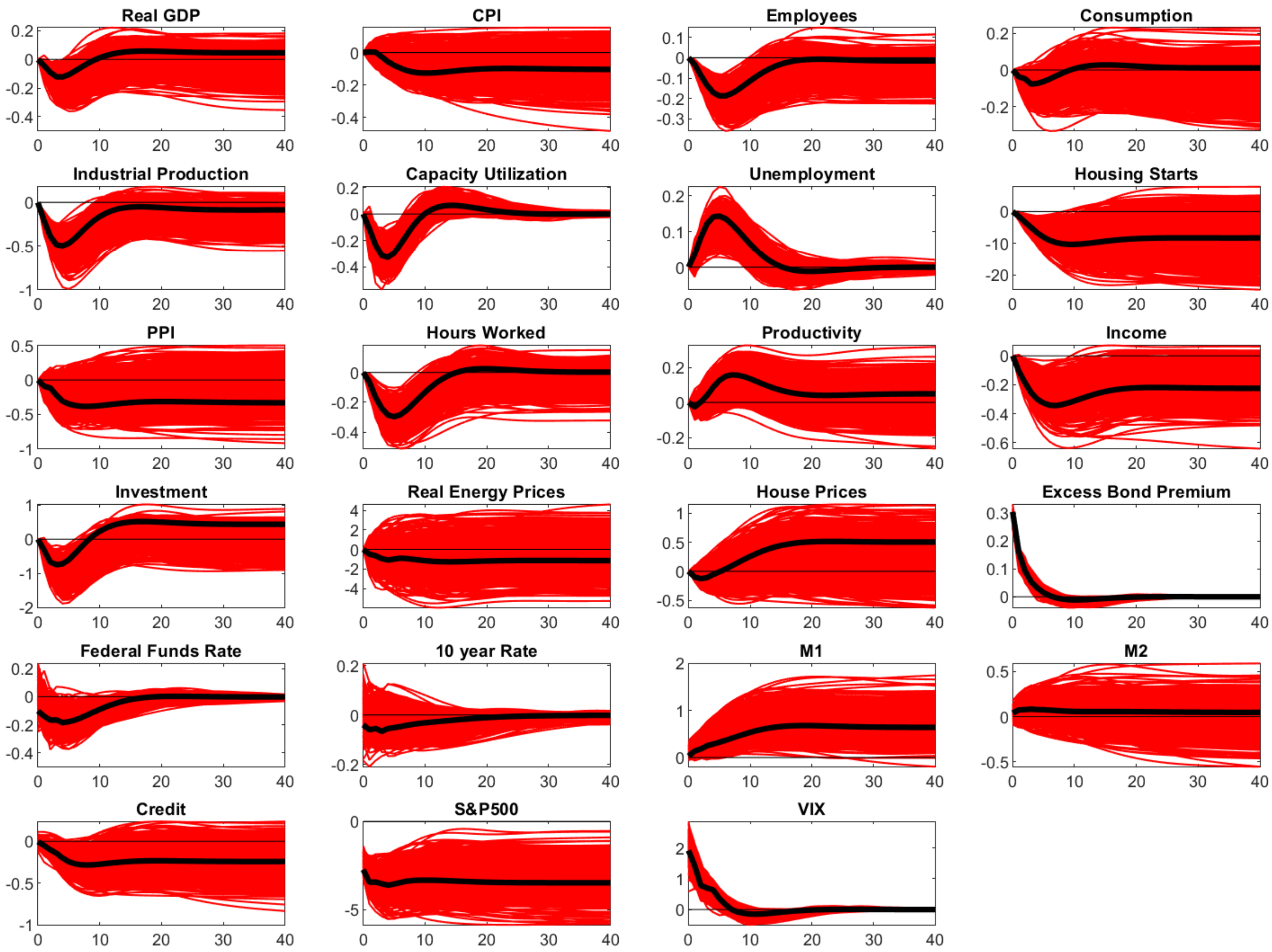


Figure E.6: Impulse response functions to a one standard deviation financial shock (Cholesky identification). Optimal estimator with 68% credible sets under optimal loss as described by Inoue and Kilian (2020). The x-axis is in terms of quarters after the shock. Capacity utilization, unemployment, Federal funds rate, and 10 years rate are in terms of percentage point deviation. VIX and excess bond premium are in their natural units. All other variables are in terms of percentage change.

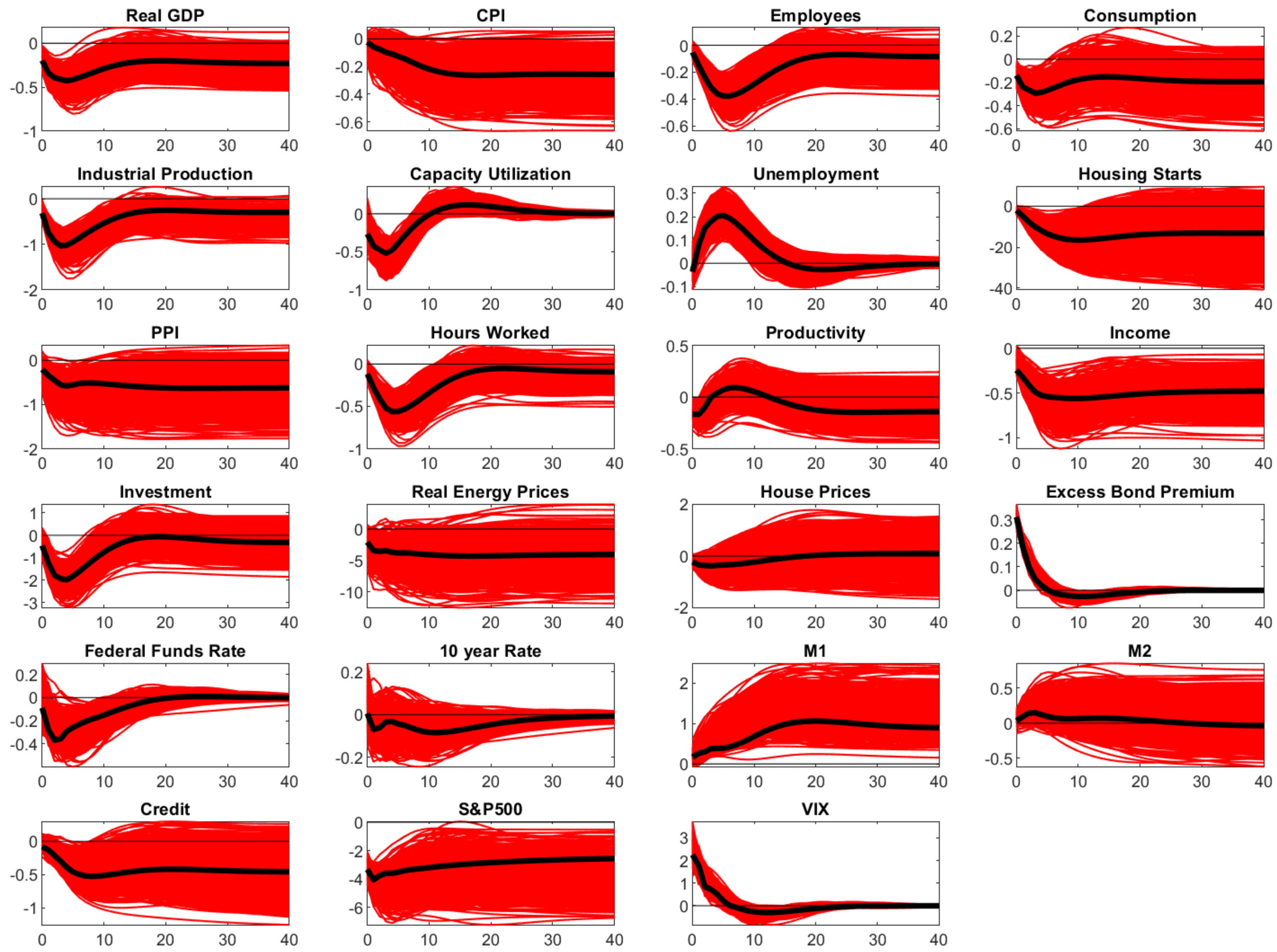


Figure E.7: Impulse response functions to a one standard deviation financial shock (penalty function identification). Optimal estimator with 68% credible sets under optimal loss as described by Inoue and Kilian (2020). The x-axis is in terms of quarters after the shock. Capacity utilization, unemployment, Federal funds rate, and 10 years rate are in terms of percentage point deviation. VIX and excess bond premium are in their natural units. All other variables are in terms of percentage change.

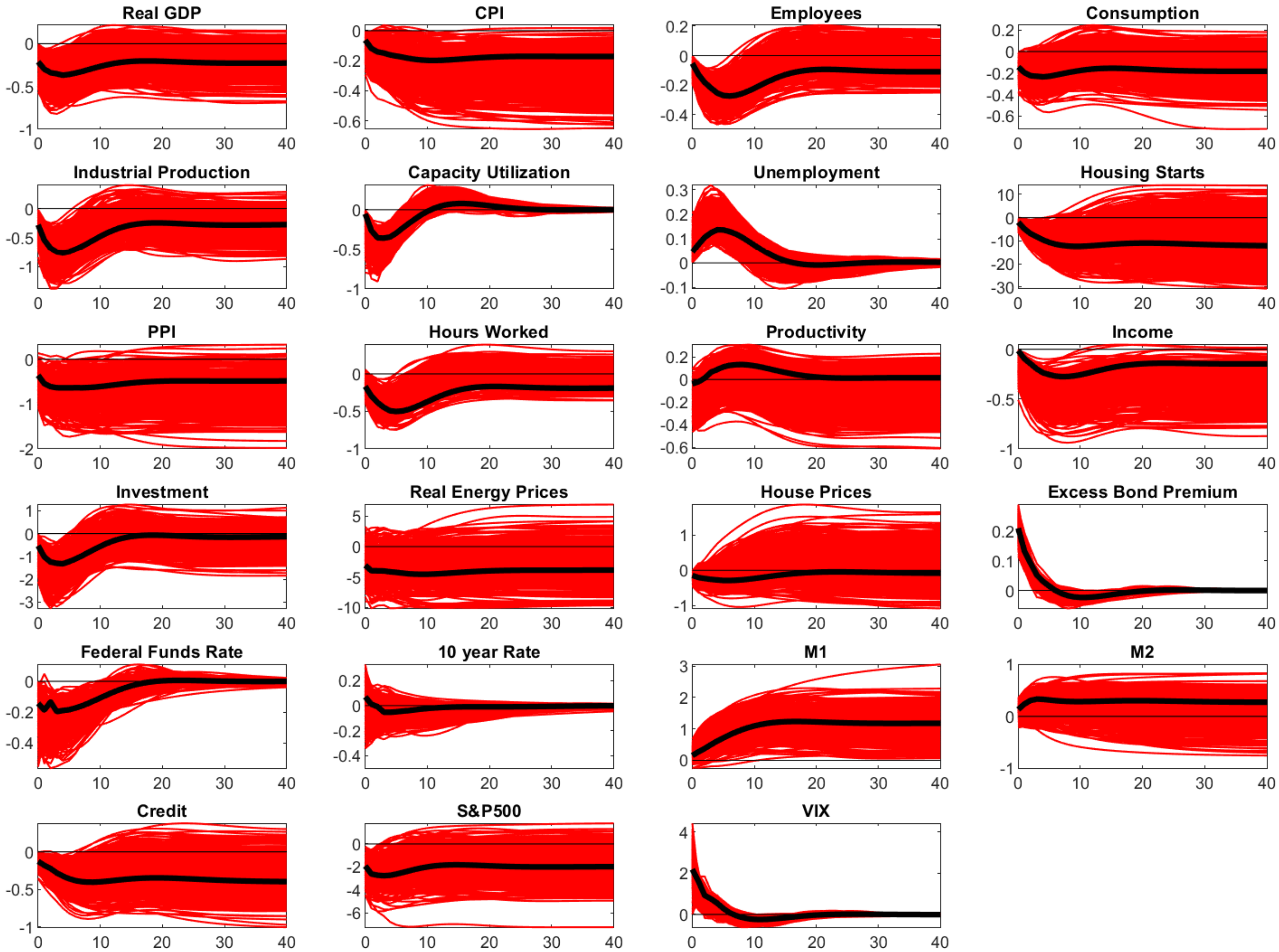


Figure E.8: Impulse response functions to a one standard deviation financial shock (sign restrictions). Optimal estimator with 68% credible sets under optimal loss as described by Inoue and Kilian (2020). The x-axis is in terms of quarters after the shock. Capacity utilization, unemployment, Federal funds rate, and 10 years rate are in terms of percentage point deviation. VIX and excess bond premium are in their natural units. All other variables are in terms of percentage change.

F Addressing Breaks

As a number of series feature breaks in their unconditional mean, we briefly discuss some alternatives. Taking guidance from Kamber and Wong (2020), our baseline approach involves using the Bai and Perron (2003) test to impose breaks in the unconditional mean. Under our baseline, we could not find evidence of a break in the drift of real GDP. However, this evidence appears fairly mixed depending on how we conducted the Bai and Perron (2003) test. We thus allowed for a possible break in 2006Q2. Figures F.9 to F.11 presents key facets of our analysis, the informational decomposition of the financial variables, the sum of the financial variables in the informational decomposition, and the role of financial shocks driving the output gap in the structural analysis. Our key results remain robust to allowing for a break in the drift of real GDP in 2006Q2; that is financial shocks appeared to play a role in heating the real economy in the 2000s, and in the informational decomposition, this role appears to be well proxied by the role of the excess bond premium.

Instead of allowing for a sharp break in the mean as identified by the Bai and Perron (2003) test, we followed the approach by Stock and Watson (2012) and demeaned all our variables using a biweight kernel with a bandwidth of 100 quarters prior to estimation. Figure F.12 presents the estimated cycle using this alternative demeaning approach relative to our baseline. While the estimated cycles do deviate slightly from our baseline, the fluctuations of the estimated cycles appear fairly comparable relative to our baseline. Figures F.13 to F.15 reproduces the analysis where we present the informational decomposition of the cycles to the financial variables and the identified financial shocks. Our key results remain robust to allowing for this alternative form of demeaning; we once again find that financial shocks appeared to play a role in heating the real economy in the 2000s, and in the informational decomposition, this role appears to be well proxied by the role of the excess bond premium.

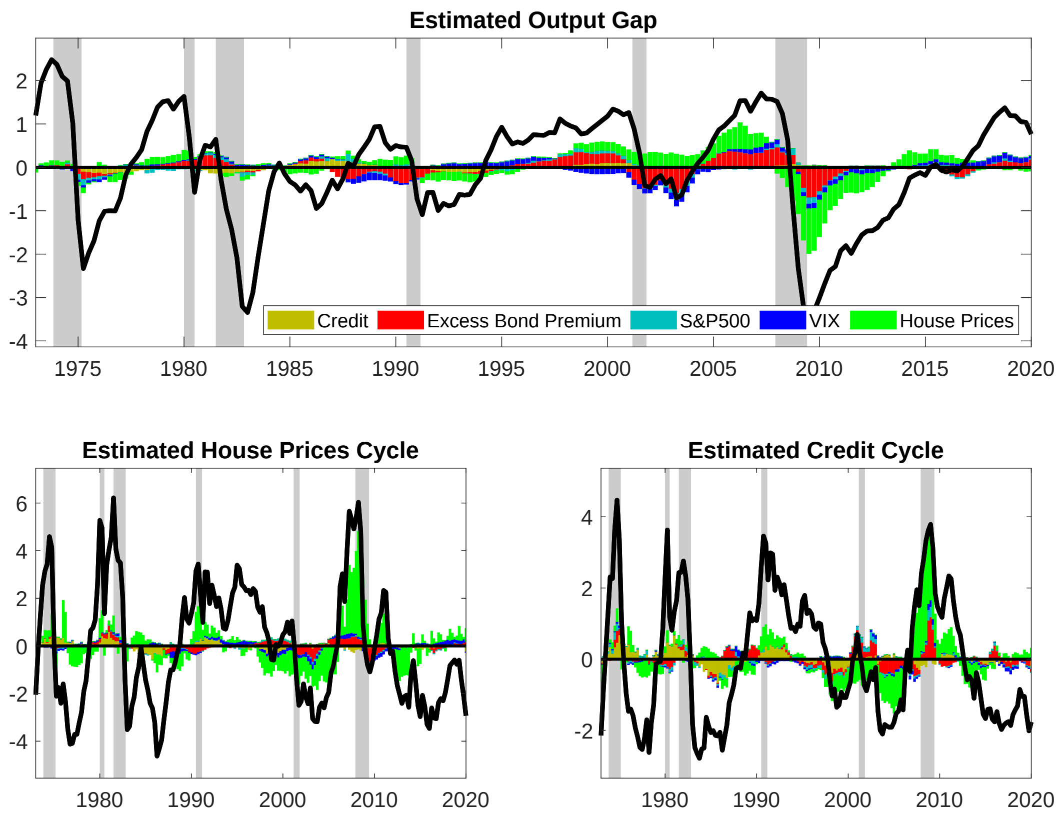


Figure F.9: Informational decomposition of the estimated cycles allowing for break in drift in real GDP in 2006Q2. The solid line denotes the estimated cycle. Cycles are measured in percent deviation from the trend. Grey shaded areas indicate NBER recessions. The bars represent the total contribution of the contribution from the BVAR forecast errors from five financial variables (credit, the excess bond premium, the S&P 500, the VIX index, and the house price).

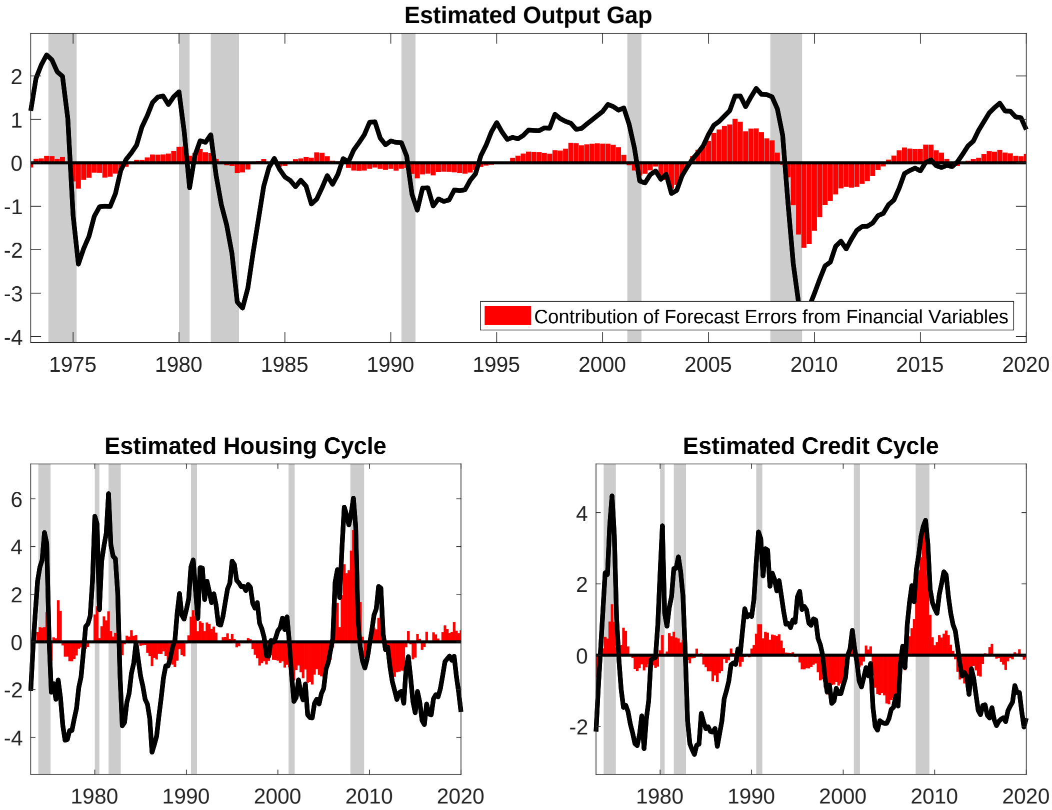


Figure F.10: Informational decomposition of the estimated cycles allowing for break in drift in real GDP in 2006Q2. The solid line denotes the estimated cycle. Cycles are measured in percent deviation from the trend. Grey shaded areas indicate NBER recessions. The bars represent the sum of the individual contribution from the BVAR forecast errors from five financial variables (credit, the excess bond premium, the S&P 500, the VIX index, and the house price). The individual contributions are presented in Figure F.9.

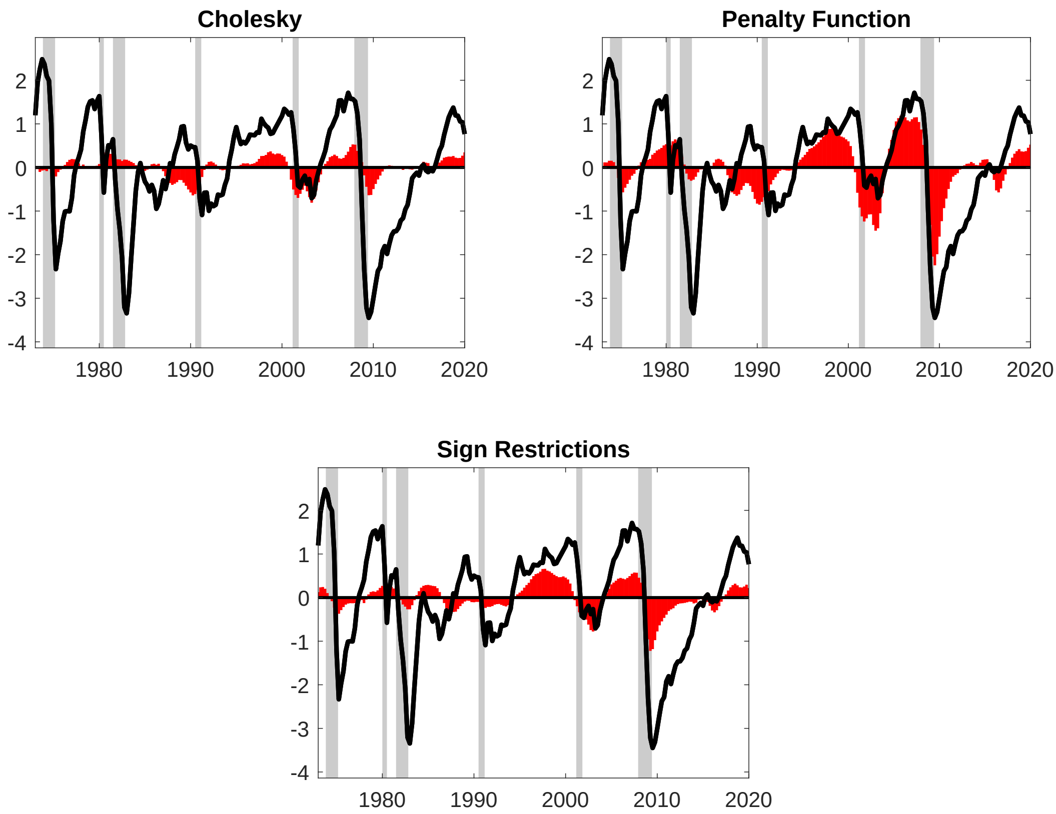


Figure F.11: Allowing for break in drift in real GDP in 2006Q2. The solid line is the estimated output gap. Output gap is measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The bars present the contribution of financial shocks to the estimated output gap. The title refers to the different identification schemes. The contribution from the sign restriction approach is averaged across draws that satisfy the sign and narrative restrictions.

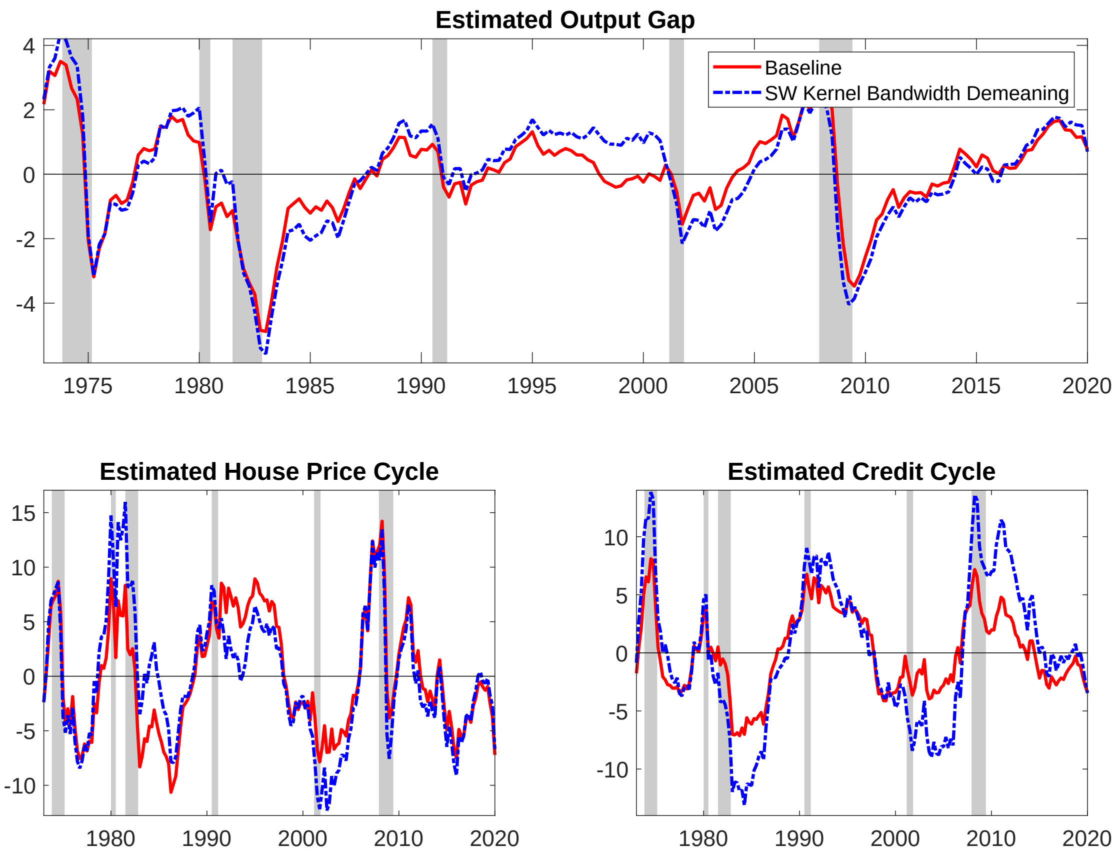


Figure F.12: Estimated cycles from the BVAR. The solid line indicates cycles estimated from our baseline. The dot-dash line presents allowing for demeaning using the biweight kernel with a bandwidth of 100 quarters as described by Stock and Watson (2012). Grey shaded areas indicate NBER recessions.

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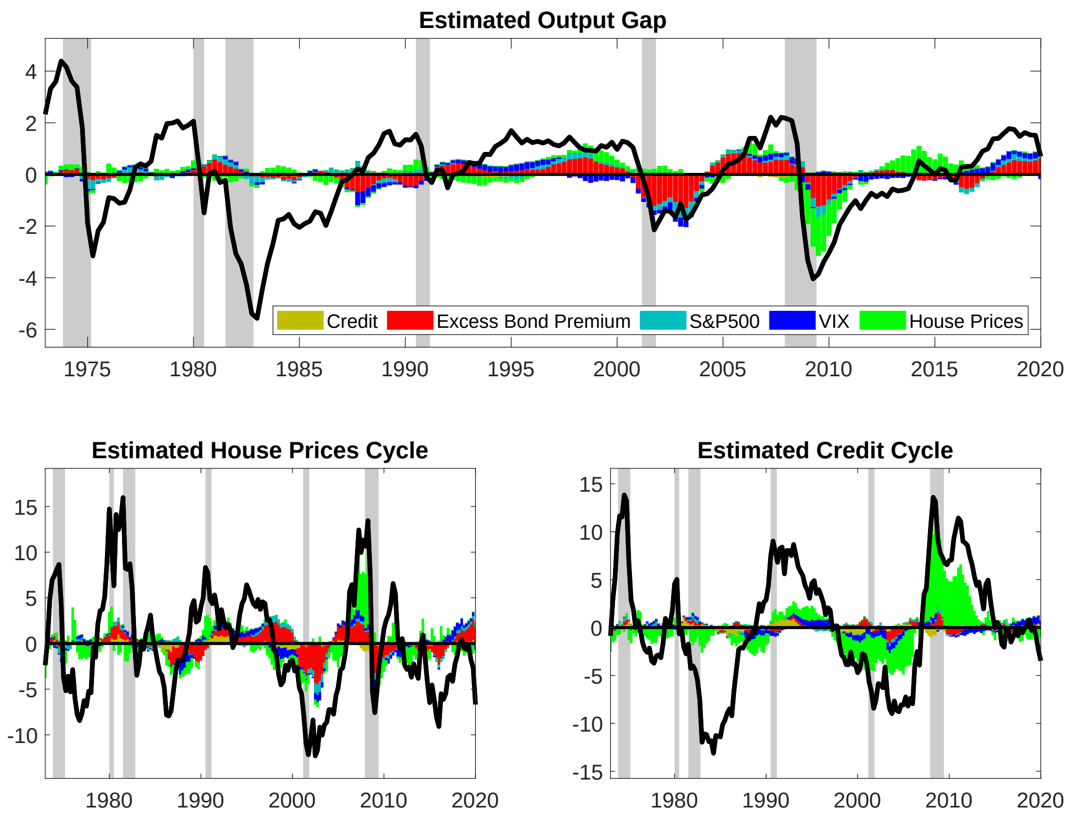

Figure F.13: Informational decomposition of the estimated cycles. The data is first demeaned using the biweight kernel with a bandwidth of 100 quarters as described by Stock and Watson (2012). The solid line denotes the estimated cycle. Cycles are measured in percent deviation from the trend. Grey shaded areas indicate NBER recessions. The bars represent the total contribution of the contribution from the BVAR forecast errors from five financial variables (credit, the excess bond premium, the S&P 500, the VIX index, and the house price).

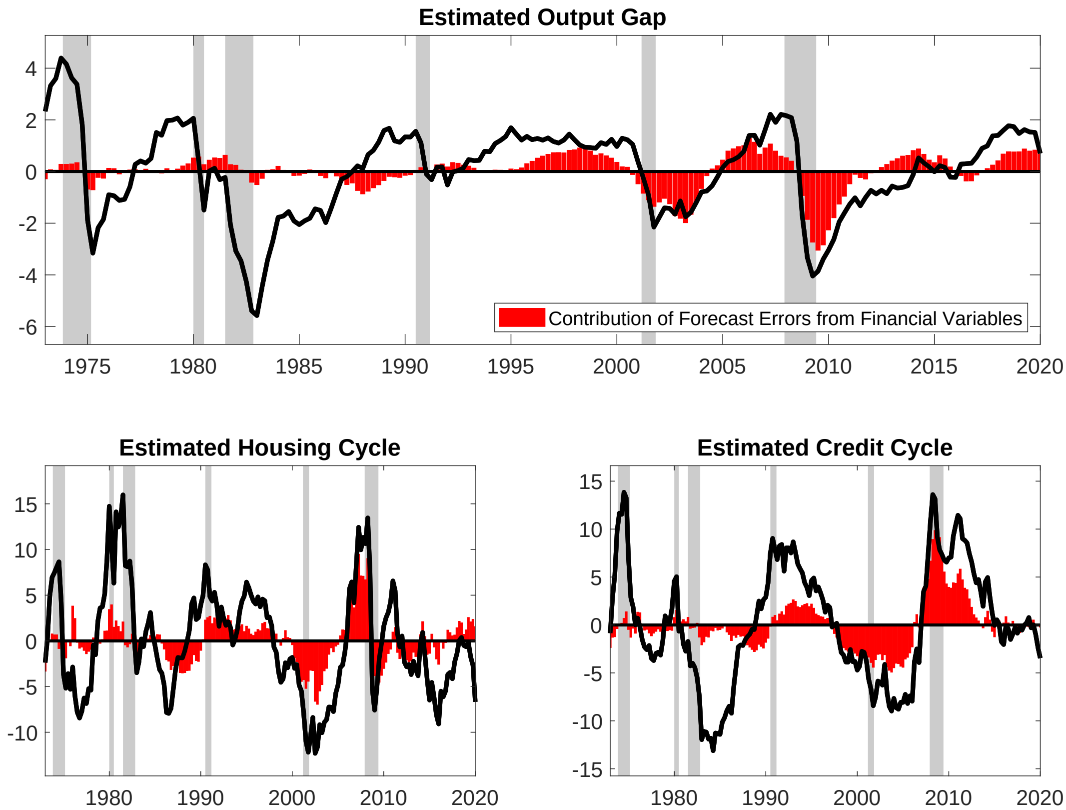

Figure F.14: Informational decomposition of the estimated cycles. The data is first demeaned using the biweight kernel with a bandwidth of 100 quarters as described by Stock and Watson (2012). Cycles are measured in percent deviation from the trend. Grey shaded areas indicate NBER recessions. The bars represent the sum of the individual contribution from the BVAR forecast errors from five financial variables (credit, the excess bond premium, the S&P 500, the VIX index, and the house price).The individual contributions are presented in Figure F.13.

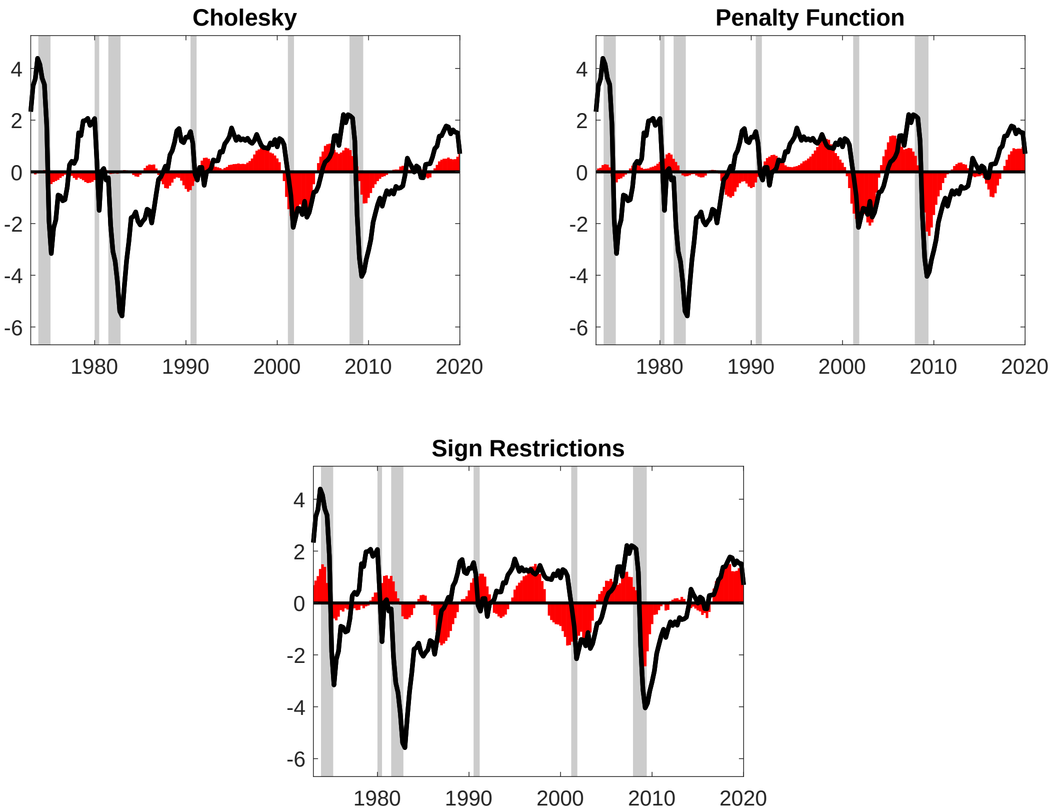

Figure F.15: Contribution of financial shocks to the estimated output gap. The data is first demeaned using the biweight kernel with a bandwidth of 100 quarters as described by Stock and Watson (2012). The solid line is the estimated output gap. Output gap is measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The bars present the contribution of financial shocks to the estimated output gap. The title refers to the different identification schemes. The contribution from the sign restriction approach is averaged across draws that satisfy the sign and narrative restrictions.

G Disentagling Uncertainty and Financial Shocks

We attempt to disentangle financial from uncertainty shocks under both our sign restriction and penalty function identification.

With the penalty function approach, our baseline approach identifies a financial shock by maximizing the four-step ahead forecast error variance decomposition of the excess bond premium.Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016) disentangle financial and uncertainty shocks by maximizing the variance decomposition of the excess bond premium and an uncertainty proxy respectively. However, the order in which one first identifies a financial shock, then uncertainty shock, or vice versa may matter. Since our approach only identifies a financial shock, the role of financial shocks in our setting would be identical to first identifying a financial shock, then uncertainty shock. We call this the baseline only because it is equivalent to identifying the shocks in this order, though noting that we never interpret the uncertainty shock in this setting within our main analysis. We thus also explored the alternative of identifying an uncertainty shock first, then financial shock. These results are presented in Figure G.16. Unsurprisingly, the order in which we identify the financial and uncertainty shock does matter since this has been well-documented by Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016). In this setting, our identified financial shock may be mixing up uncertainty and financial shocks. Nonetheless, when we take the sum of the contributions, these shares appear quite similar whether we first identify a financial shock, then uncertainty shock, or vice versa. We therefore conclude that while our conclusions using the penalty function approach may mix up uncertainty and financial shocks, to the extent that one is prepared to view the role that the two shocks together, it appeared both shocks play an important role in overheating the business cycle in the 2000s, and the subsequent bust.

With the sign restriction, we identify an uncertainty shock alongside the financial shock. That is, we also identify an uncertainty shock that has the same sign pattern as the financial shock, but the uncertainty shock sees a larger increase in the VIX/excess bond premium ratio than the financial shock. This is similar to the robustness check done by Furlanetto, Ravazzolo, and Sarferaz (2019). We present the role of financial shocks on the output gap in Figure G.17. We denote our baseline where we identify only a single financial shock. The results are almost identical to using the alternative where we identify an uncertainty shock alongside a financial shock. For completeness, we also present the role of uncertainty shocks in the bottom panel of Figure G.17. In the sign restriction case, it appears that our baseline identification of financial shocks appears to be very robust even if one includes an uncertainty shock in the system, with the role financial shocks played in overheating the business cycle in the 2000s remaining largely unchanged.

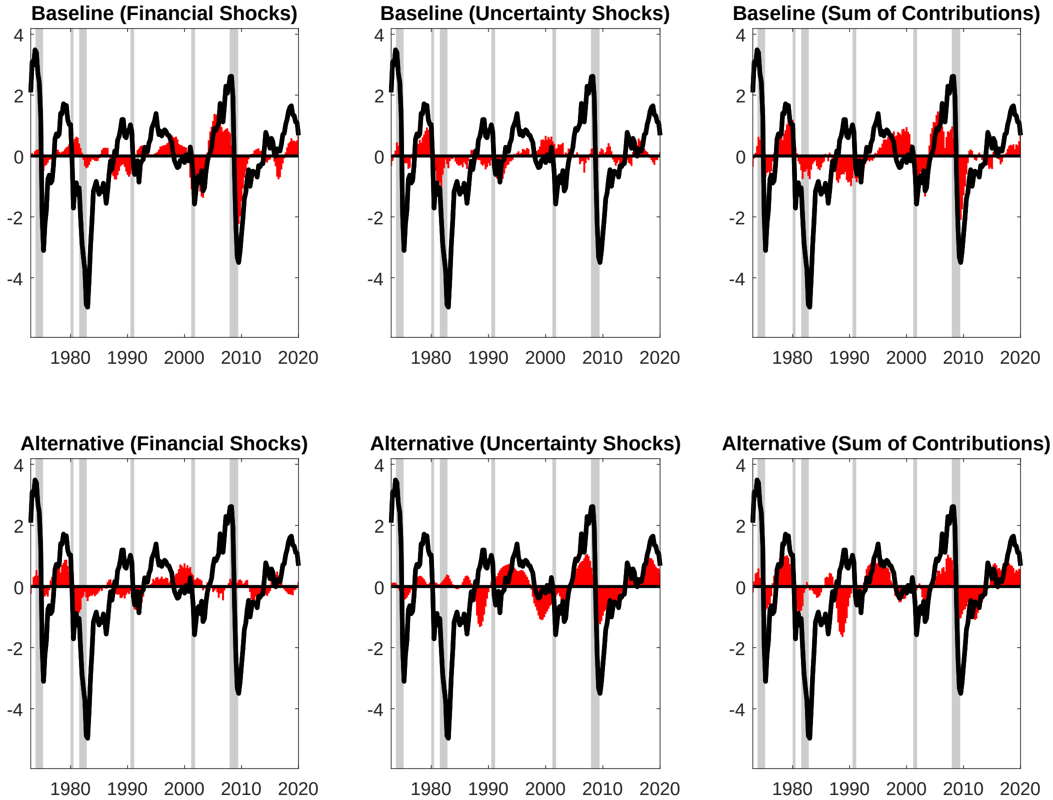

Figure G.16: Contribution of the financial and uncertainty shocks to the estimated output gap using the penalty function identification. The baseline identifies the financial shock, then uncertainty shock. The alternative identifies the uncertainty shock, then financial shock. The solid line represents the estimated output gap. The output gap is measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The title refers to the different identification schemes. The bars represent the contribution of financial shocks to the estimated output gap.

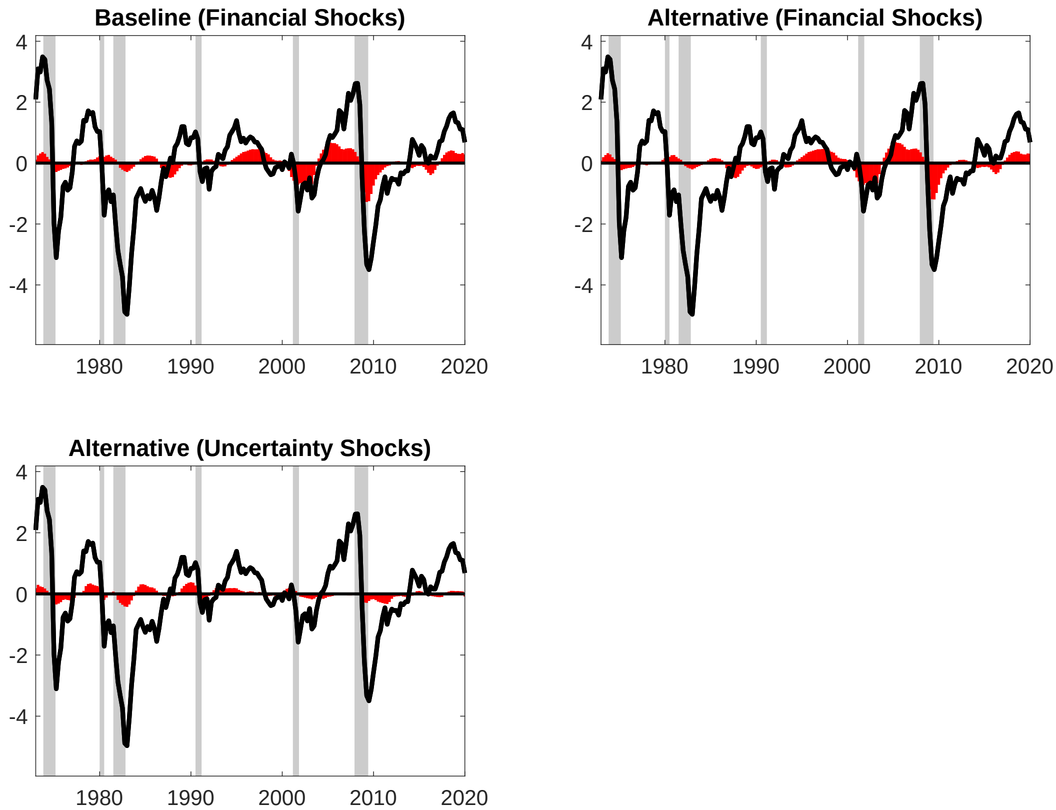

Figure G.17: Contribution of the financial and uncertainty shocks to the estimated output gap using sign and narrative restrictions. The baseline identifies only a single financial shock. The alternative identifies both a financial and uncertainty shock. The output gap is measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The title refers to the different identification schemes. The bars represent the contribution of financial shocks to the estimated output gap. The contribution from the sign restriction approach is averaged across draws that satisfy the sign and narrative restrictions.

H Using Alternative Financial and Housing Indicators

We explore using alternative financial and housing indicators. We used loans instead of credit.[5] The use of loans mimics the choice by Aikman, Haldane, and Nelson (2015) who used loans to study the financial cycle. We also explored using house prices from the Federal Housing Finance Agency (FHFA) and OECD rather than the BIS. Note that the FHFA series starts later in 1975, relative to our baseline which starts in 1973.

Figure H.18 plots the different cycles obtained using the other indicators relative to our baseline. In general, the choice of variable which we use in our analysis does not appear to affect our estimated cycle. Figures H.19 and H.20 present the informational decomposition of the output gap when we change the credit series or change the house price series. Figures H.21 and H.23 present the share of financial shocks under the different identification schemes in driving the output gap obtained in a model using the alternative house price and credit series. In general, we do not find the change in house price or credit series changes our conclusions. In particular, in the informational decomposition, it appears that for much of the overheating of the output gap in the 2000s, it seems the excess bond premium features prominently. In the structural analysis, the financial shock did matter for the overheating of the output gap.

[5] We used "Loans and Leases in Bank Credit, All Commercial Banks" from FRED.

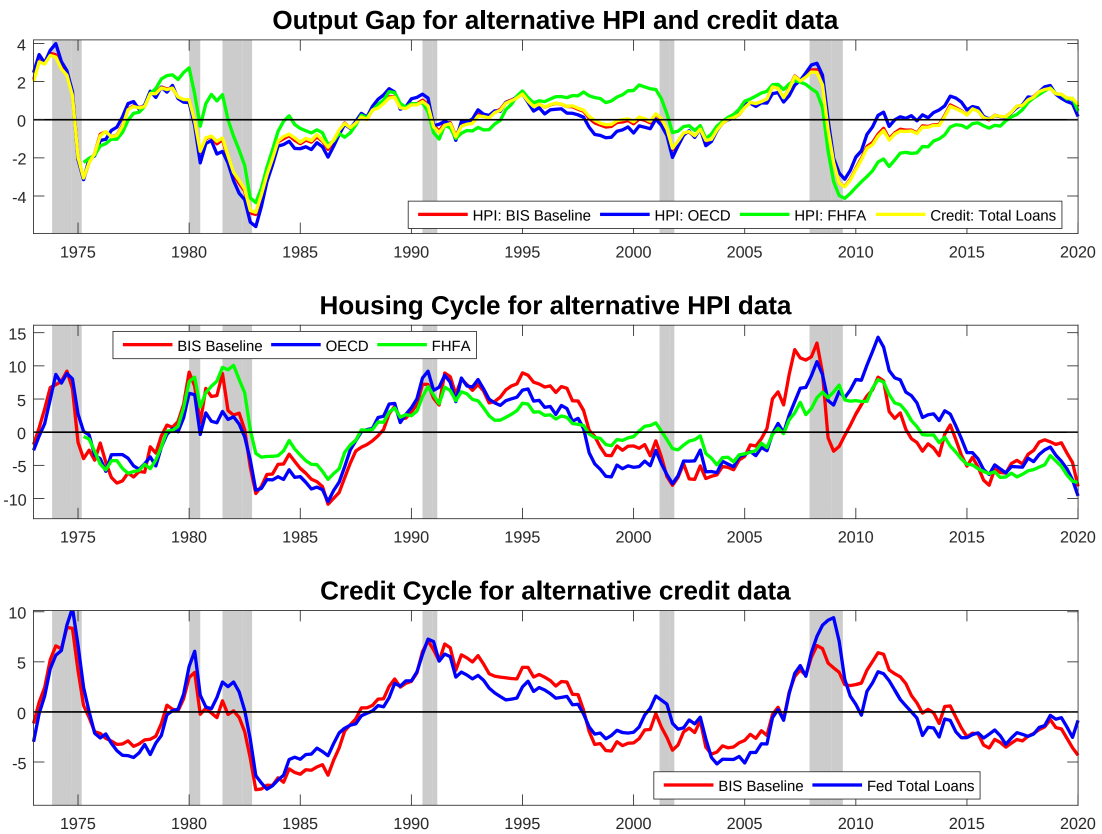

Figure H.18: Estimated cycles from the BVAR using different indicators. Units are in percent deviation from trend. Grey shaded areas indicate NBER recessions. Our baseline uses house prices from the BIS. OECD and FHFA indicate alternative sources for house price data. Total loans indicates that total loans is used in the model in place of credit.

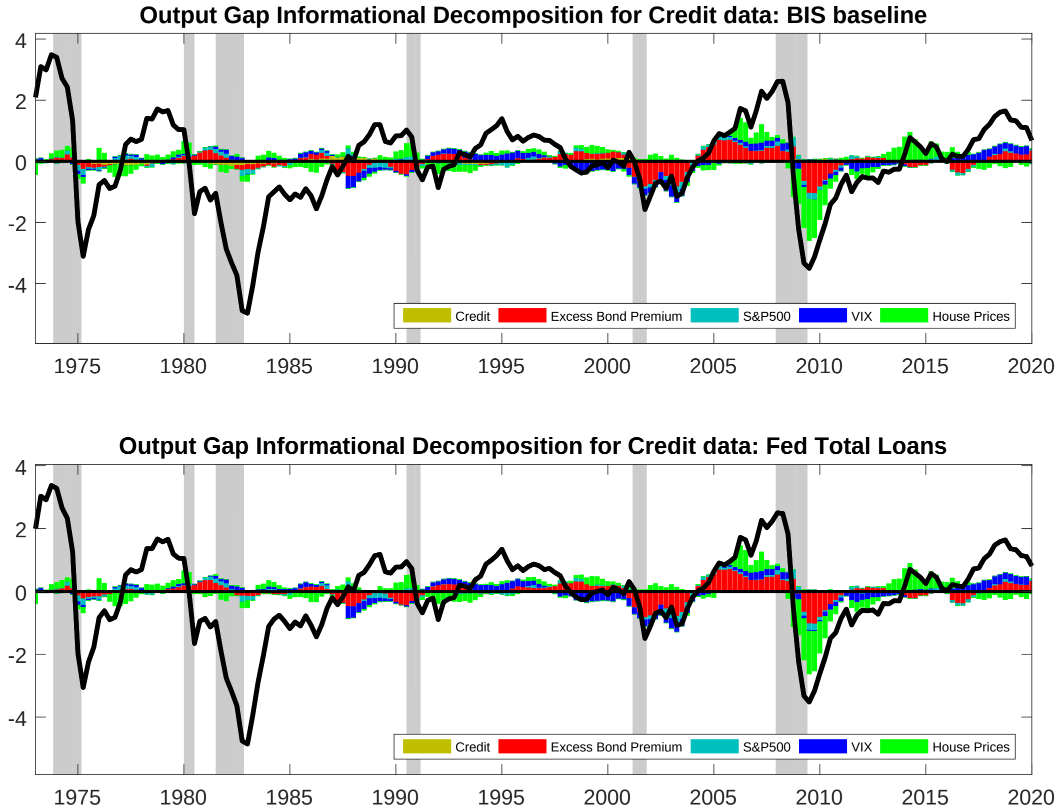

Figure H.19: Informational decomposition of the estimated cycles under our baseline and using total loans in place of credit. The solid line denotes the estimated cycle. Cycles are measured in percent deviation from the trend. Grey shaded areas indicate NBER recessions. The bars represent the total contribution of the contribution from the BVAR forecast errors from five financial variables (credit, the excess bond premium, the S&P 500, the VIX index, and the house price).

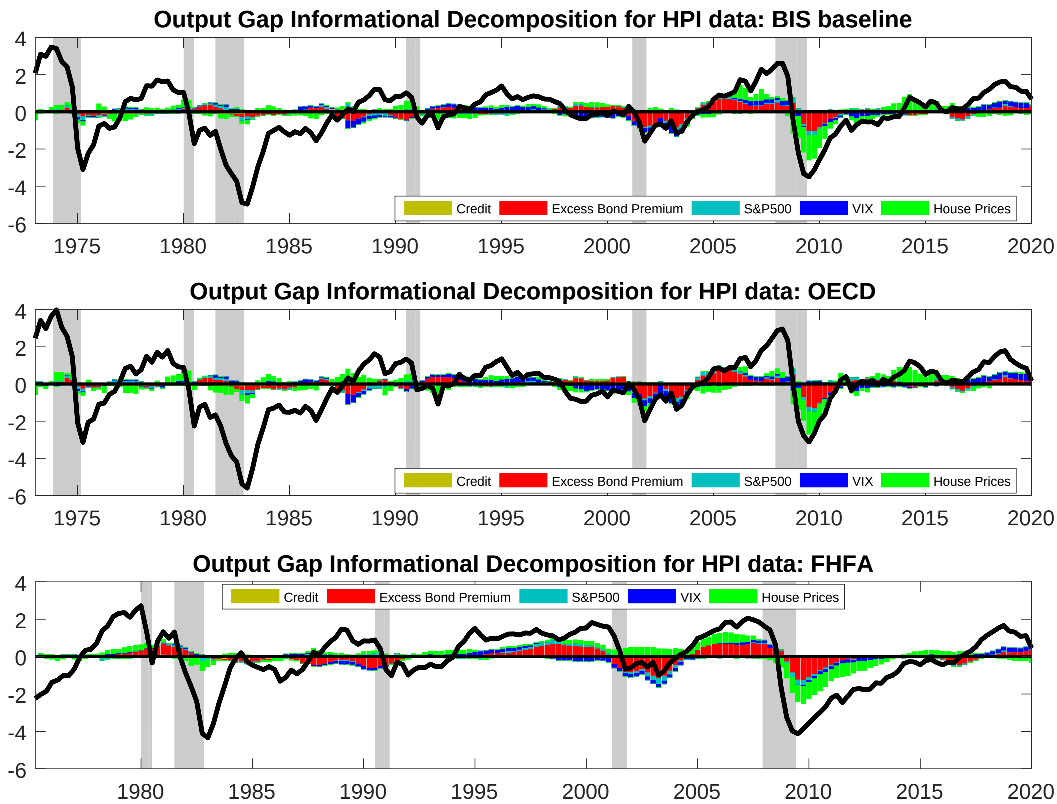

Figure H.20: Informational decomposition of the estimated cycles under our baseline and using house prices from the OECD and FHFA. The solid line denotes the estimated cycle. Cycles are measured in percent deviation from the trend. Grey shaded areas indicate NBER recessions. The bars represent the total contribution of the contribution from the BVAR forecast errors from five financial variables (credit, the excess bond premium, the S&P 500, the VIX index, and the house price).

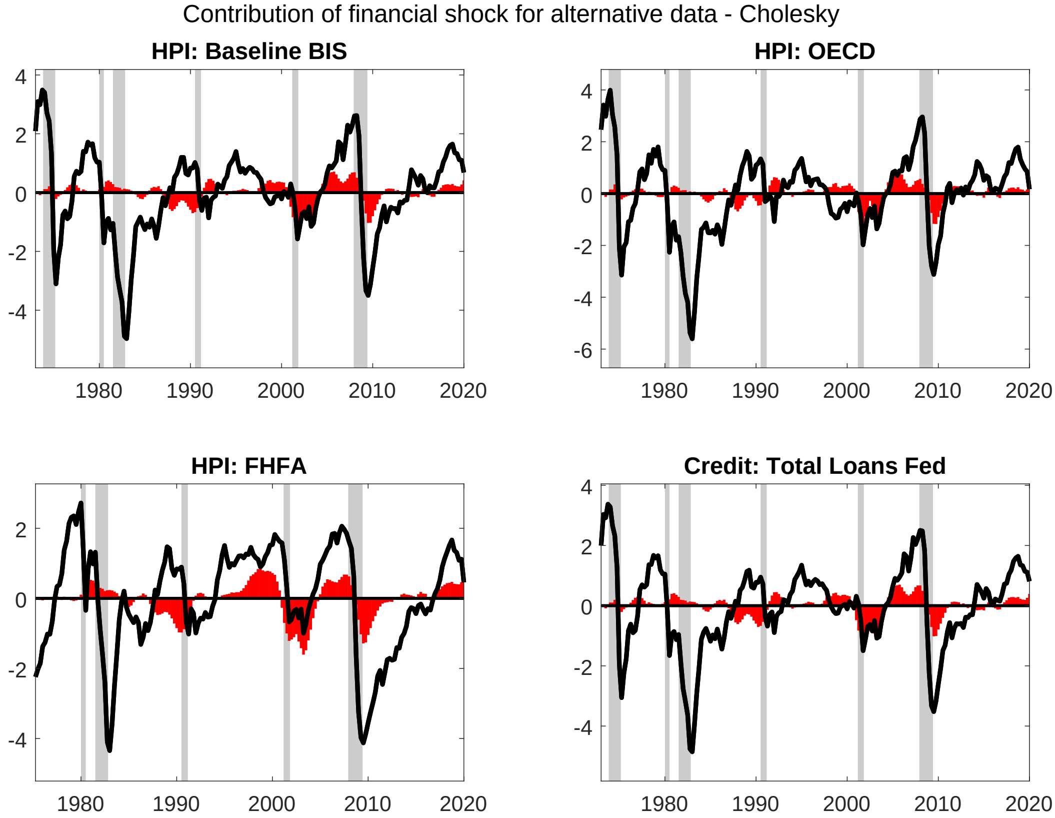

Figure H.21: Contribution of financial shocks to the estimated output gap using alternative house price and credit data in the model using Cholesky identification. The solid line is the estimated output gap. Output gap is measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The bars present the contribution of financial shocks to the estimated output gap. The title refers to the alternative house price and credit data used in the model.

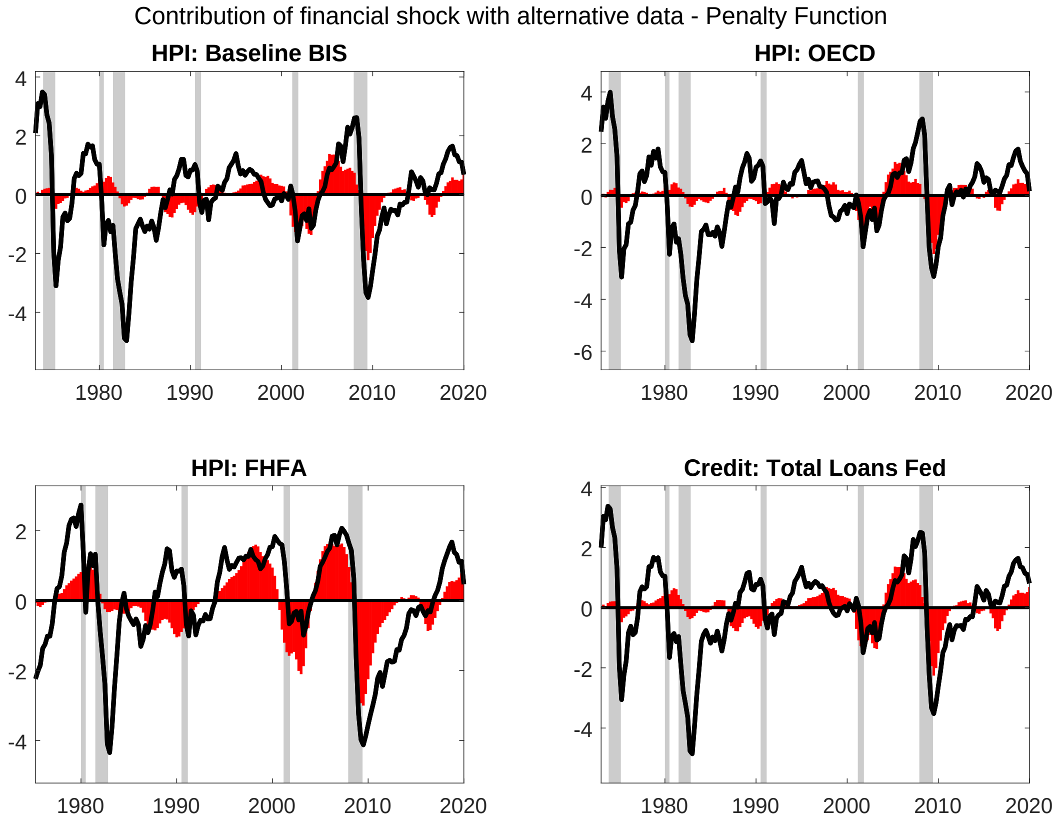

Figure H.22: Contribution of financial shocks to the estimated output gap using alternative house price and credit data in the model using penalty function identification. The solid line is the estimated output gap. Output gap is measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The bars present the contribution of financial shocks to the estimated output gap. The title refers to the alternative house price and credit data used in the model.

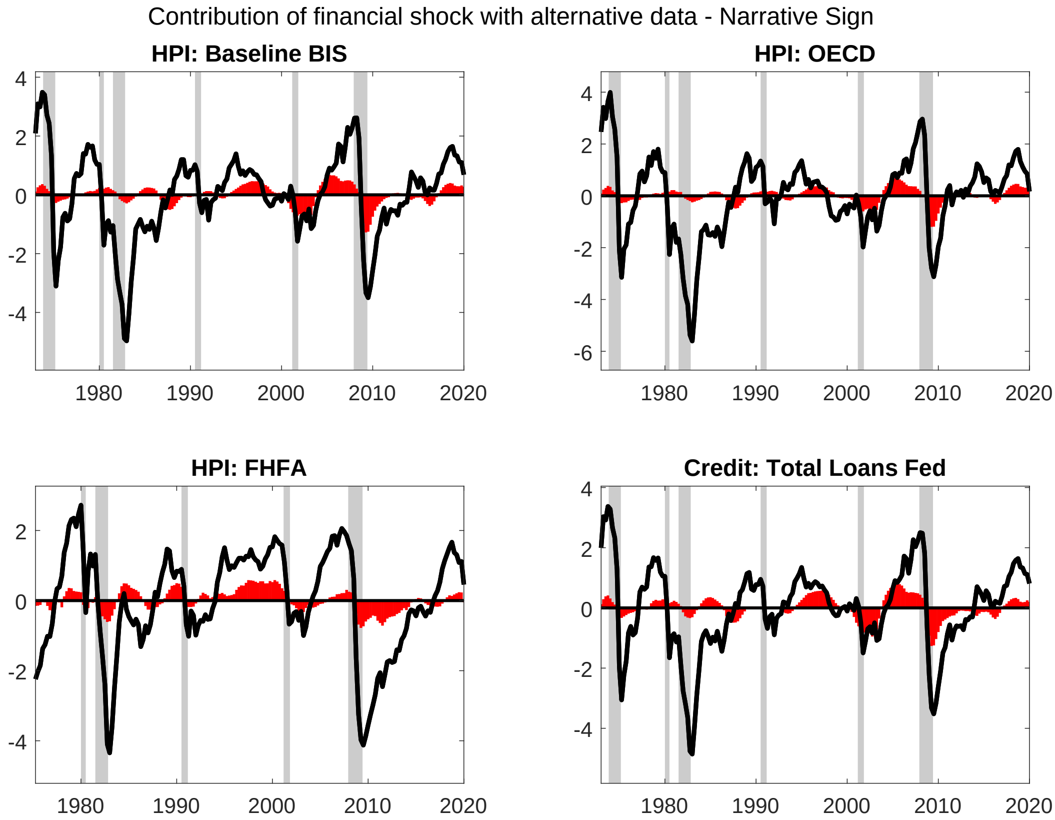

Figure H.23: Contribution of financial shocks to the estimated output gap using alternative house price and credit data in the model using sign and narrative identification. The solid line is the estimated output gap. Output gap is measured in percent deviation from trend. Grey shaded areas indicate NBER recessions. The bars present the contribution of financial shocks to the estimated output gap. The title refers to the alternative house price and credit data used in the model. The contribution from the sign restriction approach is averaged across draws that satisfy the sign and narrative restrictions.

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A | Declaration of Co-authorship

The following table shows my contribution to the three papers underlying the chapters of this dissertation.

Table A.1: Declaration of Co-authorship

Paper 1	*What has Caused Global Business Cycle Decoupling: Smaller Shocks or Reduced Sensitivity?*, joint with Tino Berger
Idea	contributed
Concept	contributed
Literature work	leading
Data work	leading
Empirical work	contributed substantially
Writing	leading
Paper 2	*Estimating Macro-Financial Linkages in Advanced Economies – A Unified Approach*, joint with Tino Berger
Idea	contributed
Concept	contributed substantially
Literature work	leading
Data work	leading
Empirical work	leading
Writing	leading
Paper 3	*A Unified Approach for Jointly Estimating the Business and Financial Cycle, and the Role of Financial Factors*, joint with Tino Berger and Benjamin Wong
Idea	contributed
Concept	contributed
Literature work	contributed substantially
Data work	contributed
Empirical work	contributed
Writing	contributed

Julia Richter　　　　　　Date

B Assurance according to §12 PStO

Ph.D. program in Economics

Declaration for admission to the doctoral examination

I confirm

1. that the dissertation that I submitted

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was produced independently without assistance from external parties, and not contrary to high scientific standards and integrity,

2. that I have adhered to the examination regulations, including upholding a high degree of scientific integrity, which includes the strict and proper use of citations so that the inclusion of other ideas in the dissertation are clearly distinguished,

3. that in the process of completing this doctoral thesis, no intermediaries were compensated to assist me neither with the admissions or preparation processes, and in this process,
- No remuneration or equivalent compensation were provided
- No services were engaged that may contradict the purpose of producing a doctoral thesis

4. that I have not submitted this dissertation or parts of this dissertation elsewhere.

I am aware that false claims (and the discovery of those false claims now, and in the future) with regards to the declaration for admission to the doctoral examination can lead to the invalidation or revoking of the doctoral degree.

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Date, Signature