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Corporate valuation with mixed financing strategies and cross-border relations

Dissertation

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Abstract

Corporate valuation is a theoretically challenging but practically highly relevant field. It is usually conducted using discounted cash flow (DCF) methods. One important part of corporate valuation with DCF methods are assumptions about future debt levels and the adjustment of costs of capital to the corresponding financial risk. In corporate valuation practice, it is usually assumed that active or passive debt management is pursued in all future periods. However, theoretical and empirical findings show that pure active or pure passive debt management are usually not accurate reflections of the financing behavior of firms. Consequently, corporate valuation theory started combining active and passive debt management to approach the real financing behavior of firms, resulting in mixed financing strategies. Thus, in a first step, this thesis addresses problems of implementing the real financing behavior of firms by analyzing mixed financing strategies. The aim is to extend existing theory to account for empirical and theoretical findings but at the same time keep the theory simple enough to be practically relevant. In particular, the characteristics of hybrid and discontinuous financing in a two-phase model are investigated and compared, where discontinuous financing considers a lagged adjustment of debt levels. Furthermore, terminal value calculation is analyzed more closely and the shortcomings of the assumption of discontinuous financing in a steady-state phase are addressed. By presenting a solution to resulting problems, debt categories as a new mixed financing strategy are developed.

Furthermore, due to the ongoing globalization, theorists and practitioners face additional problems that arise in a valuation of cross-border investments and international companies. For example, effects of different currencies, international taxation and legal regulations have to be considered. If quantities are converted to another currency, it is ambiguous whether forward or spot exchange rates should be used. Both approaches are described in the literature but a joint analysis and a comparison does not exist so far. Moreover, a consistent adjustment of costs of capital to exchange rate risk is not clear. In addition, debt financing and the implementation of a two-phase model are mostly not considered. Thus, in a second step, the more fundamental problems of cross-border DCF valuation are examined with the objective to derive a consistent valuation

framework in which debt financing can be integrated. Overall, this thesis contributes to the literature on corporate valuation by expanding and sorting the research on mixed financing strategies, and by providing answers to problems of international valuation.

Zusammenfassung

Die Bewertung von Unternehmen ist ein theoretisch anspruchsvolles, aber praktisch sehr relevantes Thema. Die Bewertung wird in der Regel mit Hilfe von Discounted Cashflow (DCF) Methoden durchgeführt. Ein wichtiger Teil der Unternehmensbewertung mit DCF-Methoden sind Annahmen über zukünftige Fremdkapitalbestände und die Anpassung von Kapitalkostensätzen an das entsprechende Finanzierungsrisiko. In der Praxis der Unternehmensbewertung wird grundsätzlich davon ausgegangen, dass eine wertabhängige oder autonome Finanzierung in allen zukünftigen Perioden angewendet wird. Theoretische und empirische Studien zeigen jedoch, dass eine rein wertabhängige oder rein autonome Finanzierungspolitik das Finanzierungsverhalten von Unternehmen oft nicht präzise widerspiegelt. Aus diesem Grund wurden in der theoretischen Literatur zur Unternehmensbewertung verschiedene Ansätze entwickelt, die eine wertabhängige und autonome Finanzierung kombinieren, um sich dem tatsächlichen Finanzierungsverhalten von Unternehmen anzunähern. Dies führte zu gemischten Finanzierungsstrategien. Diese Dissertation befasst sich daher in einem ersten Schritt mit den Problemen der Implementierung des realen Finanzierungsverhaltens von Unternehmen, durch die Analyse von gemischten Finanzierungsstrategien. Ziel ist es, die bestehende Theorie zu erweitern, indem empirische und theoretische Erkenntnisse berücksichtigt werden, gleichzeitig aber die Theorie einfach genug zu halten, um für die Praxis noch relevant zu sein. Insbesondere werden die Eigenschaften von hybrider und diskontinuierlicher Finanzierung in einem Zwei-Phasen-Modell untersucht und verglichen, wobei die diskontinuierliche Finanzierung eine verzögerte Anpassung von Fremdkapitalbeständen berücksichtigt. Außerdem wird die Restwertberechnung genauer analysiert und die Besonderheiten der Annahme einer diskontinuierlichen Finanzierung in einem eingeschwungenen Zustand werden aufgezeigt. Zur Lösung der sich daraus ergebenden Probleme werden Fremdkapitalkategorien als eine neue gemischte Finanzierungsstrategie entwickelt.

Aufgrund der fortschreitenden Globalisierung werden Theoretiker und Praktiker mit zusätzlichen Problemen konfrontiert, die bei der Bewertung von internationalen Investitionen und internationalen Unternehmen auftreten. Beispielsweise müssen die Auswirkungen unterschiedlicher

Währungen, internationaler Besteuerung und gesetzlicher Vorgaben berücksichtigt werden. Wenn Größen in eine andere Währung umgerechnet werden, ist es unklar, ob Termin- oder Kassawechselkurse verwendet werden sollten. Beide Ansätze werden in der Literatur beschrieben, aber eine gemeinsame Analyse und einen Vergleich gibt es bisher nicht. Zudem ist die konsistente Anpassung von Kapitalkostensätzen an das Wechselkursrisiko unklar. Darüber hinaus werden Fremdfinanzierung und die Implementierung eines Zwei-Phasen-Modells oft nicht berücksichtigt. Deswegen untersucht diese Dissertation in einem zweiten Schritt die grundlegenden Probleme der internationalen DCF-Bewertung, mit dem Ziel, einen konsistenten Bewertungsrahmen zu schaffen, in den Fremdfinanzierung integriert werden kann. Insgesamt leistet diese Dissertation einen Beitrag zur Literatur der Unternehmensbewertung, indem die Forschung über gemischte Finanzierungsstrategien erweitert und sortiert wird, und Antworten auf Probleme der internationalen Bewertung aufgezeigt werden.

Contents

List of Abbreviations

APV	adjusted present value
CAPM	capital asset pricing model
CFO	chief financial officer
CIP	covered interest parity
D	debt
DCF	discounted cash flow
FC	foreign currency
FCF	free cash flow
FtE	flow to equity
GCAPM	global capital asset pricing model
HC	home currency
HP	Harris and Pringle
ICAPM	international capital asset pricing model
L	leverage
LCAPM	local capital asset pricing model
ME	Miles and Ezzel
PPP	purchasing power parity
TV	terminal value
WACC	weighted average cost of capital

List of Figures

List of Tables

1 Introduction

1.1 Motivation and objectives

Corporate valuation is a theoretically challenging but practically highly relevant field. It has been addressed in the literature since Modigliani and Miller's theory of investment (Modigliani and Miller 1958, 1963). Nowadays, corporate valuation is usually conducted using discounted cash flow (DCF) methods. There, expected future cash flows are discounted using capital-market-based costs of capital. To reflect the uncertainty of future cash flows, the cost of capital is adjusted to the risks of the cash flow. For an unlevered firm, business risk has to be taken into account. In the valuation of a levered firm, additionally, financial risk has to be considered (Kruschwitz and Löffler 2020, p.47–48). It follows that one important part of corporate valuation with DCF methods are assumptions about future debt levels. Since interest on debt is deductible from taxable income, debt financing has an immediate influence on the market value of a firm (Kruschwitz and Löffler 2020, p. 73). Furthermore, due to the ongoing globalization, theorists and practitioners face new problems that arise in a valuation of cross-border investments and international companies. For example, effects of different currencies, international taxation and legal regulations have to be considered. Regarding the risk-adjustment of the cost of capital, in addition to business risk and financial risk, exchange rate risk has to be taken into account.

In a frictionless world without taxes, Modigliani and Miller (1958) showed that debt financing does not have an influence on the market value of the firm. As soon as taxes are considered, debt financing yields so-called tax shields. Depending on the definition of future debt levels, the risk of future tax shields is different resulting in different market values of the firm. Consequently, in the process of corporate valuation, financing strategies have to be carefully specified. The most prevalent financing strategies are passive and active debt management (Kruschwitz and Löffler 2020, p. 73). It is usually assumed that one of these two pure financing strategies is pursued in all future periods.

If passive debt management is assumed, debt levels are defined deterministically for all future periods. In particular, debt levels are not adjusted to the development of the firm such that the debt-to-equity ratio is stochastic (Ashton and Atkins 1978). In contrast, under the assumption of active debt management, leverage ratios are defined deterministically (Miles and Ezzell 1980, 1985; Harris and Pringle 1985). It follows that future debt levels are stochastic since they depend on the development of the firm. Comparing the assumptions of passive and active debt management, Lewellen and Emery (1986) conclude that active debt management is the "most logically consistent" (Lewellen and Emery 1986, p. 415) assumption of debt financing. However, "neither purely active nor passive debt management assumptions are accurate reflections of corporate financial practice" (Clubb and Doran 1995, p. 690). To formulate a more appropriate assumption, we need to consider a wide variety of theories on the capital structure behavior of firms.

One prevalent theory is the trade-off theory that weighs the tax benefits of debt against the costs of financial distress: While the tax benefits increase with an increase in debt levels, the risk of default and therefore, the costs of financial distress also increase (Myers 1993, Berk and DeMarzo 2020, pp. 600–602). Consequently, based on the trade-off theory, it is possible to derive an optimal debt ratio by balancing these two influences. However, in addition to costs of financial distress, agency costs occur that increase with leverage (Berk and DeMarzo 2020, pp. 603–615). Furthermore, asymmetric information among managers, stockholders, and creditors have to be taken into account, resulting in the pecking order theory (Myers and Majluf 1984, Myers 1993, Berk and DeMarzo 2020, pp. 615–622). Theoretical models have tried to incorporate those findings in the search of an optimal capital structure (Scott 1976; Abel 2018).

Furthermore, empirical analyses are important to establish findings about the real financing behavior of firms. There are a wide variety of empirical studies that examine the capital structure of firms and corresponding financing decisions. Some studies weigh the tax benefits of debt financing against the costs of financial distress: Graham (2000) concludes that debt is used conservatively, whereas Blouin et al. (2010) reassessed the tax benefits of debt financing with the result that most firms have indeed tax-efficient capital structures. Similar results can be found in Kayhan and Titman (2007), who examined the effects of a firm's history on its capital structure and conclude that firms have target debt ratios that are consistent with the trade-off theory. Deviating from the previous results, DeAngelo et al. (2011) implemented a dynamic capital structure model to show that firms diverge from a target capital structure in order to fund investment. Flannery and Rangan (2006) examined the partial adjustment toward target capital structures and found that "the typical firm closes about one-third of the gap between its actual and its target debt ratios each year" (Flannery and Rangan 2006, p. 469). Other studies show that macroeconomic conditions also play an important role for financing decisions (Bhamra et al. 2010; Graham et al. 2015).

Besides empirical studies, surveys are used to investigate the real financing behavior of companies. Graham and Harvey (2001) conducted a survey with chief financial officers (CFO) from the U.S. and Canada including questions about the capital structure. They found "moderate support that firms follow the trade-off theory and target their debt ratio" (Graham and Harvey 2001, p. 232). Brounen et al. (2006) found similar results in a survey of CFOs from several countries in Europe. A comparison of different theories and empirical results on a firm's capital structure can be found in Barclay and Smith (2005). The authors conclude that firms should have a target capital structure goal and adjust their capital structure towards this target "whenever the costs of adjustment [...] are less than the costs of deviating from the target" (Barclay and Smith 2005, p. 17).

Due to the wide variety of theoretical and empirical findings on the capital structure behavior of firms, it is difficult to build a theoretical valuation model that is simple enough to be practically relevant but at the same time considers all findings. Consequently, corporate valuation practice has to make appropriate simplifications. A central simplification is the assumption of a financing strategy that specifies future debt levels. As mentioned above, theoretical and empirical findings show that pure active or pure passive debt management are usually not accurate. Consequently, corporate valuation theory started combining active and passive debt management to approach the real financing behavior of firms, resulting in mixed financing strategies.

One approach to define mixed financing strategies is using a two-phase model. Planning future quantities is usually conducted with such a two-phase model that consists of an explicit forecast phase and a steady-state phase. Whereas a detailed planning is possible in the explicit forecast phase, afterwards, it is usually assumed that a firm has reached a steady state in which the expectation of all input parameters grows at a constant rate (Koller et al. 2020). The assumptions regarding the steady-state phase are important since the terminal value accounts for a large part of the overall market value (Holland 2018, p. 70, Koller et al. 2020, pp. 285–286). To compute the market value in the steady-state phase, the Gordon-Shapiro formula can be applied (Gordon and Shapiro 1956). In addition to the constant growth of the expected cash flows of the firm, the Gordon-Shapiro formula requires the cost of capital to be constant. For a levered firm, this implies that financing risk has to be constant in the steady-state phase.

Regarding the definition of future debt levels, it is natural to also apply this two-phase model. Kruschwitz et al. (2007) examined debt financing in such a two phase model and argue that debt levels can be planned in detail in the explicit forecast phase, but not in the steady-state phase (Kruschwitz et al. 2007, p. 427). Consequently, they investigated a model with the assumption of passive debt management in the explicit forecast phase, and active debt management in the steady-state phase. The resulting financing strategy is called hybrid financing (Kruschwitz et al. 2007). Dierkes and Gröger (2010) picked up on the findings of Kruschwitz et al. (2007) by examining the transition between the explicit forecast phase and the steady-state phase more closely. In particular, they argued that defining the leverage ratio for the steady-state phase at the valuation date can result in substantial refinancing at the end of the explicit forecast phase. They propose a refinement of hybrid financing by using the resulting leverage ratio at the end of the explicit forecast phase for the steady-state phase. The resulting financing strategy is called D-hybrid financing where D stands for debt. The hybrid financing strategy, originally presented by Kruschwitz et al. (2007), is denoted by L-hybrid financing where L stands for leverage (Dierkes and Gröger 2010).

Clubb and Doran (1995) picked up on empirical results that debt levels are adjusted to the development of the firm only slowly (Fama and French 2002) and with a time lag (Leary and Roberts 2005; Huang and Ritter 2009). They developed an analytical model based on consecutive planning phases where every planning phase has a predetermined length. Debt levels are defined for a planning phase by multiplying the specified debt-to-market value ratio by the expected market value. After a planning phase, debt levels are adjusted to the updated market value, and are again defined deterministically for the subsequent planning phase. Despite the use of a debt ratio, debt levels are certain within each planning phase since they are linked to the expected, and not to the realized, market value of a firm (Ashton and Atkins 1978). Thereby, Clubb and Doran (1995) have combined active and passive debt management to obtain “intermediate ‘partially active’ debt management policies” (Clubb and Doran 1995, p. 682). A second approach of Clubb and Doran (1995) is to hold debt constant within a planning phase. This was also analyzed by Arnold et al. (2018) who referred to this approach as discontinuous financing and examined the case of a perpetuity. Arnold et al. (2019) enhanced the model to a perpetuity with a constant growth rate.

Notwithstanding the existing literature on discontinuous financing, several key questions so far remained unanswered. In particular, differences in the assumptions of these three studies on discontinuous financing and resulting consequences are not clear. Furthermore, the implementation of discontinuous financing in a two-phase model using passive debt management in the explicit forecast phase has not been examined. Additionally, characteristics of terminal value calculation with discontinuous financing have not been addressed in the literature. In particular, the effects of the assumption of consecutive planning phases on the overall growth of the market value in the steady-state phase are ambiguous. Moreover, the derivation of valuation equations for discontinuous financing in Clubb and Doran (1995) and Arnold et al. (2018, 2019) are intricate.

Other mixed financing strategies analyze financing based on book values (Scholze 2009, pp. 131–185, Kruschwitz and Löffler 2020, pp.114–127) or assume that active and passive debt management can be mixed in each period to account for empirical results that firms diverge from the target debt ratio to fund investment (Dierkes and Schäfer 2017). Grinblatt and Liu (2008) implemented dynamic debt policies by deriving closed-form solutions for a partial differential equation that describes the value of the tax shield. Correspondingly, there are several approaches of mixed financing strategies to consider theoretical and empirical findings about the capital structure of firms. However, in corporate valuation practice, passive and active debt management are still the dominating financing strategies. Despite the presented theoretical results, discontinuous financing and other mixed financing strategies are so far of little practical relevance.

Another big task is the the integration of debt financing in international valuation. This is an increasingly important topic since nowadays, the corporate landscape is multinational (Bekaert and Hodrick 2018, p. 1). In particular, when valuing cross-border investments or an international company, compared to national valuation, theorists and practitioners face several additional problems. If different currencies occur in the valuation, in addition to business risk and financial risk, exchange rate risk has to be included. Moreover, it is not clear which currency should be used in the valuation: On the one hand, it is possible to compute a discount rate in foreign currency (FC) to discount cash flows in FC. The resulting market value at the valuation date can be converted to home currency (HC) by multiplying it by the current spot exchange rate. This approach is similar to national valuation. On the other hand, it is also possible to convert all cash flows to HC first and use a discount rate in HC (see, for example, Berk and DeMarzo 2020, p. 1099). The literature agrees that both approaches should yield the same result (see, for example, Koller et al. 2020, p. 507) but details like underlying assumptions and advantages or disadvantages of both approaches are not considered.

Furthermore, if cash flows are converted from one currency to another, it is ambiguous whether forward or spot exchange rates should be used. Both approaches are described in the literature

(Erasmus and Ernst 2014; Bekaert and Hodrick 2018, Section 15–16; Berk and DeMarzo 2020, Chapter 31; Brealey et al. 2020, Chapter 27; Holthausen and Zmijewski 2020, Chapter 17; Koller et al. 2020, Chapter 27, Appendix G) but a joint analysis and a comparison is so far elusive. Moreover, in the literature, correlation between exchange rate risk and business risk is mostly excluded in the conversion of cash flows (Bekaert and Hodrick 2018; Holthausen and Zmijewski 2020; Koller et al. 2020). Berk and DeMarzo (2020, p. 1110) critically assess this assumption and clarify that the correlation should indeed be considered for the valuation of multinational companies but a consistent approach does not yet exist. Additionally, consequences of neglecting this correlation for the derivation of the cost of capital, and thus for the valuation equation, are not yet analyzed.

Depending on the conversion of cash flows, costs of capital have to be adjusted. Corresponding formulas and assumptions have not been consistently presented in the literature. Moreover, due to the integration of capital markets, the application of the capital asset pricing model (CAPM) has to be revised. Whereas the local CAPM (LCAPM) takes segmented markets into account (Sercu 2009, p. 679), the international CAPM (ICAPM) considers the integration of capital markets (Solnik 1974; Sercu 1980; Stulz 1981; Adler and Dumas 1983; Sercu 2009). A simplified and widely accepted model is the global CAPM (GCAPM) (O'Brien 1999; Schramm and Wang 1999; Stulz 1999) but conditions for the application in a DCF valuation framework are unclear.

Furthermore, differences of foreign and domestic tax codes have to be considered. A multinational company pays taxes to several national governments such that the determination of the corporate tax rate can be difficult (Berk and DeMarzo 2020, p. 1103). Moreover, other legal requirements and differences in international accounting standards can play an important role (Erasmus and Ernst 2014, pp. 16–25). For cross-border valuations in less developed markets, the integration of economic, political, or other types of risk may be challenging (Holthausen and Zmijewski 2020, p. 889). In general, the integration of country risk premiums are widely discussed (Bekaert et al. 2016). For the application of a DCF model, the implementation of a two-phase model has not been specifically examined: If a foreign company has reached a steady state, it is ambiguous how this setting can be transferred to HC. After all, regardless of the extensive results on debt financing for national valuation, it is not clear how to implement active or passive debt management in a cross-border DCF valuation.

Thus, in a first step, this thesis addresses problems of implementing the real financing behavior of firms by analyzing mixed financing strategies. The aim is to extend existing theory to account

Valuation with mixed financing strategies	Terminal value calculation with discontinuous financing and debt categories	Cross-border discounted cash flow valuation
Investigation of financing strategies in a two-phase model, including a simulation analysis for comparison of different financing strategies.	Analysis of discontinuous financing in a steady-state phase and implementation of debt categories.	Derivation of a consistent DCF valuation framework for the valuation of a foreign company from the view of a domestic investor.
Published in Business Research (2020), Vol. 13, No. 3, pp. 1317–1341	Published in Journal of Business Economics (2022), Vol. 92, No. 7, pp. 1207–1248	working paper

Table 1.1: Overview and objectives of the thesis. All studies are co-authored by Stefan Dierkes.

for empirical and theoretical findings but at the same time keep the theory simple enough to be practically relevant. In particular, the characteristics of mixed financing strategies in a two-phase model are investigated and compared (study one). Furthermore, terminal value calculation is analyzed more closely and the shortcomings of the assumption of discontinuous financing in a steady-state phase are addressed. By presenting a solution to resulting problems, debt categories as a new mixed financing strategy are developed (study two). In a second step, the more fundamental problems of cross-border DCF valuation are examined with the objective to derive a consistent valuation framework in which debt financing can be integrated (study three). Table 1.1 displays an overview of the three comprised studies and its objectives. Overall, this thesis contributes to the literature on corporate valuation by expanding and sorting the research on mixed financing strategies, and by providing answers to problems of international valuation.

1.2 Content

The thesis comprises three studies that examine DCF valuation with different mixed financing strategies and several cross-border problems. The first study gives an overview of existing mixed financing strategies, and incorporates discontinuous financing in a two-phase model. Furthermore, using simulations, the effects of different mixed financing strategies on the market value of the firm are examined. Thereby, mixed financing strategies are made more accessible to corporate valuation practice (Chapter 2). The second study addresses problems arising from the assumption of discontinuous financing in a steady-state phase. By implementing debt categories, a solution to the described problems is presented (Chapter 3). The third study considers the increasing importance of cross-border investments and, therefore, cross-border DCF valuation, by analyzing existing problems and deducing a consistent valuation framework (Chapter 4). Chapter 5 concludes the thesis.

Study 1: Valuation with mixed financing strategies

Although it is standard practice in corporate valuation to split the forecast of the free cash flows into an explicit forecast phase and a steady-state phase, this two-phase model receives little attention in the planning of future debt levels. Therefore, the first study analyzes financing strategies in a two-phase model. The study starts with a summary of existing results on hybrid financing. Thereby, we distinguish how the debt ratio of the steady state is determined, resulting in L- or D-hybrid financing. Afterwards, we pick up on empirical studies that find a lagged adjustment of debt levels, motivating a discontinuous financing strategy. We transfer the approaches of L- and D- hybrid financing to include discontinuous financing in a two-phase model. Thereby, we develop two new mixed financing strategies, namely L- and D-discontinuous financing. Furthermore, we provide a simplified derivation of the valuation equation. This simplification may increase the acceptance for this financing strategy in corporate valuation practice.

Furthermore, we theoretically compare different mixed financing strategies in a two-phase model and illustrate this comparison with simulations. We show differences in market values implied by the assumption of different financing strategies. Moreover, we estimate correlations to outline sensitivities of input parameters. Thereby, we investigate the influence of individual parameters on the deviations. We conclude that the outlined deviations should be examined carefully by corporate valuation practice, and that the analyzed mixed financing strategies could offer a valuable alternative to better reflect the real financing behavior of firms.

Study 2: Terminal value calculation with discontinuous financing and debt categories

The terminal value constitutes a high proportion of the total market value such that its computation is an important part of corporate valuation. It is usually assumed that the firm has reached a steady state in which the expectation of all input parameters grows at a constant rate such that the market value follows from the Gordon-Shapiro formula. When discontinuous financing is assumed in the steady-state phase, problems occur that have not been sufficiently analyzed. This study examines characteristics of discontinuous financing in a steady-state phase and presents a solution by introducing debt categories.

The study begins by comparing existing approaches of discontinuous financing and their suitability for a steady state. We show that under the assumption of discontinuous financing, the market value does not grow at a constant growth rate, even if all input parameters grow at this rate.

Consequently, financing risk is not constant and the often-used Gordon-Shapiro formula cannot be applied. We derive a formula for the period-specific levered cost of equity and present an example to illustrate our findings.

Since a single adjustment of the entire debt at the beginning of every planning phase, as it is assumed in discontinuous financing, might still not be close to the real financing behavior of firms, we modify this financing strategy by implementing debt categories. Thereby, we also derive a solution to the described problems of discontinuous financing in a steady-state phase. Instead of consecutive planning phases, that are used for discontinuous financing, we assume overlapping planning phases. Then, in every period of the steady state, a certain share of the overall debt is adjusted to the development of the market value. We label such a share as one debt category. The remainder of the overall debt level is held constant. In the subsequent period, another share, and therefore another debt category, of the overall debt is adjusted, and so on. We start by deriving valuation equations and a formula for the levered cost of equity for two debt categories and generalize our results to an arbitrary number of debt categories. Due to the continuous adjustment of a certain share of the overall debt, the financing risk and the corresponding levered cost of equity are constant. Thus, if financing based on debt categories is assumed in the steady-state phase, the market value grows at a constant rate and the Gordon-Shapiro formula can be used to compute the terminal value.

We illustrate our findings on debt categories using an example. We compare the derived market value under the assumption of debt categories with the assumption of other financing strategies. Thereby, we show that the difference in market values between the assumption of standard discontinuous financing and debt categories is small. In particular, standard discontinuous financing can be interpreted as an approximation of debt categories. Furthermore, we describe the application of debt categories in a two-phase model and elaborate on the integration of the risk of default for discontinuous financing and debt categories.

Study 3: Cross-border discounted cash flow valuation

Due to the ongoing globalization, the relevance of cross-border investments steadily increases. As a consequence, cross-border DCF valuation becomes an increasingly important tool for analyzing those investments. Compared to national valuation, additional problems arise that have not been sufficiently analyzed in the literature. This study presents a consistent cross-border DCF valuation framework, resolving existing ambiguities.

The study begins by analyzing the GCAPM. We consider two countries with currencies FC and

HC, respectively. We transfer the results of Fama (1977) for deterministic input parameters in the LCAPM to the GCAPM. Furthermore, the security market line is derived in FC and in HC, respectively. We use these results to derive deterministic costs of capital in the subsequent analysis of cross-border DCF valuation.

We start the DCF valuation by deriving valuation equations for an unlevered foreign company from the perspective of a domestic investor. Thus, the market value must be expressed in HC. We distinguish between FC- and HC-valuation approaches. In the former, cash flows in FC are discounted at a cost of capital that is based on the foreign capital market. The resulting market value is converted to HC at the current spot exchange rate. In the latter, cash flows are first converted to HC and then discounted using costs of capital that are based on the domestic capital market. We differentiate the HC-approaches further into whether spot or forward exchange rates are used for the conversion of cash flows. Since the correlation between exchange rate risk and business risk is often excluded for practical reasons, we present so-called “modified” HC-valuation approaches, where we neglect the explicit consideration of this correlation. All results are incorporated into a two-phase model with an explicit forecast phase and a steady-state phase.

Afterwards, the model is extended to debt financing. We analyze the free cash flow and flow to equity method for active debt management and the adjusted present value method for passive debt management. Again, we present valuation equations for the FC-valuation approach and the different HC-valuation approaches and incorporate our findings into a two-phase model. The study concludes with a discussion of the advantages and disadvantages of the different approaches.

2 Valuation with mixed financing strategies

Joint work with Stefan Dierkes.

Published in Business Research.[1]

Abstract

In corporate valuation, it is common to assume either passive or active debt management. However, it is questionable whether these pure financing policies reflect the real financing policies of firms with a sufficient degree of accuracy. This shortcoming has led to the development of mixed financing strategies as combinations of pure financing strategies. Whereas hybrid financing is directly linked to the two-phase model, it is unclear how to apply discontinuous financing in such a setting. In this study, according to the two versions of hybrid financing, we analyze the implementation of discontinuous financing in a two-phase model. Thereby, we present a simpler and more intuitive derivation of the valuation equation for discontinuous financing to increase its acceptance and its use for corporate valuation practice. Moreover, we compare the different mixed financing strategies with each other theoretically, and we conduct simulations to elucidate the impact on market values and the sensitivities of input parameters. The study concludes that the presented mixed financing strategies can help in the attempt to reflect the real financing behavior of firms more accurately and, therefore, constitute a valuable alternative to pure financing strategies for valuation.

[1]This chapter is a version of an article published in Business Research (2020), Vol. 13, No. 3, pp. 1317–1341, , https://doi.org/10.1007/s40685-020-00126-w.

2.1 Introduction

Corporate valuation with discounted cash flow approaches requires assumptions about the firm's financing strategy. Since interest on debt is deductible from taxable income, the financing strategy has an immediate influence on the market value of the firm. In this regard, it is generally assumed that a consistent financing strategy is pursued in each period of the forecast horizon. However, empirical findings show that it is questionable whether pure financing strategies like active or passive debt management reflect the real financing policies of firms with a sufficient degree of accuracy (see e.g. Lewellen and Emery 1986; Barclay and Smith 2005; Grinblatt and Liu 2008). Therefore, it is promising to consider a two-phase model that differentiates between financing policies in the explicit forecast and the steady-state phase. The interaction between passive debt management in the explicit forecast phase and active debt management in the steady-state phase has already been examined as hybrid financing (Kruschwitz et al. 2007; Dierkes and Gröger 2010). Furthermore, discontinuous financing as another mix of active and passive debt management was developed (Clubb and Doran 1995; Arnold et al. 2018, 2019), but it has been of little relevance for corporate valuation practice so far and it is unclear how to apply this financing strategy in a two-phase model.

The contribution of this paper is threefold. First, we transfer the approach of hybrid financing to a two-phase model with passive debt management and discontinuous financing, which results in two new mixed financing strategies. Second, in the course of analyzing these new mixed financing strategies, we present a much-simplified derivation of the valuation equation for discontinuous financing. This more intuitive derivation could increase the acceptance of discontinuous financing and, therefore, its use for corporate valuation practice. Third, we compare hybrid and discontinuous financing as possible mixed financing strategies. We determine the deviations of the market values theoretically and analyze the distribution of the deviations and the influence of input parameters to investigate the impact and the relevance of these financing strategies for corporate valuation practice using a Monte Carlo simulation.

Although active and passive debt management are popular in corporate valuation practice, empirical studies indicate that these or other pure financing strategies are not suitable for modeling a firm's financing policy with a sufficient degree of accuracy; see, for example, Lewellen and Emery (1986); Barclay and Smith (2005); Grinblatt and Liu (2008). In particular, there exists a wide variety of theories on the capital structure behavior of firms (for a summary of empirical results on the capital structure research, see e.g. Graham and Leary 2011). Theories on capital structure weigh the tax benefits that result from debt financing (see e.g. Graham

2000) against the costs of financial distress (see e.g. Molina 2005; Glover 2016) or agency and information costs (see e.g. Copeland et al. 2014, pp. 413–462). In addition, the payout policy is a relevant factor since firms want to choose the optimal method to return capital to their investors (see e.g. Berk and DeMarzo 2017, pp. 519–669). Furthermore, for a company, not only the amount of debt borrowed but also the type of debt is essential (see e.g. Brealey et al. 2020, pp. 631–662).

The actual financing behavior of firms is also influenced by a large number of other circumstances. For example, Bhamra et al. (2010) and Graham et al. (2015) showed that the development of financing policies depends on a wide range of macroeconomic factors, while Kayhan and Titman (2007) concluded that a firm's history has an important influence on its capital structure. Moreover, Graham and Harvey (2001) conducted a survey on 392 chief financial officers in the US and showed that only 10% of all firms have a strict target debt ratio whereas 34% "have a somewhat tight target or range" (Graham and Harvey 2001, p. 211). The remaining firms either have a flexible target or have no target debt ratio at all (Graham and Harvey 2001, p. 211). Brounen et al. (2006) continued this research by comparing its results to those of selected countries in Europe. Their study showed "that in each of the countries merely 10% of all firms maintain a strict target" (Brounen et al. 2006, p. 1430), supporting the findings of Graham and Harvey (2001). Similar results can be found in de Jong and Verwijmeren (2010), who conducted a survey on 235 firms in the US, Canada, and Europe and used it for empirical model testing. They found that 55% of firms have a mostly flexible target (de Jong and Verwijmeren 2010, p. 220).

This variety of theories and findings plays an important role for corporate valuation since the assumption of a financing strategy should depict the real financing behavior of firms as accurately as possible. Grinblatt and Liu (2008) summarized these and other results as follows: "The actual debt policies of firms tend to deviate from those specified by the Modigliani–Miller and Miles–Ezzell models" (Grinblatt and Liu 2008, p. 226). For corporate valuation, it follows that one can either accept the resulting valuation inaccuracy or attempt to depict a firm's financing policy more accurately to achieve more precise valuation results. This yields the concept of mixed financing strategies that combine two or more pure financing strategies and, therefore, have more degrees of freedom to describe a firm's financing policy. However, an additional requirement for a financing strategy is that it is intuitive and applicable. It follows that mixed financing strategies are developed to get closer to the real financing behavior of firms but are still a simplified representation and cannot consider all theories and findings. Otherwise, the resulting model would be too complex. In this study, we concentrate on mixed financing strategies in a two-phase model with passive debt management in the explicit forecast period.

Kruschwitz et al. (2007) were the first to discuss the application of passive debt management in the explicit forecast phase and active debt management in the steady-state phase under the term hybrid financing. They outlined that debt levels of firms are observed to be largely fixed in the early years of the planning phase, particularly due to fixed investment planning. It follows that debt financing can be adjusted only to a limited extent following active debt management in the first T periods. Meanwhile, a deterministic definition of debt levels at the time of valuation in periods further away appears to be equally unrealistic, thereby impairing the plausibility of passive debt management in these periods (Kruschwitz et al. 2007). Dierkes and Gröger (2010) continued this research by pointing out that a distinction can be made regarding the definition of the debt-to-market value ratio of active debt management in the steady state. On the one hand, it is possible to define the leverage ratio deterministically at the time of valuation, which is referred to as L-hybrid financing and complies with the financing strategy of Kruschwitz et al. (2007). On the other hand, D-hybrid financing is possible, whereby the debt level in period T is defined deterministically at the valuation date. Therefore, the leverage ratio of the steady-state phase results from the deterministic debt level and the uncertain market value at the end of the explicit forecast phase. The abbreviations L and D stand for leverage and debt, respectively. These combinations of active and passive debt management yield different financing strategies, which in turn yield different valuation results (Dierkes and Gröger 2010).

Discontinuous financing considers other shortcomings of pure active and passive debt management. Particularly, discontinuous financing picks up on empirical research that indicates that firms adjust their debt levels very slowly (Fama and French 2002) and only with a time lag (Leary and Roberts 2005; Huang and Ritter 2009). Originally, Clubb and Doran (1995) introduced discontinuous financing under the term lagged debt management policy. This financing strategy consists of passive debt management that is adapted to the development of the firm after a limited number of periods. We call these periods a planning phase. Thus, debt levels are defined deterministically only for one planning phase. They are derived by multiplying the expected market values of the firm by a debt-to-market value ratio. After such a planning phase, the debt levels are again defined deterministically considering changes in the economic environment by using the updated expected market values. It is important to observe that despite the use of a debt ratio, the debt levels are certain within a planning phase, since they are linked to the debt-to-market value ratio according to the expected—not realized—market values of the firm

(Ashton and Atkins 1978). Furthermore, contrary to hybrid financing, which is characterized by one switch from passive to active debt management, discontinuous financing “allows for shifts in both directions for several times” (Arnold et al. 2018, p.151) and thereby relaxes the assumption of hybrid financing. An extension of the lagged debt management policy of Clubb and Doran (1995) was introduced by Arnold et al. (2018) who referred to it as discontinuous financing since it extends active debt management according to Miles and Ezzel (ME) “by a discontinuous refinancing sequence” (Arnold et al. 2018, p. 150). Specifically, they pursued the second approach of Clubb and Doran (1995) that keeps debt constant between rescheduling, and conducted their analysis for the case of a perpetuity. Arnold et al. (2019) enhanced the valuation formula to a perpetuity with a constant growth rate. Since we pick up on the research of Arnold et al. (2018) we use the term discontinuous financing instead of lagged debt management.

Although discontinuous financing is a recognized concept in the literature of corporate valuation, it is so far of little relevance for corporate valuation practice. In this study, we present a simplified derivation of the valuation equation for the approach that Arnold et al. (2018) pursued. This simplified derivation is more intuitive and enhances the understanding of discontinuous financing. Therefore, it might make this concept more accessible for corporate valuation practice. The key is to use a recursive setting and apply the relation of the market values at the beginning of each planning phase. To consider the detailed forecast analysis of firms in early periods of their planning horizon, we combine discontinuous financing with an explicit forecast phase where passive debt management is used. To do so, we transfer the approaches of L- and D-hybrid financing to discontinuous financing. This approach leads to the development of two new mixed financing strategies, which we refer to as L-discontinuous and D-discontinuous financing. As above, the abbreviations L and D stand for leverage and debt, respectively. L-discontinuous financing is characterized by a deterministic definition of the leverage ratio at the time of valuation. In the case of D-discontinuous financing, the debt level in period T is defined deterministically at the valuation date.

We analyze the effects on the market value of these new financing strategies theoretically and with the help of a simulation. Furthermore, we compare L- and D-discontinuous financing to L- and D-hybrid financing, and to pure active and passive debt management, respectively. With the help of the simulation analysis, we can not only determine the distributions of the deviations of the market values but also estimate the influence of different input parameters. Thereby, we expand the example of Arnold et al. (2018) who compare firm values under discontinuous financing for various lengths of planning phases and three different debt-to-market value ratios.

We conclude that the use of mixed financing strategies constitutes a reasonable and promising alternative to pure financing strategies in depicting the financing behavior of a firm. Particularly, hybrid financing or discontinuous financing in a two-phase model solves some shortcomings of active and passive debt management.

The remainder of this study is structured as follows. The next section offers an analysis of passive and active debt management in a two-phase model by presenting valuation equations under L- and D-hybrid financing. This constitutes the basis for the development of a two-phase model that includes discontinuous financing, which results in L- and D-discontinuous financing. In Section 2.3, the mixed financing strategies are compared theoretically and the results are illustrated by simulations. Finally, the possibility of using mixed financing is discussed from theoretical and practical viewpoints.

2.2 Mixed financing strategies in a two-phase model

2.2.1 Valuation with hybrid financing

Since we want to transfer the approach of L- and D-hybrid financing to discontinuous financing, we need to analyze the concept of hybrid financing first. The construction is based on a two-phase model. In the explicit forecast phase, passive debt management is used, whereas in the steady-state phase, active debt management is assumed (Kruschwitz et al. 2007). The distinction between L- and D-hybrid is made by different ways of determining the debt-to-market value ratio of the steady-state phase (Dierkes and Gröger 2010).

We assume that the explicit forecast phase consists of T periods. After this first phase, the firm is situated in a steady state. In this second phase, all variables associated with the valuation increase at a uniform and constant growth rate g. In addition, we suppose that the business risk does not change over time, which results in a constant cost of equity of the unlevered firm ρ^u. Since it is not the focus of our analysis, the costs of financial distress and the possibility of default are not considered such that the cost of debt corresponds to the risk-free interest rate r. This strong assumption can easily be relaxed by considering the cost of debt instead of the risk-free interest rate as it is done in Clubb and Doran (1995) and Arnold et al. (2018).[2] Moreover, it is assumed that interest on debt is fully deductible from taxable income.

[2]The discount rate to compute the market value of the tax shield depends on assumptions regarding the tax treatment. Depending on the taxation in the case of default, differences in the market value occur, see, for example, Sick (1990); Kruschwitz et al. (2005); Rapp (2006); Krause and Lahmann (2016); Baule (2019). A more explicit consideration of the insolvency risk for discontinuous financing can be found in Arnold et al. (2019).

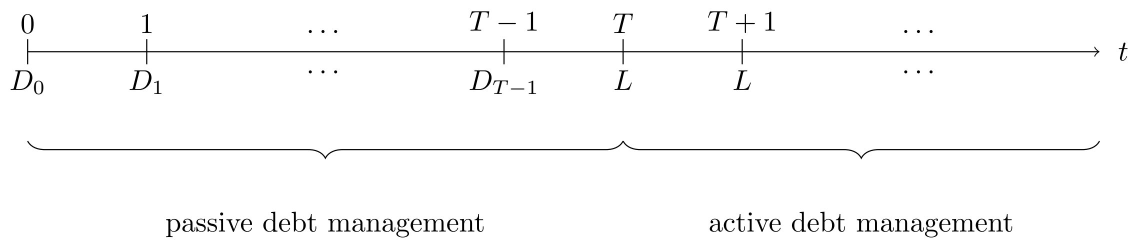

Figure 2.1: L-hybrid financing: the leverage of the steady-state phase is determined at the valuation date.

We start with the analysis of L-hybrid financing. The abbreviation L refers to leverage. Under this financing strategy, we suppose that the debt-to-market value ratio θ and therefore, the leverage L of active debt management, which is used in the steady-state phase, is determined at the time of valuation; see Fig. 2.1. By combining the adjusted present value (APV) approach (Kruschwitz and Löffler 2020, p. 90) for the explicit forecast phase with the free cash flow (FCF) approach (Kruschwitz and Löffler 2020, pp. 101–103) for the steady-state phase, the following valuation equation for L-hybrid financing (Kruschwitz et al. 2007, p. 429; Dierkes and Gröger 2010, p. 60) is obtained:

$$V_0^{\ell,\mathrm{LH}} = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_t]}{(1+\rho^u)^t} + \sum_{t=1}^{T} \frac{\tau \cdot r \cdot D_{t-1}}{(1+r)^t} + \frac{\mathbb{E}[\widetilde{FCF}_{T+1}]}{(k^\tau - g) \cdot (1+\rho^u)^T} \;, \tag{2.1}$$

where τ denotes the corporate tax rate; FCF_t the free cash flow, which is the cash flow of the unlevered firm; D_t the amount of debt; and V_t^ℓ the market value of the levered firm at the end of period t. The abbreviation LH refers to L-hybrid financing.

The valuation equation can be interpreted as follows. In the first term, the market value of the unlevered firm in the explicit forecast phase is computed by discounting the FCFs at the cost of equity of an unlevered firm. In the second term, we add the value of the tax shields of the first T periods. Since we assume passive debt management in this phase, the tax shields can be discounted at the risk-free interest rate. The term $\frac{\mathbb{E}[\widetilde{FCF}_{T+1}]}{k^\tau - g}$ determines the terminal value of a perpetual annuity with growth under active debt management at the beginning of the steady-state phase. In accordance with the FCF approach, the discounting is conducted with the weighted average cost of capital $k^\tau = \rho^\ell \cdot (1-\theta) + r \cdot (1-\tau) \cdot \theta$, where ρ^ℓ represents the cost of equity of a levered firm. Since ME showed that the market value of a levered firm and the market value of an unlevered firm of one period differ only by a factor that is already known at the valuation date, we can use the cost of equity of an unlevered firm to discount the terminal value to the valuation date (Miles and Ezzell 1980; Kruschwitz et al. 2007, p. 429; Dierkes and

Gröger 2010, pp. 60–61). This formula applies to both active debt management according to ME and active debt management according to HP, since one can select the corresponding adjustment formula for calculating the cost of equity of the levered firm.[3]

Defining the leverage L and, therefore, the debt-to-market value ratio θ at the time of valuation can lead to substantial refinancing at the beginning of the steady-state phase, since the debt level at the end of the explicit forecast phase D_{T-1} may differ significantly from the debt level $\widetilde{D}_T = \theta \cdot \widetilde{V}_T^{\ell,\mathrm{LH}}$, which is determined according to active debt management in the first period of the steady-state phase. For an example that illustrates this refinancing, see Dierkes and Gröger (2010). This disadvantage of L-hybrid financing is compensated by D-hybrid financing, in which the debt level of period T is defined deterministically. Thus, the leverage of the steady-state phase results from this deterministic debt level and the uncertain market value at the end of the explicit forecast phase. The abbreviation D refers to debt. This approach has the advantage that no refinancing is necessary at the end of the first forecast phase, such that a smoother transition from the explicit forecast phase to the steady-state phase is achieved. However, it has the disadvantage that the debt-to-market value ratio of the steady-state phase is uncertain from the perspective of the valuation date and can vary depending on the realized state (Dierkes and Gröger 2010, pp. 59, 63–64). Figure 2.2 shows that in the case of D-hybrid financing, the debt level of period T has to be additionally determined autonomously to calculate the debt-to-market value ratio. It follows that the debt-to-market value ratio of the steady-state phase is defined deterministically at the beginning of the steady-state phase rather than at the beginning of the explicit forecast phase, as in the case of L-hybrid financing.

By applying the APV approach for the explicit forecast and the steady-state phase, one obtains the valuation equation for D-hybrid financing (Dierkes and Gröger 2010, p. 63)

$$V_0^{\ell,\mathrm{DH}} = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_t]}{(1+\rho^u)^t} + \sum_{t=1}^{T} \frac{\tau \cdot r \cdot D_{t-1}}{(1+r)^t} + \frac{\mathbb{E}[\widetilde{FCF}_{T+1}]}{(\rho^u - g) \cdot (1+\rho^u)^T} + \frac{\tau \cdot r \cdot D_T}{(\rho^u - g)(1+r)^T}\,, \tag{2.2}$$

where DH stands for D-hybrid financing. Analogous to L-hybrid financing, the value of the firm in the explicit forecast phase is calculated in the first two terms. The computation of the terminal value is divided into the computation of the terminal value of the unlevered firm in the third term and the computation of the terminal value of the tax shields in the last term. The

[3]In the case of active debt management according to HP, all tax shields are uncertain which yields the adjustment formula for the levered cost of equity $\rho^\ell = \rho^u + (\rho^u - r) \cdot L$ (Harris and Pringle 1985). Under active debt management of ME, the tax shields are certain in the period of their emergence and uncertain in all other periods, which yields $\rho^\ell = \rho^u + (\rho^u - r) \cdot \frac{1+r\cdot(1-\tau)}{1+r} \cdot L$ (Miles and Ezzell 1985).

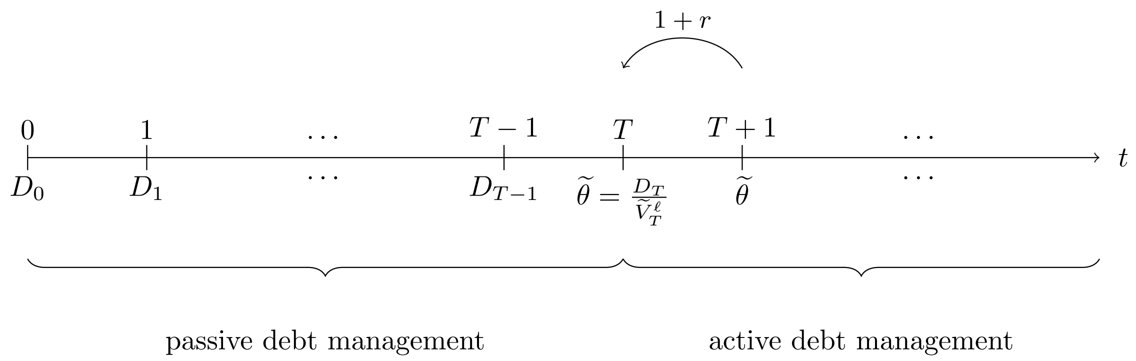

Figure 2.2: D-hybrid financing: the debt-to-market value ratio of the steady-state phase results from the deterministic debt level and the uncertain market value at the end of the explicit forecast phase.

former is calculated by discounting the constantly growing FCF at the unlevered cost of equity ρ^u, which results in the formula for a perpetual annuity with growth $\rho^u - g$. The denominator is multiplied by $(1+\rho^u)^T$ to obtain the value at the valuation date. Unlike Eq. (2.1) that applies to active debt management according to ME and HP, this valuation equation only applies to active debt management according to HP. In this case, the tax shields of the steady state are uncertain in all periods and have to be discounted at the cost of equity of an unlevered firm. Furthermore, the tax shields grow at the constant growth rate g such that they are also discounted using the formula $\rho^u - g$. Since the debt level D_T is already known at the valuation date, the terminal value of the tax shields at time T is discounted to the valuation date at the risk-free interest rate. If active debt management according to ME was used in the steady-state phase, discounting the tax shield of one period to the preceding period can be conducted using the risk-free interest rate r instead of ρ^u. It follows that the terminal value of the tax shields needs to be multiplied by the factor $\frac{1+\rho^u}{1+r}$ (Miles and Ezzell 1980; Dierkes and Gröger 2010, p. 63).

In the remainder of this subsection, we theoretically compare the market values in the case of L- and D-hybrid financing. To do so, we require that either active debt management of ME or active debt management of Harris and Pringle (HP) is used for the steady-state phase in both cases. Furthermore, we assume that the expected debt-to-market value ratio of D-hybrid financing coincides with the deterministic debt-to-market value ratio of L-hybrid financing, that is,

$$\theta = \frac{D_T}{\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DH}}]} \,. \tag{2.3}$$

It follows that the tax shields of the steady-state phase coincide such that the terminal value of the tax shields at the beginning of the steady-state phase is the same for both financing strategies:

$$\mathbb{E}[\widetilde{V}_T^{TS,\mathrm{LH}}] = V_T^{TS,\mathrm{DH}} \,. \tag{2.4}$$

The value of the unlevered firm does not depend on the financing strategy, such that the terminal value of the levered firm is identical:

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LH}}] = \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DH}}] \,. \tag{2.5}$$

Although, the market values coincide at the beginning of the steady-state phase, the market values differ at the valuation date. The value difference of L- and D-hybrid financing lies in the discounting of the terminal values to the valuation date. This results in a higher value under D-hybrid financing than under L-hybrid financing, since not the entire residual value is discounted at the unlevered cost of equity but only the value of the unlevered firm. The value of the tax shield at the beginning of the steady-state phase can be discounted using the lower risk-free interest rate, which yields a higher tax shield at the valuation date in the case of D-hybrid financing than in the case of L-hybrid financing. We exclude the explicit forecast phase and consider only value differences that result from the discounting of the terminal value to the valuation date. To do so, we introduce the notation TV_0 and deduce

$$TV_0^{\ell,\mathrm{DH}} > TV_0^{\ell,\mathrm{LH}} \,. \tag{2.6}$$

Thus, the terminal value under D-hybrid financing is always higher than that under L-hybrid financing at the valuation date if the debt-to-market value ratios coincide. Table 2.1 summarizes these results.

2.2.2 Valuation with discontinuous financing

In this subsection, we examine the possibility of using discontinuous financing in the steady-state phase and passive debt management in the explicit forecast phase. To specify discontinuous financing, we follow the approach of Arnold et al. (2018) but present a simpler and more intuitive derivation of the valuation equation, which could increase its acceptance and its use for corporate valuation practice. Discontinuous financing consists of consecutive planning phases in which passive debt management is used. At the beginning of each planning phase, a refinancing is carried out. We determine the debt level at some refinancing date by multiplying the debt-to-market value ratio by the expected market value of the levered firm. Since we consider a steady state, it is assumed that this debt level, as well as the FCF, grows at a constant growth rate within

		LH		DH
Time T				
	TV unlevered firm	$\mathbb{E}[\widetilde{V}_T^u]$	$=$	$\mathbb{E}[\widetilde{V}_T^u]$
$+$	TV tax shields	$\mathbb{E}[\widetilde{V}_T^{TS,\mathrm{LH}}]$	$=$	$V_T^{TS,\mathrm{DH}}$
$=$	TV levered firm	$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LH}}]$	$=$	$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DH}}]$
TV TS is discounted at		ρ^u		r
Time 0				
	TV unlevered firm	TV_0^u	$=$	TV_0^u
$+$	TV tax shields	$TV_0^{TS,\mathrm{LH}}$	$<$	$TV_0^{TS,\mathrm{DH}}$
$=$	TV levered firm	$TV_0^{\ell,\mathrm{LH}}$	$<$	$TV_0^{\ell,\mathrm{DH}}$

Table 2.1: Comparison of the terminal values (TV) of the unlevered firm, the tax shields and the levered firm under L- and D-hybrid financing for the case of coinciding debt-to-market value ratios at time T and at the valuation date.

the subsequent planning phase. After the planning phase, the next refinancing is carried out by adjusting the debt levels according to the updated expected market values.

If we combine passive debt management in the explicit forecast phase with discontinuous financing, the debt levels are defined deterministically for the first T periods. After these periods, the firm reaches a steady state, and the debt levels must be defined deterministically for the upcoming planning phase. Although the number of periods of this planning phase can generally be chosen arbitrarily, it is plausible that it is again possible to define the debt levels deterministically for T periods and so on. Therefore, we link the number of periods of a planning phase to the number of periods T of the explicit forecast phase, which can then be interpreted as the first planning phase, see Fig. 2.3. Regarding the specification of the debt-to-market value ratio of the steady-state phase, we can make the same distinction as in the previous subsection. On the one hand, the debt-to-market value ratio can be determined at the time of valuation analogous to L-hybrid

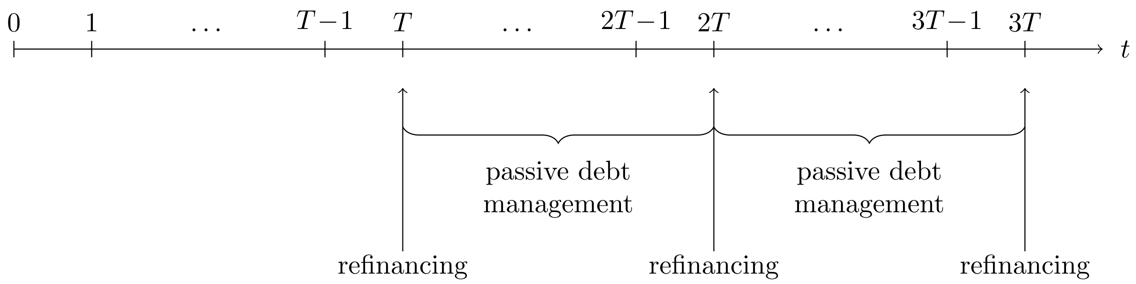

Figure 2.3: Discontinuous financing in the steady-state phase.

financing, which is referred to as L-discontinuous financing, where L stands again for leverage. On the other hand, analogous to a D-hybrid financing strategy, the debt-to-market value ratio that arises at the end of the explicit forecast phase can be used, which yields the development of a D-discontinuous financing strategy. As in the previous subsection, D stands for debt. The deterministic debt-to-market value ratio or the arising debt-to-market value ratio in the case of L- or D-discontinuous financing, respectively, is then used for the definition of debt levels in all subsequent planning phases.

First, we consider the case of L-discontinuous financing, which implies that the debt-to-market value ratio is defined deterministically at the valuation date. In contrast to Arnold et al. (2018) we choose a recursive approach to compute the market value under discontinuous financing. By using the APV method, we obtain the value of the levered firm at the beginning of the steady-state phase as

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LD}}] = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_{T+1}] \cdot (1+g)^{t-1}}{(1+\rho^u)^t} + \sum_{t=1}^{T} \frac{\tau \cdot r \cdot \theta \cdot \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LD}}] \cdot (1+g)^{t-1}}{(1+r)^t} + \frac{\mathbb{E}[\widetilde{V}_{2T}^{\ell,\mathrm{LD}}]}{(1+\rho^u)^T}\ , \quad (2.7)$$

where LD stands for L-discontinuous financing. The valuation equation can be interpreted as follows. In the first term, the value of the unlevered firm in the first planning phase of the steady state is determined by discounting the FCFs at the cost of equity of an unlevered firm. In the second term, the value of the tax shields in the first planning phase is computed. Since the debt levels are certain within a planning phase, the risk-free interest rate is the appropriate discount factor. Finally, the value of the levered firm at the beginning of the second planning phase is added and, according to ME, discounted at the cost of equity of an unlevered firm. This expression can be simplified by using the annuity present value factor for a constantly growing cash flow to

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LD}}] = \mathbb{E}[\widetilde{FCF}_{T+1}] \cdot \mathrm{APV}(\rho^u, g, T) + \tau \cdot r \cdot \theta \cdot \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LD}}] \cdot \mathrm{APV}(r, g, T) + \frac{\mathbb{E}[\widetilde{V}_{2T}^{\ell,\mathrm{LD}}]}{(1+\rho^u)^T}\ , \quad (2.8)$$

where

$$\mathrm{APV}(k, g, T) = \frac{1}{k-g} \cdot \left(1 - \frac{(1+g)^T}{(1+k)^T}\right) \quad (2.9)$$

is the annuity present value factor. So far, this valuation equation is of little use since it contains the market value at the beginning of the second phase, that is, the market value in period $2T$. However, since the free cash flow as well as the debt level grow at a constant growth rate g, the value of the levered firm also increases at this rate such that we obtain

$$\mathbb{E}[\widetilde{V}_{2T}^{\ell,\mathrm{LD}}] = \mathbb{E}[\widetilde{V}_{T}^{\ell,\mathrm{LD}}] \cdot (1+g)^T \ . \tag{2.10}$$

This relation is crucial for our analysis, and its use makes the derivation of the valuation equation much easier compared to Arnold et al. (2018). Inserting Eq. (2.10) into Eq. (2.8) and solving the circularity problem, that is, solving for the market value of the firm $\mathbb{E}[\widetilde{V}_{T}^{\ell,\mathrm{LD}}]$, results in

$$\mathbb{E}[\widetilde{V}_{T}^{\ell,\mathrm{LD}}] = \frac{\mathbb{E}[\widetilde{FCF}_{T+1}] \cdot \mathrm{APV}(\rho^u, g, T)}{1 - \tau \cdot r \cdot \theta \cdot \mathrm{APV}(r,g,T) - \frac{(1+g)^T}{(1+\rho^u)^T}} \ . \tag{2.11}$$

By multiplying the numerator by $\frac{\rho^u - g}{\rho^u - g}$, Eq. (2.11) can be reduced to

$$\mathbb{E}[\widetilde{V}_{T}^{\ell,\mathrm{LD}}] = \frac{\mathbb{E}[\widetilde{V}_{T}^{u}]}{1 - \theta \cdot \Gamma^{T,g}} \ , \tag{2.12}$$

where

$$\Gamma^{T,g} = \frac{\tau \cdot r \cdot \mathrm{APV}(r,g,T)}{\mathrm{APV}(\rho^u, g, T) \cdot (\rho^u - g)} \ . \tag{2.13}$$

Since the annuity present value factor is the reciprocal of the annuity factor, this result is consistent with the valuation equation of Arnold et al. (2018) and Arnold et al. (2019). Thus, by applying a recursive approach and the help of Eq. (2.10), we obtain a simplified and more intuitive derivation for this valuation equation, which easily shows how the factor $\Gamma^{T,g}$ is derived.

Note that the marginal cases of valuation Eq. (2.11) display well-known pure financing strategies. For $T \to \infty$, the valuation equation simplifies to the valuation equation of passive debt management, since there is only one infinitely long planning phase in which the debt levels are defined deterministically. For $T = 1$, the discontinuous financing strategy is equivalent to active debt management according to ME, because the debt levels are defined following a deterministic debt-to-market value ratio θ at the beginning of every period. The limit $T \to 0$ displays a continuous adjustment of the debt levels and, therefore, constitutes active debt management according to HP (for more detailed explanations on the marginal cases, see Clubb and Doran 1995, pp. 687, 690; Arnold et al. 2018, p. 165; Arnold et al. 2019, pp. 352–353). It follows that discontinuous financing can be used to depict a wide range of financing behaviors.

To obtain a valuation equation at the valuation date, we assume passive debt management in the explicit forecast phase and deduce

$$V_0^{\ell,\mathrm{LD}} = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_t]}{(1+\rho^u)^t} + \sum_{t=1}^{T} \frac{\tau \cdot r \cdot D_{t-1}}{(1+r)^t} + \frac{\mathbb{E}[\widetilde{FCF}_{T+1}]}{(\rho^u - g)\cdot(1+\rho^u)^T} \cdot \frac{1}{1-\theta\cdot\Gamma^{T,g}} \,. \tag{2.14}$$

In the first two terms, the market value of the levered firm in the explicit forecast phase is determined analogously to valuation Eq. (2.1) of L-hybrid financing. In the third term, the value of the firm at the beginning of the steady-state phase is calculated according to Eq. (2.12) and is discounted to the valuation date using the cost of equity of an unlevered firm.

The disadvantage of L-hybrid financing can be transferred to L-discontinuous financing: the debt level that is defined at the beginning of the steady-state phase according to discontinuous financing $\widetilde{D}_T = \theta \cdot \widetilde{V}_T^{\ell,\mathrm{LD}}$ can deviate considerably from the deterministically defined debt level D_{T-1}. Thus, the determination of the debt-to-market value ratio at the time of valuation implies that possibly unrealizable refinancing must be carried out at the end of the last period of the explicit forecast phase. This can be compensated by D-discontinuous financing in which—analogously to D-hybrid financing—the debt level of period T is defined deterministically. It follows that no substantial refinancing is required at the beginning of the steady-state phase. However, in period $2T$, $3T$, and so on, refinancing is still required; but since the firm is in a steady state in these periods, we consider these refinancing activities as less severe. To derive a valuation equation for D-discontinuous financing, we use the deterministically defined debt level D_T instead of $\theta \cdot \mathbb{E}[\widetilde{V}_T^{\ell}]$ in Eq. (2.7) and apply an analogous relationship, as in Eq. (2.10). At the end of the explicit forecast phase, we obtain

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DD}}] = \mathbb{E}[\widetilde{FCF}_{T+1}] \cdot \mathrm{APV}(\rho^u, g, T) + \tau \cdot r \cdot D_T \cdot \mathrm{APV}(r, g, T) + \frac{\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DD}}]\cdot(1+g)^T}{(1+\rho^u)^T} \,, \tag{2.15}$$

which can be simplified by solving the circularity problem to

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DD}}] = \frac{\mathbb{E}[\widetilde{FCF}_{T+1}] \cdot \mathrm{APV}(\rho^u, g, T) + \tau \cdot r \cdot D_T \cdot \mathrm{APV}(r, g, T)}{1 - \frac{(1+g)^T}{(1+\rho^u)^T}} \,, \tag{2.16}$$

where the abbreviation DD stands for D-discontinuous financing. This expression can be further reduced, similar to L-discontinuous financing, by using the factor $\Gamma^{T,g}$, see Eq. (2.13), to

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DD}}] = \mathbb{E}[\widetilde{V}_T^{u}] + D_T \cdot \Gamma^{T,g} \,. \tag{2.17}$$

Contrary to L-discontinuous financing, this valuation equation contains the deterministic debt level D_T instead of the debt-to-market value ratio θ. Furthermore, it is an additive instead of a multiplicative composition between the value of the unlevered firm and the factor $\Gamma^{T,g}$. This is because, in the case of D-discontinuous financing, the value of the tax shield does not contain the value of the levered firm, such that its calculation does not involve a circularity problem. In the explicit forecast phase, we again assume passive debt management, which yields at the valuation date

$$V_0^{\ell,\mathrm{DD}} = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_t]}{(1+\rho^u)^t} + \sum_{t=1}^{T} \frac{\tau \cdot r \cdot D_{t-1}}{(1+r)^t} + \frac{\mathbb{E}[\widetilde{FCF}_{T+1}]}{(\rho^u - g)\cdot(1+\rho^u)^T} + \frac{D_T \cdot \Gamma^{T,g}}{(1+r)^T} \; . \tag{2.18}$$

The first three terms correspond to valuation Eq. (2.2) of D-hybrid financing. Only the calculation of the terminal value of the tax shields in the last term differs, since discontinuous financing instead of active debt management is used in the steady-state phase. The deduced value of the tax shield at the beginning of the steady-state phase, see Eq. (2.17), is discounted to the valuation date using the risk-free interest rate, since it depends only on the debt level D_T, which is defined deterministically at the valuation date.

If the emerging debt-to-market value ratio of D-discontinuous financing $\frac{D_T}{\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DD}}]}$ coincides with the deterministic debt-to-market value ratio θ of L-discontinuous financing, we again obtain the same results for both financing strategies at the beginning of the steady-state phase, that is,

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LD}}] = \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DD}}] \; . \tag{2.19}$$

To obtain the relationship at the valuation date, we again exclude the explicit forecast phase. Analogous to hybrid financing, the terminal values of the tax shields are discounted differently to the valuation date such that the terminal value of the levered firm in the case of D-discontinuous financing is higher than in the case of L-discontinuous financing at the valuation date, that is,

$$TV_0^{\ell,\mathrm{DD}} > TV_0^{\ell,\mathrm{LD}} \, . \tag{2.20}$$

Table 2.2 summarizes these results. In the next section, we compare L- and D-discontinuous financing with L- and D-hybrid financing and conduct simulations to illustrate the differences in firm value.

		LD		DD
Time T				
	TV unlevered firm	$\mathbb{E}[\widetilde{V}_T^u]$	$=$	$\mathbb{E}[\widetilde{V}_T^u]$
$+$	TV tax shields	$\mathbb{E}[\widetilde{V}_T^{TS,\text{LD}}]$	$=$	$V_T^{TS,\text{DD}}$
$=$	TV levered firm	$\mathbb{E}[\widetilde{V}_T^{\ell,\text{LD}}]$	$=$	$\mathbb{E}[\widetilde{V}_T^{\ell,\text{DD}}]$
TV	TS is discounted at	ρ^u		r
Time 0				
	TV unlevered firm	TV_0^u	$=$	TV_0^u
$+$	TV tax shields	$TV_0^{TS,\text{LD}}$	$<$	$TV_0^{TS,\text{DD}}$
$=$	TV levered firm	$TV_0^{\ell,\text{LD}}$	$<$	$TV_0^{\ell,\text{DD}}$

Table 2.2: Comparison of the terminal values of the unlevered firm, the tax shields and the levered firm under L- and D-discontinuous financing for the case of coinciding debt-to-market value ratios at time T and at the valuation date.

2.3 Comparison of mixed financing strategies in a two-phase model

2.3.1 Theoretical comparison of mixed financing strategies in a two-phase model

In the previous section, we compared L- and D-hybrid financing, as well as L- and D-discontinuous financing. Now we compare these financing policies among each other to outline value differences that occur at the beginning of the steady-state phase and at the valuation date. We start with differences of the terminal values at time T. These differences result from different assumptions about the financing strategy of the steady-state phase. If discontinuous financing is assumed, the tax shields are certain within a planning phase and, therefore, can be discounted at the risk-free interest rate for T periods. Otherwise, if active debt management according to ME is used, the tax shields are certain only in the period of their emergence and can be discounted at the risk-free interest rate for only one period. Under active debt management according to HP, the adjustment occurs continuously such that all tax shields are discounted at the unlevered cost of equity. The longer the tax shields can be discounted at the risk-free interest rate, the higher is the value of the tax shields. For an explicit forecast phase, and therefore planning phases, that are composed of more than one period, that is, $T > 1$, follows that the terminal value of the tax shields is ceteris paribus higher in the case of discontinuous financing than in the case of active debt management of ME or HP which is used for hybrid financing. We obtain

$$\mathbb{E}[\widetilde{V}_T^{TS,\text{D}}] > \mathbb{E}[\widetilde{V}_T^{TS,\text{H}}]\,, \tag{2.21}$$

where D and H shorten discontinuous and hybrid financing, respectively. Since the terminal value of the unlevered firm is independent of the financing policy, the relationship is preserved for the market value of the firm, that is,

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{D}}] > \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{H}}] \; . \tag{2.22}$$

In the case of passive debt management, the debt level of each period is defined deterministically at the valuation date such that all tax shields are certain and can be discounted at the risk-free interest rate. Thus, the terminal value of the tax shields is considerably higher under passive debt management than under discontinuous financing, which yields a higher market value of the firm. Overall, we conclude

$$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{a}}] = \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LH}}] = \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DH}}] < \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LD}}] = \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DD}}] \ll \mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{p}}] \; , \tag{2.23}$$

where a and p stand for active and passive debt management, respectively. For this comparison, a consistent assumption regarding active debt management is again necessary. Active debt management of either ME or HP needs to be used for both cases of hybrid financing. The upper half of Table 2.3 summarizes these findings. The result that the difference between passive debt management, active debt management, and discontinuous financing depends on the length of the planning phases T is not new. It was already illustrated by an example in Arnold et al. (2018). However, they did not consider a two-phase model with a distinction in L- and D-financing. Hence, we expand their example by these aspects and additionally, quantify the influence of the parameter T and the influence of other input parameters in the next section.

It remains to outline deviations of the market values at the valuation date, which are outlined in the lower half of Table 2.3. All comparisons apply for active debt management of ME and HP. By excluding the explicit forecast phase, we consider again only disparities that result from the discounting of the terminal value depending on whether L- or D-financing is assumed. We start with a comparison of L-hybrid and L-discontinuous financing. In both cases, the entire market value of period T is discounted at the cost of equity of an unlevered firm, see Eq. (2.1) and Eq. (2.14), respectively, since the tax shields of the steady-state phase are uncertain. It follows that value differences that occur at the end of the explicit forecast phase are transferred to the valuation date such that the terminal value under L-discontinuous financing is higher than

		LH		DH		LD		DD
Time *T*								
	TV unlevered firm	$\mathbb{E}[\widetilde{V}_T^u]$	$=$	$\mathbb{E}[\widetilde{V}_T^u]$	$=$	$\mathbb{E}[\widetilde{V}_T^u]$	$=$	$\mathbb{E}[\widetilde{V}_T^u]$
$+$	TV tax shields	$\mathbb{E}[\widetilde{V}_T^{TS,\mathrm{LH}}]$	$=$	$V_T^{TS,\mathrm{DH}}$	$<$	$\mathbb{E}[\widetilde{V}_T^{TS,\mathrm{LD}}]$	$=$	$V_T^{TS,\mathrm{DD}}$
$=$	TV levered firm	$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LH}}]$	$=$	$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DH}}]$	$<$	$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{LD}}]$	$=$	$\mathbb{E}[\widetilde{V}_T^{\ell,\mathrm{DD}}]$
TV TS is discounted at		ρ^u		r		ρ^u		r
Time 0								
	TV unlevered firm	TV_0^u	$=$	TV_0^u	$=$	TV_0^u	$=$	TV_0^u
$+$	TV tax shields	$TV_0^{TS,\mathrm{LH}}$	$<$	$TV_0^{TS,\mathrm{DH}}$	?	$TV_0^{TS,\mathrm{LD}}$	$<$	$TV_0^{TS,\mathrm{DD}}$
$=$	TV levered firm	$TV_0^{\ell,\mathrm{LH}}$	$<$	$TV_0^{\ell,\mathrm{DH}}$	?	$TV_0^{\ell,\mathrm{LD}}$	$<$	$TV_0^{\ell,\mathrm{DD}}$

Table 2.3: Comparison of the terminal values of the unlevered firm, the tax shields and the levered firm under L-Hybrid, D-hybrid, L-discontinuous, and D-discontinuous financing for the case of coinciding debt-to-market value ratios at time T and at the valuation date.

that under L-hybrid financing at the valuation date. Considering Eq. (2.20) yields

$$TV_0^{\ell,\mathrm{DD}} > TV_0^{\ell,\mathrm{LD}} > TV_0^{\ell,\mathrm{LH}} . \tag{2.24}$$

Under both D-hybrid and D-discontinuous financing, the terminal value of the tax shields depends on the debt level at time T that is defined deterministically at the valuation date. Thus, the terminal value of the tax shields is certain and can be discounted at the risk-free interest rate; see Eq. (2.2) and Eq. (2.18). The value advantage of discontinuous financing over hybrid financing is again transferred to the valuation date, which yields a higher value under D-discontinuous financing. By additionally considering Eq. (2.6), we conclude

$$TV_0^{\ell,\mathrm{DD}} > TV_0^{\ell,\mathrm{DH}} > TV_0^{\ell,\mathrm{LH}} . \tag{2.25}$$

Whereas these relationships apply for every specification of the input parameters, the relationship of L-discontinuous and D-hybrid financing is still unclear. Although the terminal value under L-discontinuous financing is always higher than that under D-hybrid financing at the beginning of the steady-state phase, see Eq. (2.22), this value advantage of L-discontinuous financing is countered by the value advantage of D-hybrid financing through the discounting of the terminal value of the tax shields to the valuation date at the risk-free interest rate. Depending on which effect is dominant, a higher firm value under L-discontinuous financing is conceivable and vice versa. Furthermore, it remains unclear whether the outlined differences are severe or negligible, that is, whether these theoretical findings have a considerable impact on the market value. In the following subsection, we conduct a simulation analysis to analyze the distribution of the deviations between the different financing strategies and to quantify the influence of all input parameters. This analysis enables us to draw conclusions under which conditions these mixed financing are relevant for the practice of corporate valuation.

2.3.2 Simulation analysis

In this subsection, we use a Monte Carlo simulation to analyze the distribution of the theoretically outlined differences in the market value of the firm and the sensitivity of input parameters to illustrate the economic relevance. We assume a population of 100,000 firms that pursue mixed financing. We model the necessary input parameters as independent and uniformly distributed as follows. For the firms' unlevered cost of equity, cost of debt, and corporate tax rate, we define $\rho^u \sim U[8\%; 12\%]$, $r \sim U[2\%; 5\%]$, and $\tau \sim U[25\%; 35\%]$, respectively. We assume a consistent debt-to-market value ratio for all financing strategies, which is distributed according to $\theta \sim U[40\%; 80\%]$. For D-hybrid and D-discontinuous financing, we determine again the debt level D_T such that the debt-to-market value ratio that results from this debt level and the uncertain market value at the end of the explicit forecast phase equals θ. For the growth rate of the steady-state phase, we suppose $g \sim U[0.5\%; 2.0\%]$ and for the length of the explicit forecast phase, we consider $T \sim U\{5; 6; 7\}$.[4] Note that we apply active debt management according to HP in the case of hybrid financing.

The percentage valuation deviation is defined as

$$p(A, B) = \frac{TV_0^{\ell,A} - TV_0^{\ell,B}}{TV_0^{\ell,B}} , \tag{2.26}$$

where $A, B \in \{\mathrm{H}, \mathrm{LH}, \mathrm{DH}, \mathrm{D}, \mathrm{LD}, \mathrm{DD}\}$. We need not specify the value of the FCF, since it does not affect the valuation deviation. In the simulation, we analyze the distribution of the percentage valuation deviation by computing the mean, the standard deviation, the minimum, and the maximum. Moreover, we quantify the influence of the input parameters on this deviation. Particularly, we use Spearman's rank correlation coefficient (Charnes 2007, p. 63-65) to analyze which parameters influence this deviation the most and which the least. Table 2.4 summarizes the results.

[4]The length of the explicit forecast phase is company-specific and should be extended until the assumption of a steady state seems realistic (Ballwieser and Hachmeister 2016, p. 52). Brealey et al. (2020) assume a length of six periods (Brealey et al. 2020, p. 97) and (Koller et al. 2015) recommend five to seven periods (Koller et al. 2015, p. 230), which is why we decided on this distribution for T.

	at time T	at the valuation date					
	$p(\mathrm{D}, \mathrm{H})$	$p(\mathrm{DH}, \mathrm{LH})$	$p(\mathrm{DD}, \mathrm{LD})$	$p(\mathrm{LD}, \mathrm{LH})$	$p(\mathrm{DD}, \mathrm{DH})$	$p(\mathrm{LD}, \mathrm{DH})$	$p(\mathrm{DD}, \mathrm{LH})$
distribution							
mean	1.7%	3.1%	3.8%	1.7%	2.4%	-1.3%	5.6%
standard deviation	0.5%	0.9%	1.2%	0.5%	0.8%	0.4%	1.8%
min; max	0.6%; 4.3%	1.0%; 7.5%	1.2%; 9.7%	0.6%; 4.3%	0.8%; 6.4%	−3.0%; −0.4%	1.8%; 14.3%
sensitivity analysis							
θ	52.0%	43.2%	38.4%	52.0%	43.9%	−27.7%	43.7%
T	16.2%	29.8%	35.0%	16.2%	28.0%	−49.1%	29.1%
τ	11.7%	9.8%	8.8%	11.7%	10.0%	−6.4%	9.9%
r	14.2%	7.9%	3.2%	14.2%	3.8%	−2.0%	5.9%
ρ^u	2.4%	6.6%	12.2%	2.4%	11.2%	−13.6%	8.6%
g	3.6%	2.6%	2.5%	3.6%	3.1%	−1.3%	2.9%

Table 2.4: Simulation results: mean, standard deviation, minimum, and maximum of the distribution of the deviations between different financing strategies and sensitivities of each input parameter. For each observation j, we determine the percentage valuation deviation $p(A, B)$ for $A \in \{\mathrm{H}, \mathrm{D}, \mathrm{LH}, \mathrm{DH}, \mathrm{LD}, \mathrm{DD}\}$. We compute the mean as $\bar{y} = \frac{1}{n}\sum_{j=1}^{n} y_j$, the standard deviation as $s_y = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(y_j - \bar{y})}$ and the sensitivities as Spearman's rank correlation coefficients (Charnes 2007).

As outlined above, at the beginning of the steady-state phase, the values of L- and D-financing coincide. Thus, at time T we need only compare the values of hybrid and discontinuous financing. As illustrated in the previous subsection, the value under discontinuous financing is higher than that under hybrid financing. For the outlined intervals, we obtain a minimal deviation of 0.6% and a maximal deviation of 4.3% with a mean of 1.7%. The debt-to-market value ratio θ has the greatest influence on this difference. The correlation coefficient amounts to more than 50%. If a firm pursues a high debt ratio, the importance of the value of the tax shields increases. Furthermore, the length of the explicit forecast phase T, which determines the length of the planning phases, has the second biggest influence, namely around 16%. For a high value of T, the planning phases are longer such that the difference of the number of periods for which the tax shields are certain becomes larger. By comparison, the other value drivers of the tax shields, the risk-free interest rate r and the tax rate τ, account for 14.2% and 11.7% of the valuation deviation, respectively. A higher value of r and τ increases the tax shields. The growth rate g and unlevered cost of equity ρ^u each has a sensitivity of under 4%, which is a negligible effect.

At the valuation date, we compare each one of the presented mixed financing strategies with every other mixed financing strategy. The results are also outlined in Table 2.4. We obtain similar deviations for a comparison of L- and D-hybrid or L- and D-discontinuous financing, for which we obtain an average deviation of 3.1% and 3.8%, respectively. In both cases the debt-to-market value ratio θ and the length of the explicit forecast phase T are most important. The higher the debt-to-market value ratio and the longer the planning phases, the higher terminal value of the tax shields. Furthermore, the longer the explicit forecast phase, the larger the value advantage of D-financing over L-financing. The influence of every other parameter is considerably smaller.

Under L-financing, both the terminal value of the unlevered firm and the terminal value of the tax shields are discounted at the cost of equity of the unlevered firm to obtain the value at the valuation date. It follows that comparing L-hybrid and L-discontinuous financing at the valuation date yields the same deviations as a comparison of hybrid and discontinuous financing at time T. Comparing D-hybrid and D-discontinuous financing shows a larger valuation deviation of between 0.8% and 6.4%. Again, the debt-to-market value ratio θ has the highest impact, of about 44%, followed by the length of the explicit forecast phase, which explains about 28%. The remarkable higher sensitivity of T, compared to the deviation of L-hybrid and L-discontinuous financing, is due to an increasing importance of the length of the explicit forecast phase since the terminal value of the tax shields is discounted at the risk-free interest rate.

In the previous subsection, 2.3.1, we were not able to make a general statement about the relationship of D-hybrid and L-discontinuous financing. However, our simulations show that for the specified definition areas, the market value under D-hybrid financing is always higher than that under L-discontinuous financing. The percentage valuation deviation has a mean of -1.3% and a standard deviation of 0.4%. The length of the explicit forecast phase has the highest impact, accounting for more almost 50% of the valuation deviation. It follows that the discounting of the terminal value of the tax shields to the valuation date at the risk-free interest rate for T periods has a high impact and compensates for the value advantage of L-discontinuous financing at the beginning of the steady-state phase. For other specifications of the parameters, a value advantage of L-discontinuous financing is conceivable but not plausible. For example, for an unusual short explicit forecast phase that comprises only one period and otherwise unaffected definition ranges, we find that the market value under L-discontinuous financing is always higher than that under D-hybrid financing. For an explicit forecast phase of length two, there are only very few parameter constellations for which the market value under L-discontinuous financing is higher; and for an explicit forecast phase of three periods, there are no constellations where this case appears any more. It follows that under most parameter ranges, we record that the market value under D-hybrid financing is higher than that under L-discontinuous financing.

The value differences become considerably larger if we compare L-hybrid and D-discontinuous financing, whereby we deduce valuation deviations of almost 15%. Whereas the deviations that occur in a comparison of L-discontinuous financing and hybrid financing are negligible, a deviation of more than 10% can be considered economically relevant. Even deviations of 7.5% or 9.7%, as in the comparisons of L- and D-hybrid or L- and D-discontinuous financing, respectively, can lead to considerable disparities.

The results that the debt-to-market value ratio and the length of the planning phases have the highest influence are not surprising. If a firm had no or only little debt financing and no or very short planning phases of passive debt management, there would not be significant deviations. However, we were able to describe the distributions of the deviations and the influences of these parameters. Furthermore, to be able to analyze for which length of planning phases our model is economically relevant, we conducted the above simulation three more times each with a different fixed T. The results can be found in Table 2.5.

The deviations increase for longer planning phases, that is, for a larger T but even in the simulation with $T = 5$, we obtain deviations of up to 9.5%. For planning phases of length $T = 7$,

		at time T	at the valuation date					
		$p(\mathrm{D}, \mathrm{H})$	$p(\mathrm{DH}, \mathrm{LH})$	$p(\mathrm{DD}, \mathrm{LD})$	$p(\mathrm{LD}, \mathrm{LH})$	$p(\mathrm{DD}, \mathrm{DH})$	$p(\mathrm{LD}, \mathrm{DH})$	$p(\mathrm{DD}, \mathrm{LH})$
$T = 5$	mean	1.5%	2.5%	2.9%	1.5%	1.9%	-1.0%	4.5%
	standard deviation	0.4%	0.6%	0.8%	0.4%	0.5%	0.2%	1.2%
	min; max	0.5%; 3.2%	0.9%; 5.0%	1.1%; 6.0%	0.5%; 3.2%	0.7%; 4.2%	−1.8%; −0.4%	1.6%; 9.5%
$T = 6$	mean	1.7%	3.1%	3.7%	1.7%	2.4%	-1.3%	5.5%
	standard deviation	0.5%	0.8%	1.0%	0.5%	0.7%	0.3%	1.5%
	min; max	0.6%; 3.8%	1.2%; 6.3%	1.5%; 7.9%	0.6%; 3.8%	0.9%; 5.3%	−2.4%; −0.6%	2.1%; 12.0%
$T = 7$	mean	2.0%	3.7%	4.6%	2.0%	2.9%	-1.7%	6.7%
	standard deviation	0.6%	0.9%	1.2%	0.6%	0.8%	0.4%	1.8%
	min; max	0.7%; 4.3%	1.4%; 7.4%	1.8%; 9.6%	0.7%; 4.3%	1.1%; 6.3%	−3.0%; −0.7%	2.6%; 14.2%

Table 2.5: Results of three simulation analyses, each with a different fixed T.

we obtain deviations of more than 5% in almost all comparisons. Overall, we conclude that especially for firms with a high leverage and a long explicit forecast phase the outlined deviations should be examined carefully and the analyzed mixed financing strategies should be considered for the valuation to depict a wide range of financing behavior.

2.4 Conclusions

Empirical research indicates that active or passive debt management as pure financing strategies can explain the capital structure decisions of firms only to a limited extent. In response, corporate valuation theory has introduced various forms of mixed financing strategies. In this study, we analyzed discontinuous financing and hybrid financing as the main mixed financing policies and clarified their use and impacts on the market value in a two-phase model.

With passive debt management in the explicit forecast phase and active debt management in the steady-state phase, hybrid financing is directly linked to the two-phase model. Discontinuous financing, on the contrary, is characterized by the possibility of refinancing according to updated expected market values after a certain number of periods, independently of the separation of the planning horizon into two phases. To use this mixed financing strategy in a two-phase model, we linked the number of periods after which a refinancing can be carried out to the number of periods of the explicit forecast phase. Therefore, at the end of the explicit forecast phase with T periods and, accordingly, at the end of every T periods, the firm has the option of refinancing. This study improved the comprehensibility of the previous derivation of a valuation equation under discontinuous financing by applying a simpler and more intuitive recursive valuation approach.

Analogous to L- and D-hybrid financing, we differentiated between L- and D-discontinuous financing. On the one hand, under L-hybrid and L-discontinuous financing, the leverage of the steady state is defined at the time of valuation. On the other hand, under D-hybrid and D-discontinuous financing, the debt-to-market value ratio that results from the deterministic debt level and the uncertain market value at the end of the explicit forecast phase is used. The difference between these financing policies lies in the necessity for refinancing at the end of the explicit forecast phase. In the case of L-hybrid and L-discontinuous financing, the debt level at the end of the explicit forecast phase has to be adjusted according to the deterministic leverage. By contrast, this is not necessary in the case of D-hybrid and D-discontinuous financing, since the debt level is defined deterministically at the valuation date. However, in this case, the leverage of the steady state is uncertain at the valuation date.

Furthermore, we showed that differences occur if a firm's financing behavior corresponds to one of these mixed financing strategies but a pure financing strategy is applied in the valuation. The difference is smaller if pure active debt management is used instead of pure passive debt management. The comparison of the four mixed financing strategies yields the result that for a consistent debt-to-market value ratio, the terminal values at the beginning of the steady state coincide in the case of L- and D-hybrid financing and in the case of L- and D-discontinuous financing. Due to the greater uncertainty of the tax shields in the case of hybrid financing, the terminal value under this financing policy is lower than that under discontinuous financing. Regarding the market values at the valuation date, we elucidated that this difference is transferred in the case of D- or L-financing. It follows that the market value under D-discontinuous financing is higher than that under D-hybrid financing. Analogously, the market value under L-discontinuous financing is higher than the market value under L-hybrid financing. Only the relationship of D-hybrid and L-discontinuous financing remained unclear. However, for most cases, we found that D-hybrid financing yields a higher firm value than L-discontinuous financing does.

The differences between the mixed financing strategies become larger if the leverage increases or the explicit forecast phase becomes longer. Therefore, especially when the leverage is high and the explicit forecast phase is long, the deviations between the terminal values can be considerable, since they can amount to more than 10%. In these cases, one should examine a firm's financing strategy carefully and consider the application of a mixed financing strategy to avoid valuation inaccuracies. It follows that the valuation formulas presented in this study could offer a valuable alternative for corporate valuation practice to better reflect the financing behavior of firms and could lead to a more sophisticated valuation result. Thus, the analysis in this study contributes to both the practice and the theory of corporate valuation.

Further research could address other forms of mixed financing strategies. These could be generated by combining passive and active debt management differently, or by replacing passive or active debt management with other pure financing strategies. In addition, a mixed financing strategy that consists of more than two pure financing strategies is conceivable. Moreover, one could consider adding an additional phase to secure the transition from the explicit forecast phase to the steady state. In such a three-phase model, it would be possible, for example, to use the simultaneous mixed financing of Dierkes and Schäfer (2017) to obtain a gradual transition from passive debt management to the financing strategy of the steady-state phase. Furthermore, the assumption that debt is risk-free can be relaxed. Arnold et al. (2019) show that for longer planning phases, the probability of default increases and outline ideas on how to consider this in the valuation. Thus, further research could also attempt to develop a model that includes the costs of financial distress and the probability of default for these mixed financing strategies.

3 Terminal value calculation with discontinuous financing and debt categories

Joint work with Stefan Dierkes.

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Abstract

Empirical analyses indicate that active and passive debt management have limited power to explain the financing behavior of firms. Therefore, discontinuous financing, as a combination of active and passive debt management, might be a more realistic financing strategy. However, the properties of this financing strategy for the steady state have not yet been sufficiently analyzed. For this reason, we investigate analytically terminal value calculation with discontinuous financing and derive adjustment formulas for the period-specific levered cost of equity. Since a single adjustment of the entire debt at the beginning of every planning phase might still not be close to the real financing behavior of firms, we modify discontinuous financing by introducing debt categories, which are adjusted successively and include the maturity of debt. For this new financing strategy, we derive valuation equations and an adjustment formula for the constant cost of equity. Finally, we discuss the relevance and applicability of discontinuous financing with debt categories and its impact on the market value of a firm.

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3.1 Introduction

Terminal value calculation is based on the assumption that a firm has reached a steady state after an explicit forecast phase (Koller et al. 2020, pp. 186–188; Brealey et al. 2020, pp. 95–99). To realistically depict the characteristics of a firm in a steady state, the choice of the financing strategy is a core issue. In corporate valuation practice, active or passive debt management are typically considered. However, empirical studies indicate that these financing strategies cannot model a firm's real financing policy with sufficient accuracy (see, e.g., Lewellen and Emery 1986; Barclay and Smith 2005; Grinblatt and Liu 2008). Consequently, discontinuous financing was developed as an alternative to active and passive debt management (Clubb and Doran 1995; Arnold et al. 2018, 2019; Dierkes and de Maeyer 2020). In particular, Arnold et al. (2018) derived valuation equations for the use of discontinuous financing in a steady state, but the effects of this financing strategy on the development of the market value of a firm, financial risk, and the cost of equity have not yet been investigated.

The aim of the paper is to analyze and further develop discontinuous financing as a more realistic financing policy for the steady state. First, we examine the properties of a steady state under discontinuous financing. We show that financial risk is inconstant under this financing policy, analyze the risk-free part of the tax shield, and derive an adjustment formula for the period-specific levered cost of equity. Furthermore, we outline that the sole adjustment of the entire debt level at the beginning of every planning phase could be improved by a consecutive adjustment of debt levels. Second, to model this consecutive adjustment, we develop a modified discontinuous financing policy, where every period a predetermined part of the overall debt level is adapted. Specifically, we introduce so-called debt categories, which include debt maturity and derive valuation equations with an adjustment formula for the levered cost of equity. Finally, we discuss its relevance, applicability, and impact on market value of a firm.

Findings on active and passive debt management are well known in the literature on corporate valuation (see, e.g., Kruschwitz and Löffler 2020, pp. 104–109). However, "neither purely active nor passive debt management assumptions are accurate reflections of corporate financial practice" (Clubb and Doran 1995, p. 690). In particular, empirical research showed that firms adjust their debt levels slowly (Fama and French 2002) and with a time lag (Leary and Roberts 2005; Huang and Ritter 2009). Therefore, Clubb and Doran (1995) introduced a lagged debt management policy that consists of consecutive planning phases in which passive debt management is used. In their first approach, debt levels are derived by multiplying the expected market value of a firm by the debt-to-market value ratio. After a predetermined number of periods, that is, after a planning phase, debt levels are adapted to the development of the firm and are redefined deterministically for the next planning phase, considering the updated expected market value of the firm. Despite the use of the debt ratio, debt levels are certain within a planning phase because they are linked to the expected and not the realized market value of the firm (Ashton and Atkins 1978). An extension of this mixed financing strategy was introduced by Arnold et al. (2018), who referred to it as discontinuous financing. Specifically, they pursued the second approach of Clubb and Doran (1995) that holds debt levels constant between rescheduling and analyzed the case of a perpetuity. Arnold et al. (2019) enhanced the valuation formula to a perpetuity with a constant growth rate, while Dierkes and de Maeyer (2020) examined the effects of discontinuous financing using a two-phase model.

In this study, we identify the differences between the approaches of Clubb and Doran (1995) and Arnold et al. (2018) and examine the properties of a steady state under discontinuous financing. It is apparent that financial risk cannot be deterministic given that passive debt management is used during a planning phase. However, the property of constant expected financial risk does not transfer from a steady state under passive debt management to a steady state under discontinuous financing. We highlight that financial risk varies depending on the remaining number of periods until the next planning phase. Furthermore, following the line of Inselbag and Kaufold (1997), we derive an adjustment formula for the period-specific cost of equity, which is also necessary for the unlevering and relevering of beta factors. We show that inconstant financial risk yields an inconstant growth of the market value of the firm. This effect was already briefly mentioned by Clubb and Doran (1995), who stated that "it does illustrate an interesting point, even if expectations [...] do not change [...] the value of the firm will still fluctuate" (Clubb and Doran 1995, p. 693). However, this effect has not been examined further thus far. Moreover, we outline that, compared to active and passive debt management, discontinuous financing might indeed be a more realistic depiction of a firm's financing behavior but the assumption that the entire debt level is adjusted according to the development only at the end of each planning phase might still not be practical. In particular, we find a partial adjustment in every period more realistic.

We implement this partial adjustment of debt by introducing debt categories, which constitutes the core contribution of our study. The resulting financing strategy is a modification of discontinuous financing that might come closer to a firm's real financing behavior. We assume that a firm adapts only a certain share of its overall debt in each period, requiring debt categories to be successively adjusted. That is, in each period, only one debt category is adjusted according to the development of the firm, and no other debt category is adapted to new information. In the subsequent period, another debt category is adjusted, and so on. Therefore, the planning phases overlap, and we, thus, consider debt maturity. At every point in time, a firm has various debt categories, characterized by different remaining maturities. First, we deduce a valuation equation for two debt categories and derive the adjustment formula for the levered cost of equity. Second, we extend the valuation and adjustment formulas to an arbitrary number of debt categories. Encouragingly, independent of the number of debt categories, the expected financial risk under this modified discontinuous financing policy is constant. Therefore, we obtain a constant levered cost of equity, making the application of the Gordon-Shapiro formula (Gordon and Shapiro 1956) possible. Although this successive adjustment of debt levels might be more realistic than a sole adjustment at the beginning of each planning phase, compared to discontinuous financing, its effect on the market value of the firm is small. Nevertheless, the analysis of this financing strategy is instructive for corporate valuation theory, and the valuation approach is also applicable in valuation practice. We discuss the application of both versions of discontinuous financing and outline that standard discontinuous financing can be used as an approximation for discontinuous financing with debt categories.

Overall, this study contributes to the literature on valuation by examining terminal value calculation with discontinuous financing and deriving a period-specific adjustment formula for the levered cost of equity. Furthermore, the modification of standard discontinuous financing into discontinuous financing with debt categories yields a new financing policy with a constant levered cost of equity, which might be more suitable to describe the financing behavior of firms in a steady state.

The remainder of this study is organized as follows. In Section 3.2, discontinuous financing is analyzed, and the costs of equity under discontinuous financing are derived. The results are illustrated using an example. In Section 3.3, debt categories are introduced. First, the valuation formula for the special case of two debt categories is determined before generalizing it to an arbitrary number of debt categories. Second, the example from Section 3.2 is extended to debt categories to outline the impact of the new financing strategy on the market value of a firm and to compare it with other financing strategies. Third, it is outlined how discontinuous financing with debt categories can be applied in a two-phase model and how the risk of default can be included. Finally, we summarize and discuss the contributions of the study in Section 3.4.

3.2 Discontinuous financing

3.2.1 Fundamentals of discontinuous financing

Discontinuous financing is defined via planning phases in which debt levels are specified deterministically. The definition of debt levels within a planning phase is linked to the expected market value of a firm. After a planning phase, debt levels are adjusted according to the development of the firm and are again deterministically defined for the next planning phase. Fig. 3.1 illustrates this structure for planning phases of length T.

There are different approaches on how to link debt levels to the expected market value of a firm. Initially, Clubb and Doran (1995) multiplied the debt-to-market value ratio by the expected market value of the firm and used a finite planning horizon for analysis. A second approach that Clubb and Doran (1995) discussed was to hold debt constant between rescheduling. This approach was also pursued by Arnold et al. (2018), who extended it to a steady state without growth. Arnold et al. (2019) examined this concept in a steady state with a constant growth rate g, and Dierkes and de Maeyer (2020) included this structure in a two-phase model. Table 3.1 summarizes the different debt level definitions, where θ is the debt-to-market value ratio, and $\mathbb{E}_T[\cdot]$ is the expectation depending on the available information at time T. The tilde represents uncertain variables. Furthermore, V_t^ℓ and D_t denote the market value of the levered firm and the debt level at time t for $t \in \{0, \ldots, T-1\}$, respectively. Superscript ℓ indicates the levered firm. Note that $T = 1$ constitutes active debt management according to Miles and Ezzell (ME), $T \to 0$ complies with active debt management according to Harris and Pringle (HP), and $T \to \infty$ represents passive debt management (Clubb and Doran 1995, pp. 687, 690; Arnold et al. 2018, p. 165; Arnold et al. 2019, pp. 352–353).

For all approaches in Table 3.1, the financial risk of a period depends on the remaining number of periods until the next refinancing date. If the firm is at a refinancing date, the tax shield is certain for the next T periods. However, if the firm will refinance at the end of the period,

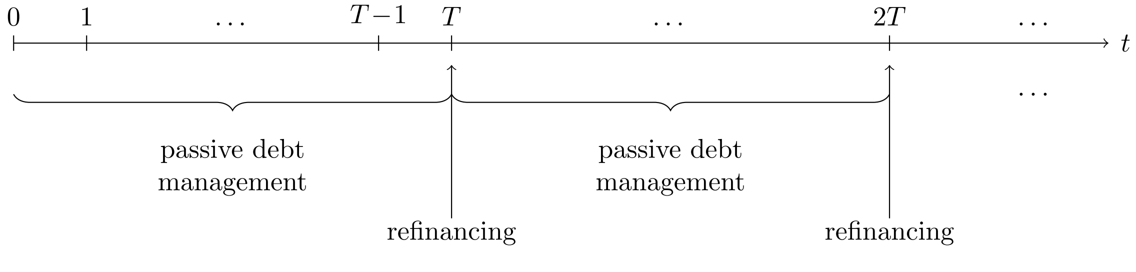

Figure 3.1: Basic structure of discontinuous financing.

	1st planning phase	2nd planning phase
	$0, \dots, T-1$	$T, \dots, 2T-1$
Clubb and Doran (1995) 1st appr.	$D_t = \theta \cdot \mathbb{E}[\widetilde{V}_t^\ell]$	$D_{T+t} = \theta \cdot \mathbb{E}_T[\widetilde{V}_{T+t}^\ell]$
Clubb and Doran (1995) 2nd appr.; Arnold et al. (2018)	$D_t = \theta \cdot V_0^\ell$	$D_{T+t} = \theta \cdot V_T^\ell$
Arnold et al. (2019); Dierkes and de Maeyer (2020)	$D_t = \theta \cdot V_0^\ell \cdot (1+g)^t$	$D_{T+t} = \theta \cdot V_T^\ell \cdot (1+g)^t$

Table 3.1: Overview of different debt level definitions in the case of discontinuous financing with $t \in \{0, \dots, T-1\}$.

only the tax shield of the subsequent period is certain, which implies a higher financial risk. Therefore, in a steady state, in which the free cash flow (FCF) grows at a constant rate g, only the financial risks for periods in the same section of a planning phase coincide, being minimal at refinancing dates and increasing as the next refinancing date approaches. Such inconstant financial risk leads to a fluctuation in the market value of the firm even if cash flow expectations do not change. It follows that

$$\mathbb{E}[\widetilde{V}_{2T+t}^\ell] = \mathbb{E}[\widetilde{V}_{T+t}^\ell] \cdot (1+g)^T \qquad \text{but} \qquad \mathbb{E}[\widetilde{V}_{T+t}^\ell] \neq \mathbb{E}[\widetilde{V}_T^\ell] \cdot (1+g)^t, \tag{3.1}$$

for $t \neq n \cdot T$, $n \in \mathbb{N}$. Therefore, although the concepts in Table 3.1 may appear identical, there exists an important difference between the first and second approach of Clubb and Doran (1995) for $T \notin \{0, 1, \infty\}$. If the initial approach of Clubb and Doran (1995) was transferred to a steady state, debt levels would not grow at the constant growth rate g. For this reason, Arnold et al. (2018, 2019) and Dierkes and de Maeyer (2020) used the second approach of Clubb and Doran (1995) for their analysis of a steady state. While Arnold et al. (2018) examined a steady state without growth, Arnold et al. (2019) and Dierkes and de Maeyer (2020) included growth and allowed the debt levels to grow at the same constant growth rate as the FCFs within every planning phase. We build our analysis upon these results and use the second approach of Clubb and Doran (1995) with a constant growth of debt levels within planning phases.

To derive valuation equations for discontinuous financing on a clear theoretical basis, in a first step, we do not consider the costs of financial distress or the possibility of default. This is common in comparable basic analyses for other financing strategies (see, e.g., Miles and Ezzell 1985). It follows that debt is risk-free and can be discounted at the risk-free interest rate r, which is constant over time. In a second step, consequences and relaxations of this assumption are discussed in Section 3.3.5. Moreover, in the presented valuation equations, we do not consider an explicit forecast phase, which means that the steady-state phase starts at the valuation date. A detailed analysis on how to link standard discontinuous financing to an explicit forecast phase with passive debt management can be found in Dierkes and de Maeyer (2020).

Thus far, to investigate the market value of a firm under discontinuous financing, the adjusted present value (APV) approach has been theoretically analyzed. As for all financing strategies, it is instructive to also analyze other discounted cash flow (DCF) methods (for DCF approaches, see, e.g., Kruschwitz and Löffler 2020). We start by citing existing results, which we then use to analyze the adjustment formula for the cost of equity in Section 3.2.2, which is required for the FCF and the Flow to Equity (FtE) approach. Dierkes and de Maeyer (2020) used a recursive procedure to derive the valuation equation for a steady state and derived for the terminal value (Dierkes and de Maeyer 2020, p. 1327)

$$V_0^\ell = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF_1}] \cdot (1+g)^{t-1}}{(1+\rho^u)^t} + \sum_{t=1}^{T} \frac{\tau \cdot r \cdot \theta \cdot V_0^\ell \cdot (1+g)^{t-1}}{(1+r)^t} + \frac{\mathbb{E}[\widetilde{V}_T^\ell]}{(1+\rho^u)^T} \ , \tag{3.2}$$

where τ denotes the corporate tax rate and ρ^u the cost of equity of an unlevered firm. Superscript u indicates the unlevered firm. The market value of the firm in the first planning phase is computed in the first and second terms by calculating the values of the unlevered firm and of the tax shields separately. As tax shields are certain over the planning phase, they can be discounted using the risk-free interest rate. The value of the levered firm at the beginning of the second planning phase is added in the third term. Dierkes and de Maeyer (2020, pp. 1327–1328) used the relation $\mathbb{E}[\widetilde{V}_T^\ell] = V_0^\ell \cdot (1+g)^T$, solved the circularity problem, and deduced the expression

$$V_0^\ell = \frac{\mathbb{E}[\widetilde{FCF_1}] \cdot \mathrm{PVA}(\rho^u, g, T)}{1 - \tau \cdot r \cdot \theta \cdot \mathrm{PVA}(r, g, T) - \frac{(1+g)^T}{(1+\rho^u)^T}} \ , \tag{3.3}$$

where

$$\mathrm{PVA}(k, g, T) = \frac{1}{k-g} \cdot \left(1 - \frac{(1+g)^T}{(1+k)^T}\right) \ , \tag{3.4}$$

for $k \in \{\rho^u, r\}$, determines the present value of a growing annuity. Therefore, there exists a valuation equation for discontinuous financing that can be used to calculate the terminal value at the valuation date without circularity problems.

3.2.2 Cost of equity derivation

A thorough analysis of a financing strategy does not only consist of a valuation formula based on the APV approach but includes also an analysis of the levered cost of equity. In particular, an adjustment formula for the levered cost of equity is necessary to apply other DCF approaches. Therefore, in this section, we first extend existing research by calculating the market value of a firm within a planning phase. Second, we use this result to derive an adjustment formula for period-specific levered costs of equity. Moreover, this analysis functions as a theoretical foundation for the extension of discontinuous financing in Section 3.3.

We identified that financial risk is inconstant under discontinuous financing, which yields inconstant levered costs of equity. Furthermore, since the debt-to-market value ratios vary with the development of the firm, they generally are random variables. Consequently, to calculate the levered cost of equity, we need to determine the market value of the firm between refinancing dates. As in Eq. (3.2), we compute the market values of the firm in the first planning phase by partitioning it into the planning phase value and the value of the firm at the end of the planning phase. For $t \in \{0, \ldots, T-1\}$, the market value of the firm is given by

$$\begin{aligned}\mathbb{E}[\widetilde{V}_t^\ell] &= \sum_{k=1}^{T-t} \frac{\mathbb{E}[\widetilde{FCF}_1] \cdot (1+g)^{t+k-1}}{(1+\rho^u)^k} + \sum_{k=1}^{T-t} \frac{\tau \cdot r \cdot \theta \cdot V_0^\ell \cdot (1+g)^{t+k-1}}{(1+r)^k} + \frac{\mathbb{E}[\widetilde{V}_T^\ell]}{(1+\rho^u)^{T-t}} \\ &= \mathbb{E}[\widetilde{FCF}_1] \cdot (1+g)^t \cdot \mathrm{PVA}(\rho^u, g, T-t) + \tau \cdot r \cdot \theta \cdot V_0^\ell \\ &\quad \cdot (1+g)^t \cdot \mathrm{PVA}(r, g, T-t) + V_0^\ell \cdot \frac{(1+g)^T}{(1+\rho^u)^{T-t}} \ . \end{aligned} \tag{3.5}$$

The market value of the firm of the current planning phase can be divided into the value of the unlevered firm and that of the tax shields. The planning phases consist of T periods, which implies that the firm will refinance in $T-t$ periods. The computation of the tax shield includes the market value of the firm at the valuation date. This value is known in period t so that we can use the risk-free interest rate as discount rate. The market value of the firm at the end of the first planning phase is added in the third term. The equity value can be derived by subtracting the debt level of period t, that is,

$$\mathbb{E}[\widetilde{E}_t^\ell] = \mathbb{E}[\widetilde{V}_t^\ell] - \theta \cdot V_0^\ell \cdot (1+g)^t \ . \tag{3.6}$$

Note that $t = 0$ results in valuation Eq. (3.2) for the levered firm at the valuation date. By the following proposition, we derive an adjustment formula for the levered cost of equity and the weighted average cost of capital (WACC).

Proposition 1. *Under the assumption of discontinuous financing, the levered cost of equity is given by*

$$\rho_t^\ell = \rho^u + (\rho^u - r) \cdot (1 - \tau \cdot r \cdot PVA(r, g, T - t)) \cdot L_t \ , \tag{3.7}$$

where

$$L_t = \frac{D_t}{\mathbb{E}[\widetilde{E}_t^\ell]} \tag{3.8}$$

is the leverage in period t for $t \in \{0, \dots, T-1\}$. Furthermore, the WACC is given by

$$k_t^\tau = (1 - \tau \cdot r \cdot \theta_t \cdot PVA(r, g, T - t)) \cdot \rho^u - \tau \cdot r \cdot \theta_t \cdot (1 - r \cdot PVA(r, g, T - t)) \ , \tag{3.9}$$

where $\theta_t = \frac{D_t}{\mathbb{E}[\widetilde{V}_t^\ell]}$ is the debt-to-market value ratio in period t.

Note that the leverage is defined as a quotient of a deterministic quantity and an expectation of a random variable. Thereby, we use a similar definition of the leverage as for passive debt management. However, due to Jensen's inequality, the leverage is generally not equal to the expression $\mathbb{E}\left[\frac{D_t}{\widetilde{E}_t^\ell}\right]$ since the equity value is a random variable. The same argumentation holds for the debt-to-market value ratio.

In the proof of Proposition 1, we derive the WACC by using the definition of the cost of capital. We define the cost of capital as the amount by which the expected value of the sum of the FCF and the market value of the firm at the end of the period is to be discounted to obtain the expected market value of the firm at the beginning of the period:

$$k_t^\tau = \frac{\mathbb{E}[\widetilde{FCF}_1] \cdot (1+g)^t + \mathbb{E}[\widetilde{V}_{t+1}^\ell]}{\mathbb{E}[\widetilde{V}_t^\ell]} - 1 \ . \tag{3.10}$$

Note that the expectations are not conditioned on the level of information, that is, we consider the cost of capital at the valuation date. Subsequently, deducing the cost of equity is straightforward. A detailed computation is provided in the Appendix.

Proposition 1 shows that the formula for the derivation of the costs of equity under discontinuous financing has a similar structure as that for the levered cost of equity under other financing strategies (for the adjustment formulas under active or passive debt management, see, e.g., Miles and Ezzell 1985; Inselbag and Kaufold 1997; Kruschwitz and Löffler 2020). The first term is the unlevered cost of equity and depicts operational risk. To consider the financial risk that results from debt financing, a risk premium is added, which depends on the difference between the unlevered cost of equity and the risk-free interest rate. The term $\tau \cdot r \cdot \mathrm{PVA}(r, g, T-t)$ considers that the tax shields are certain until the end of the planning phase, that is, for $T-t$ periods. The risk premium increases linearly with leverage. For $T = 1$, the formula simplifies to the adjustment formula for active debt management of Miles and Ezzel (see Inselbag and Kaufold 1997, Eq. (10)) and $T \to \infty$ results in the adjustment formula for passive debt management (see Inselbag and Kaufold 1997, Eq. (7))

$$\rho_t^\ell = \rho^u + (\rho^u - r) \cdot \frac{D_t - VTS_t}{\mathbb{E}[\widetilde{E}_t^\ell]} \ . \tag{3.11}$$

Following the line of Inselbag and Kaufold (1997) and adapting the adjustment formula of passive debt management is another possibility to derive the adjustment formula of Proposition 1. In a steady state with passive debt management, all future tax shields are certain such that the value of the tax shield is calculated as $VTS_t = \frac{\tau \cdot r \cdot D_t}{r-g}$. For discontinuous financing, the tax shield is only certain for the next $T-t$ periods. The value of the risk-free part of the tax shields VTS_t^R, that is, the part that can be discounted at the risk-free interest rate, amounts to

$$\begin{aligned} VTS_t^R &= \tau \cdot r \cdot \theta \cdot V_0^\ell \cdot (1+g)^t \cdot \mathrm{PVA}(r, g, T-t) \\ &= \tau \cdot r \cdot D_t \cdot \mathrm{PVA}(r, g, T-t) \ , \end{aligned} \tag{3.12}$$

see Eq. (3.5). The remaining part of the value of the tax shield is uncertain and, therefore, discounted at the unlevered cost of equity (Miles and Ezzell 1985). The value of the risk-free part of the tax shield lowers the risk premium that is added to the business risk in the formula for the levered cost of equity. We conclude

$$\rho_t^\ell = \rho^u + (\rho^u - r) \cdot \frac{D_t - VTS_t^R}{\mathbb{E}[\widetilde{E}_t^\ell]} \ . \tag{3.13}$$

Plugging VTS_t^R, see Eq. (3.12), into Eq. (3.13) yields the adjustment formula from Eq. (3.7). It follows that the basic structure of the adjustment formula does not change, but the share of the risk-free tax shield is adjusted (for similar observations, see Inselbag and Kaufold 1997). The factor that displays the risk-free part of the tax shield can be expressed in percentage of V_0^ℓ.

Similar observations can be conducted for the formula of the WACC. In the case of passive debt management, the WACC can be computed as (Inselbag and Kaufold 1997, Eq. (8))

$$k_t^\tau = \rho^u \cdot \left(1 - \frac{VTS_t}{\mathbb{E}[\widetilde{V}_t^\ell]}\right) + r \cdot \frac{VTS_t - \tau \cdot D_t}{\mathbb{E}[\widetilde{V}_t^\ell]} \,. \tag{3.14}$$

To derive the WACC for discontinuous financing we can again replace the value of the tax shield VTS_t by the risk-free part of the value of the tax shield VTS_t^R, see Eq. (3.12), to obtain the WACC from Eq. (3.9).

At $t = T - 1$, the firm is in the last period of the first planning phase and will refinance in the next period. Therefore, the tax shield is only certain for the subsequent period, which complies with an active debt management according to ME. Note that Eq. (3.7) and (3.9) indeed simplify to the cost of equity and WACC under active debt management with leverage L_{T-1} for $t = T-1$, respectively (Miles and Ezzell 1985). At the valuation date, that is, $t = 0$, the leverage L_0 complies with the specified ratio $L = \frac{\theta}{1-\theta}$. It follows that the cost of equity is exceptionally a deterministic variable during this period. Furthermore, for $t = 0$, Eq. (3.7) is in line with the formula for unlevering β in Arnold et al. (2018). However, in their analysis, it remains unclear how to adjust β or the cost of equity in the other periods of a planning phase. Insofar, we extend the study of Arnold et al. (2018) by analyzing periodic-specific costs of equity and by deriving the corresponding adjustment formulas.

As outlined above, the debt-to-market value ratio in period t differs from the specified ratio θ, and the adjusted leverage L_t needs to be used. This ratio depends on the equity value at time t. It follows that the computation of both the levered cost of equity and the WACC contains a circularity problem. They cannot be computed without determining first the debt level and expected market value of the firm at time t. However, solving circularity problems is a common and recurring procedure in corporate valuation practice. It is usually addressed using a spreadsheet software.

Since we derived a closed-form solution for the levered costs of equity and corresponding WACCs within the first planning phase, we can compute the levered costs of equity for all periods. We have shown that financial risk coincides for periods in the same section of a planning phase, which implies $\rho^\ell_{n\cdot T+t} = \rho^\ell_t$ for $n \in \mathbb{N}$.

3.2.3 Example

To illustrate the findings of the previous subsections, we present an example. We assume a steady state in which the input parameters grow at a constant rate of $g = 1.5\%$. The FCF of period one is 1,000, and the debt-to-market value ratio is $\theta = 60\%$.[2] The risk-free interest rate, unlevered cost of equity, and tax rate amount to $r = 4\%$, $\rho^u = 10\%$, and $\tau = 30\%$, respectively. We use Eq. (3.3) to compute the market value of the firm V_0^ℓ and deduce

$$V_0^\ell = \frac{1{,}000 \cdot 3.90}{1 - 30\% \cdot 4\% \cdot 60\% \cdot 4.58 - \frac{(1+1.5\%)^5}{(1+10\%)^5}} = 13{,}066.70 \ .$$

By multiplying this value by the debt-to-market value ratio, we obtain the debt level $D_0 = 7{,}840.02$, and by subtracting this from the market value of the firm, we obtain the equity value $E_0^\ell = 5{,}226.68$ at the valuation date. Given that the firm is situated in a steady state, we assume that the debt level grows at the constant growth rate of $g = 1.5\%$. To calculate the market values of the firm within the first planning phase, we use Eq. (3.5). In period five, refinancing is carried out so that the financial risk for this period coincides with the financial risk at the valuation date. Therefore, we obtain

$$\mathbb{E}[\widetilde{E}_5^\ell] = E_0^\ell \cdot (1+g)^5 = 5{,}226.68 \cdot (1+1.5\%)^5 = 5{,}630.62 \ .$$

Furthermore, by applying Eq. (3.8), we obtain the debt-to-market value ratios of periods 1 to 4, and, by using Eq. (3.7), we then derive the levered costs of equity, see Table 3.2.

Although the differences in the costs of equity are small, this example illustrates that financial risk differs depending on the remaining number of periods until the next refinancing date. At a refinancing date, the tax shield is certain for the next five periods so that the cost of equity is lower than that in the subsequent period in which the tax shield is only certain for the next four periods. In the fourth period, financial risk is maximal, since only the tax shield of the subsequent period is certain. Furthermore, note that

$$E_0^\ell \cdot (1+g) = 5{,}226.68 \cdot (1+1.5\%) = 5{,}305.08 \neq 5{,}295.81 = \mathbb{E}[\widetilde{E}_1^\ell] \ .$$

It follows that the equity value, and therefore the market value of the firm, grows at a smaller rate than $g = 1.5\%$ from period 0 to period 1, since financial risk increases. In terms of the equity value, this growth rate amounts to $g_1 = 1.32\%$. Conversely, for example, from period 4 to period 5, financial risk decreases so that the equity value grows at a higher rate, $g_5 = 1.70\%$. However, from period 0 to period 5, from period 1 to period 6, and so on, the growth rate is

[2]The calculations can be easily adjusted if the debt level, rather than the debt-to-market value ratio, is defined deterministically. Then the debt-to-market value ratio is obtained by dividing the debt level by the market value of the firm. This ratio is used in the subsequent periods, see Dierkes and de Maeyer (2020) for further explanations.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
$\mathbb{E}[\widetilde{FCF}_t]$		1,000.00	1,015.00	1,030.23	1,045.68	1,061.36
$\mathbb{E}[\widetilde{V}_t^\ell]$	13,066.70	13,253.43	13,447.02	13,648.20	13,857.75	14,076.55
D_t	7,840.02	7,957.62	8,076.99	8,198.14	8,321.11	8,445.93
$\mathbb{E}[\widetilde{E}_t^\ell]$	5,226.68	5,295.81	5,370.04	5,450.06	5,536.64	5,630.62
$\mathbb{E}[\widetilde{TS}_t]$		94.08	95.49	96.92	98.38	99.85
$\mathbb{E}[\widetilde{TCF}_t]$		1,094.08	1,110.49	1,127.15	1,144.06	1,161.22
$\mathbb{E}[\widetilde{VTS}_t]$	1,302.00	1,312.25	1,326.73	1,346.10	1,371.12	1,402.62
VTS_t^R	431.08	354.24	272.92	186.91	96.01	464.40
θ_t	60.00%	60.04%	60.07%	60.07%	60.05%	60.00%
ρ_t^ℓ	18.51%	18.61%	18.72%	18.82%	18.91%	18.51%
g_t		1.32%	1.40%	1.49%	1.59%	1.70%

Table 3.2: Illustration of discontinuous financing.

exactly $g = 1.5\%$ for both the market value of the firm and equity value. Since the growth rate is inconstant, the debt-to-market value ratio varies. On the one hand, at the beginning of a planning phase, the debt levels grow faster than the market value of the firm, which yields an increase in the debt-to-market value ratio. On the other hand, at the end of a planning phase, the market value of the firm grows faster than the debt levels, resulting in a decreasing debt-to-market value ratio. In period 5, the second planning phase starts, and the structure is repeated.

In Table 3.2, we also included the expected tax shield $\mathbb{E}[\widetilde{TS}_t]$, expected total cash flow $\mathbb{E}[\widetilde{TCF}_t]$ (as the sum of FCF and tax shield), expected value of the tax shield $\mathbb{E}[\widetilde{VTS}_t]$, and risk-free part of the value of the tax shield VTS_t^R. As outlined in the previous subsection, the latter can alternatively be used to compute the levered cost of equity.

This example illustrates that, in a steady state under discontinuous financing, financing risk is inconstant, which yields inconstant costs of equity and market value fluctuations. In summary, the operating, investing, and financing activities yield a constant growth of every relevant quantity, but the financing activities still do not result in a constant financial risk. This setting is the result of the sole and, therefore, big refinancing every T periods. It follows that discontinuous financing provides an opportunity to depict a broad range of financing behaviors of firms, but might still not come close to the real financing behavior. It might be more practical and more realistic to adjust a certain part of the debt level in every period. After every T periods, the entire debt level has still been adjusted, but the refinancing is partitioned into several periods. This motivates the analysis of debt categories.

3.3 Discontinuous financing with debt categories

3.3.1 Derivation of a valuation formula for two debt categories

In this section, we introduce a modification of discontinuous financing as follows. We consider a firm that has various debt categories, which are adjusted successively. In each period, some debt category is adjusted by multiplying the debt-to-market value ratio by the market value of the firm, and no other category is adjusted. Since we examine this financing strategy in a steady state, these other debt categories grow at the same constant growth rate g as the FCF. In the subsequent period, another category is adjusted according to the updated market value of the firm, and so on. Consequently, at each point in time, the debt categories reflect shares of the overall debt that have different remaining maturities. We obtain an overlapping sequence of bonds with an identical time to maturity, each of which is prolonged at an adjusted level every year. Therefore, instead of consecutive planning phases, this financing strategy incorporates overlapping planning phases and includes the maturity of debt. In particular, active and passive debt management is mixed in every period.[3] The successive adjustment of proportions of the overall debt in each period seems more practical than the sole adjustment of the entire debt level at the valuation date or at the beginning of a planning phase. It follows that the assumption of discontinuous financing with debt categories might come closer to a firm's financing behavior as opposed to the assumption of standard discontinuous financing.

In this subsection, we examine a steady state of a firm that has two debt categories. Thus, compared to Miles and Ezzell (1980), we introduce one additional layer of debt. If θ is the pursued debt-to-market value ratio, we define $\theta^{(2)} := \frac{1}{2}\theta$. Superscript (2) refers to the number of debt categories. Furthermore, let D_t^j be the amount of debt in category $j \in \{0,1\}$ over period t. Category 0 is adjusted in period $0, 2, 4, \ldots$, and category 1 is adjusted in period $1, 3, 5, \ldots$. Fig. 3.2 illustrates the concept of two debt categories.

At the valuation date, the entire debt level has to be specified, but the part D_0^1 should have been defined in the previous period. If the assumption of debt categories is used in a two-phase model

[3] A mix of active and passive debt management in every period can also be found in Dierkes and Schäfer (2017). In their study, the mix of active and passive debt management can be arbitrarily determined in each period. Furthermore, they define a deterministic part of the overall debt for all future periods at the valuation date, which is different from our study. We adjust the deterministic part successively according to the debt-to-market value ratio.

$$
\begin{array}{cccc}
0 & 1 & 2 & \dots \quad \rightarrow t \\
D_0^0 = \theta^{(2)} \cdot V_0^\ell & D_1^0 = D_0^0 \cdot (1+g) & \mathbb{E}[\widetilde{D}_2^0] = \theta^{(2)} \cdot \mathbb{E}[\widetilde{V}_2^\ell] & \dots \\
D_0^1 = \theta^{(2)} \cdot V_0^\ell & \mathbb{E}[\widetilde{D}_1^1] = \theta^{(2)} \cdot \mathbb{E}[\widetilde{V}_1^\ell] & \mathbb{E}[\widetilde{D}_2^1] = \mathbb{E}[\widetilde{D}_1^1] \cdot (1+g) &
\end{array}
$$

Figure 3.2: Discontinuous financing with two debt categories.

and an explicit forecast phase is planned before the steady state, the debt level that results from an explicit planning could be used. Since we want to concentrate on the valuation formulas for the steady-state phase, we exclude this detailed planning in our derivations and discuss possible links in Section 3.3.4. It follows that, at the valuation date, the firm exceptionally adjusts both debt categories, that is 0 and 1, according to $\theta^{(2)}$, which yields $D_0 = \theta \cdot V_0^\ell$. The expected total amount of debt, $\mathbb{E}[\widetilde{D}_t]$, at some time $t \neq 0$ can be derived by adding the two debt categories:

$$\mathbb{E}[\widetilde{D}_t] = \theta^{(2)} \cdot \mathbb{E}[\widetilde{V}_{t-1}^\ell] \cdot (1+g) + \theta^{(2)} \cdot \mathbb{E}[\widetilde{V}_t^\ell]\,. \tag{3.15}$$

We use a backward inductive approach to derive the valuation formula, which is similar to the approaches of Miles and Ezzell (1980); Inselbag and Kaufold (1997) and Dierkes and Schäfer (2017). First, we assume a finite time horizon of $T < \infty$ periods to derive valuation equations. Afterwards, we extend the formulas for $T \to \infty$. We start with the valuation formula in period $T-1$. To calculate the value of the levered firm, we use the concept of value-additivity and compute the values of the unlevered firm and of the tax shield separately. We derive the former by discounting the firm's expected FCF at time T at the unlevered cost of equity. The value of the tax shield can be computed by discounting the tax savings due to both debt categories. In period $T-1$, the firm alters one of its debt categories. The other has already been adjusted in period $T-2$ and grows at the constant growth rate g. Therefore, the value of the levered firm in period $T-1$ depends on the market value of the firm in period $T-2$. Since the market value of the firm in periods $T-2$ and $T-1$ is certain in period $T-1$, the amount of debt for both categories is certain, and it can be discounted at the risk-free interest rate r. Again, we begin with a theoretical framework and assume risk-free debt to concentrate on the derivation of the valuation equations and adjustment formulas. For a discussion of the integration of the risk of default, we refer to Section 3.3.5. We obtain

$$V_{T-1}^\ell = \frac{\mathbb{E}_{T-1}[\widetilde{FCF_T}]}{1+\rho^u} + \frac{\tau \cdot r \cdot \theta^{(2)} \cdot V_{T-2}^\ell \cdot (1+g)}{1+r} + \frac{\tau \cdot r \cdot \theta^{(2)} \cdot V_{T-1}^\ell}{1+r}\,. \tag{3.16}$$

Solving the circularity (i.e., solving for the value of the levered firm in period $T-1$) yields

$$V_{T-1}^{\ell} = \frac{\mathbb{E}_{T-1}[\widetilde{FCF}_T]}{(1+\rho^u)\cdot\eta_1^{(2)}} + \frac{\tau\cdot r\cdot\theta^{(2)}\cdot V_{T-2}^{\ell}\cdot(1+g)}{(1+r)\cdot\eta_1^{(2)}}\,, \tag{3.17}$$

where

$$\eta_1^{(2)} := 1 - \frac{\tau\cdot r\cdot\theta^{(2)}}{1+r}\,. \tag{3.18}$$

To derive the value of the levered firm in period $T-2$, we consider that one debt category has been adjusted in period $T-3$. Furthermore, we have to include the value of the levered firm in period $T-1$. Therefore, the market value of the firm in period $T-2$ is

$$V_{T-2}^{\ell} = \frac{\mathbb{E}_{T-2}[\widetilde{FCF}_{T-1}]}{1+\rho^u} + \frac{\tau\cdot r\cdot\theta^{(2)}\cdot V_{T-3}^{\ell}\cdot(1+g)}{1+r} + \frac{\tau\cdot r\cdot\theta^{(2)}\cdot V_{T-2}^{\ell}}{1+r} + \frac{\mathbb{E}_{T-2}[\widetilde{V}_{T-1}^{\ell}]}{1+d}\,. \tag{3.19}$$

The amount of debt for both debt categories in period $T-2$ is again certain, and it can be discounted at the risk-free interest rate r. Since the appropriate discount rate, d, of the value of the levered firm in period $T-1$ is not apparent, we can again apply the value-additivity principle and divide the last term into its components to obtain

$$\begin{aligned} V_{T-2}^{\ell} = {} & \frac{\mathbb{E}_{T-2}[\widetilde{FCF}_{T-1}]}{1+\rho^u} + \frac{\tau\cdot r\cdot\theta^{(2)}\cdot V_{T-3}^{\ell}\cdot(1+g)}{1+r} + \frac{\tau\cdot r\cdot\theta^{(2)}\cdot V_{T-2}^{\ell}}{1+r} \\ & + \frac{\mathbb{E}_{T-2}[\widetilde{FCF}_T]}{(1+\rho^u)^2\cdot\eta_1^{(2)}} + \frac{\tau\cdot r\cdot\theta^{(2)}\cdot V_{T-2}^{\ell}\cdot(1+g)}{(1+r)^2\cdot\eta_1^{(2)}}\,. \end{aligned} \tag{3.20}$$

The FCF reflects the cash flow of an unlevered firm and can, thus, be discounted using the unlevered cost of equity. Since the market value of the firm in period $T-2$ is certain, we discount it at the risk-free interest rate r. Solving for the market value of the firm in period $T-2$ yields

$$V_{T-2}^{\ell} = \frac{\mathbb{E}_{T-2}[\widetilde{FCF}_{T-1}]}{(1+\rho^u)\cdot\eta_2^{(2)}} + \frac{\mathbb{E}_{T-2}[\widetilde{FCF}_T]}{(1+\rho^u)^2\cdot\eta_2^{(2)}\cdot\eta_1^{(2)}} + \frac{\tau\cdot r\cdot\theta^{(2)}\cdot V_{T-3}^{\ell}\cdot(1+g)}{(1+r)\cdot\eta_2^{(2)}}\,, \tag{3.21}$$

where

$$\eta_2^{(2)} := 1 - \frac{\tau\cdot r\cdot\theta^{(2)}}{1+r} - \frac{\tau\cdot r\cdot\theta^{(2)}\cdot(1+g)}{(1+r)^2\cdot\eta_1^{(2)}}\,. \tag{3.22}$$

With these calculations, a general formula can be deduced for the sequence $(\eta_k^{(2)})_{k\in\mathbb{N}}$. For $k > 1$, we define

$$\eta_k^{(2)} := 1 - \frac{\tau \cdot r \cdot \theta^{(2)}}{1+r} - \frac{\tau \cdot r \cdot \theta^{(2)} \cdot (1+g)}{(1+r)^2 \cdot \eta_{k-1}^{(2)}} \,. \tag{3.23}$$

This sequence considers that the tax shield of a specific period depends not only on the debt level of this period but also on that of the previous period. To derive a valuation equation for a perpetuity, we need to show that this sequence converges to a limit $\eta^{(2)}$ for $k \to \infty$, which we do in Corollary 1, see the Appendix.

From these results, we can derive a valuation formula for the levered firm at the valuation date. Since we exclude an explicit planning of debt levels, we use $D_0^1 = \theta^{(2)} \cdot V_0^\ell$ and deduce a valuation formula for the market value of the firm for a perpetuity.

Proposition 2. *If $g < k^*$, the market value of the firm for two debt categories is given by*

$$V_0^\ell = \frac{E[\widetilde{FCF_1}]}{k^* - g} + \frac{\tau \cdot r \cdot D_0^1}{1+r^*} = \frac{E[\widetilde{FCF_1}]}{k^* - g} \cdot \left(1 - \frac{\tau \cdot r \cdot \theta^{(2)}}{1+r^*}\right)^{-1} \,, \tag{3.24}$$

where $k^* := (1+\rho^u) \cdot \eta^{(2)} - 1$ and $r^* := (1+r) \cdot \eta^{(2)} - 1$.

Proposition 2 exemplifies that the FCFs are discounted at the adjusted cost of capital, k^*, which we deduced by solving the emerging circularity problems. We add the tax shield that results from the amount of debt D_0^1 of the category that is again adapted in period 1. By using the relation $D_0^1 = \theta^{(2)} \cdot V_0^\ell$, we can rewrite the expression and deduce a circularity-free valuation formula for two debt categories. Multiplying the market value of the firm by the debt-to-market value ratio yields the equity value. For a detailed derivation of Eq. (3.24), see the Proof of Proposition 2 in the Appendix.

As mentioned in Section 3.2, in corporate valuation practice, it is common to use the FCF or the FtE approach. To do so, we need to derive an adjustment formula for the levered cost of equity and deduce the WACC to be able to apply a valuation formula of the form

$$V_0^\ell = \frac{\mathbb{E}[\widetilde{FCF_1}]}{k^\tau - g} \,, \tag{3.25}$$

with $k^\tau = (1-\theta) \cdot \rho^\ell + r \cdot (1-\tau) \cdot \theta$. The adjustment formula and the expression for the WACC are captured in the following proposition.

Proposition 3. *Under the assumption of two debt categories, the levered cost of equity can be obtained by*

$$\rho^\ell = \rho^u + (\rho^u - r) \cdot \left(1 - \frac{\tau \cdot r}{1+r} - \frac{1}{2} \cdot \frac{\tau \cdot r \cdot (1+g)}{(1+r)^2 \cdot \eta^{(2)}}\right) \cdot L \,, \tag{3.26}$$

where $L = \frac{\theta}{1-\theta}$ is the leverage. Furthermore, the WACC is given by

$$k^\tau = \rho^u - \tau \cdot r \cdot \theta \cdot \frac{(1+\rho^u)}{1+r} - (\rho^u - r) \cdot \frac{1}{2} \cdot \frac{\tau \cdot r \cdot \theta \cdot (1+g)}{(1+r)^2 \cdot \eta^{(2)}} \,. \tag{3.27}$$

A proof is provided in the Appendix. Proposition 3 clarifies that the derivation of the levered cost of equity can again be divided into the operational risk ρ^u and a risk premium that depends on the difference of the unlevered cost of equity and the risk-free interest rate. This difference is multiplied by a factor that incorporates the financial risk due to both debt categories. Compared to the adjustment formula for active debt management according to HP, where all tax shields are uncertain and this factor equals 1, we subtract two terms to depict the smaller financial risk. Both debt categories are certain in the subsequent period, which corresponds to active debt management of ME and yields the subtraction of $\frac{\tau \cdot r}{1+r}$. Additionally, half of the debt level is certain in the period after next; that is, it can be discounted at the risk-free interest rate for two periods. The effect of this additional certainty of the tax shield is reflected in the term $\frac{1}{2}\frac{\tau \cdot r \cdot (1+g)}{(1+r)^2 \cdot \eta^{(2)}}$. The WACC is also similar to the WACC for active debt management (for the WACC of active debt management, see e.g., Miles and Ezzell 1980, Eq. (20); Kruschwitz and Löffler 2020, p. 105). In addition to the one half of the debt that is adjusted in the next period, we must consider the effects of the other debt category.

For a comparison to passive debt management, we compute the value of the risk-free part of the tax shields at the valuation date, as we have done for standard discontinuous financing. It can be derived by similar recursive steps as above, which yields

$$VTS_0^R = \frac{\tau \cdot r \cdot \theta \cdot V_0^\ell}{1+r} + \frac{1}{2} \cdot \frac{\tau \cdot r \cdot \theta \cdot V_0^\ell \cdot (1+g)}{(1+r)^2 \cdot \eta^{(2)}} \,. \tag{3.28}$$

It displays that the entire debt level is certain for one period and half of the debt level is certain for two periods. Inserting this into the adjustment formula derived by Inselbag and Kaufold (1997) see Eq. (3.13), yields the adjustment formula from Eq. (3.26). This alternative derivation of the adjustment formula for the cost of equity highlights that the value of the tax shield compared to passive debt management is again decreased by a factor that depends on V_0^ℓ.

The same holds for the WACC compared to the WACC under passive debt management, see Eq. (3.14). It becomes clearer, if we rearrange Eq. (3.27) to

$$\begin{aligned} k^\tau = \rho^u \cdot \left(1 - \frac{\tau \cdot r \cdot \theta}{1+r} - \frac{1}{2} \cdot \frac{\tau \cdot r \cdot \theta \cdot (1+g)}{(1+r)^2 \cdot \eta^{(2)}}\right) \\ + r \cdot \left(\frac{\tau \cdot r \cdot \theta}{1+r} + \frac{1}{2} \cdot \frac{\tau \cdot r \cdot \theta \cdot (1+g)}{(1+r)^2 \cdot \eta^{(2)}} - \tau \cdot \theta\right) . \end{aligned} \tag{3.29}$$

The value of the risk-free part of the tax shields, see Eq. (3.28) can again be inserted into the formula for the WACC, see Eq. (3.14), to derive the expression from Eq. (3.29).

Note that the levered cost of equity is, generally, a random variable. In each period, the debt-to-market value ratio of the debt category that is not adjusted depends on the development of the firm. An exception displays the debt-to-market value ratio at the valuation date since, in this period, both debt categories are adjusted. However, the expected financial risk is constant for all periods. In every period, half of the tax shield is certain for two periods and the other half is certain for one period. Consequently, the construction of debt categories is similar to that of standard discontinuous financing. The difference is that by partially adjusting the debt level, we obtain a financial risk that does not vary depending on the remaining number of periods in the planning phase. It follows that this financing policy could be more suitable to model the financing behavior and financial risk of a firm. Since we were able to derive a closed-form solution for the levered cost of equity, the market value of the firm for two debt categories can be easily calculated using the Gordon–Shapiro formula (Gordon and Shapiro 1956).

3.3.2 Derivation of a valuation formula for an arbitrary number of debt categories

While the previous analysis was based on a firm with two debt categories, it is now of interest to derive a valuation formula for a firm with various debt categories. Their number can be company specific. First, we consider three debt categories. In some period t, one category is adjusted according to the debt-to-market value ratio $\theta^{(3)} := \frac{1}{3}\theta$, and the amount of debt for the other two debt categories depends on the market value of the firm in periods $t-1$ and $t-2$, respectively. At the valuation date, we again assume that categories 1 and 2 are exceptionally adjusted according to the specified ratio $\theta^{(3)}$.[4] The expected total amount of debt in some period $t \geq 2$ is

$$\mathbb{E}[\widetilde{D}_t] = \theta^{(3)} \cdot \mathbb{E}[\widetilde{V}^\ell_{t-2}] \cdot (1+g)^2 + \theta^{(3)} \cdot \mathbb{E}[\widetilde{V}^\ell_{t-1}] \cdot (1+g) + \theta^{(3)} \cdot \mathbb{E}[\widetilde{V}^\ell_t] . \tag{3.30}$$

[4]In accordance with our analysis in the previous subsection, in a first step, we only consider the steady-state phase and refer to Section 3.3.4 for possible links to the explicit forecast phase.

It follows that, to derive a valuation formula for this setting, the sequence $(\eta_k^{(2)})_{k\in\mathbb{N}}$ needs to be adjusted. By repeating the above backward iteration for three debt categories, we obtain

$$
\begin{aligned}
\eta_1^{(3)} &:= 1 - \frac{\tau \cdot r \cdot \theta^{(3)}}{1+r} \,, \\
\eta_2^{(3)} &:= 1 - \frac{\tau \cdot r \cdot \theta^{(3)}}{1+r} - \frac{\tau \cdot r \cdot \theta^{(3)} \cdot (1+g)}{(1+r)^2 \cdot \eta_1^{(3)}} \quad \text{and} \,, \\
\eta_k^{(3)} &:= 1 - \frac{\tau \cdot r \cdot \theta^{(3)}}{1+r} - \frac{\tau \cdot r \cdot \theta^{(3)} \cdot (1+g)}{(1+r)^2 \cdot \eta_{k-1}^{(3)}} - \frac{\tau \cdot r \cdot \theta^{(3)} \cdot (1+g)^2}{(1+r)^3 \cdot \eta_{k-1} \cdot \eta_{k-2}^{(3)}} \,,
\end{aligned}
\tag{3.31}
$$

for $k > 2$. This sequence considers that one third of the overall debt is certain only in the subsequent period, one part is certain for two periods, and one part is certain for three periods. The latter is displayed in the last term of $\eta_k^{(3)}$. Since deriving an analytic solution for the limit $\eta^{(3)}$ of the sequence $(\eta_k^{(3)})_{k\in\mathbb{N}}$ is difficult, a spreadsheet software or other programs should be used to calculate the limit numerically.

With these observations, we can now deduce a valuation formula for a firm with T debt categories. Let $\theta^{(T)} := \frac{1}{T}\theta$. We exclude a detailed planning of debt levels and adjust all debt levels according to $\theta^{(T)}$ at the valuation date. Thereafter, in period one, category one is adjusted and so on. For an illustration of T debt categories, see Fig. 3.3. It follows that the expected total amount of debt in some period $t \geq T-1$ is

$$
\mathbb{E}[\widetilde{D}_t] = \sum_{s=0}^{T-1} \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_{t-s}^{\ell}] \cdot (1+g)^s \,. \tag{3.32}
$$

In this setting, the total amount of debt in period t depends on the market values of the firm in periods $t-(T-1)$ to t. Accordingly, the sequence $(\eta_k^{(T)})_{k\in\mathbb{N}}$ must be derived. Following the structure for two and three debt categories, respectively, we define $\eta_1^{(T)} := 1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r}$. For $k > 1$, we derive

$$
\eta_k^{(T)} := 1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} - \sum_{t=1}^{\min\{k,T\}-1} \frac{\tau \cdot r \cdot \theta^{(T)} \cdot (1+g)^t}{(1+r)^{t+1} \cdot \prod_{s=1}^{t} \eta_{k-s}^{(T)}} \,. \tag{3.33}
$$

The sequence $(\eta_k^{(T)})_{k\in\mathbb{N}}$ considers the dependencies of the debt levels on the market value of the firm of the previous $k-(T-1)$ periods.

As in the case of two debt categories, we can show that the sequence $(\eta_k^{(T)})_{k\in\mathbb{N}}$ converges to

0	1	$\dots$	$T-1$	T	$\dots$ t
$D_0^0 = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_0^\ell]$	$D_1^0 = D_0^0 \cdot (1+g)$	$\cdots$	$D_{T-1}^0 = D_0^0 \cdot (1+g)^{T-1}$	$\mathbb{E}[\widetilde{D}_T^0] = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_T^\ell]$	$\cdots$
$D_0^1 = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_0^\ell]$	$\mathbb{E}[\widetilde{D}_1^1] = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_1^\ell]$		$\mathbb{E}[\widetilde{D}_{T-1}^1] = \mathbb{E}[\widetilde{D}_1^1] \cdot (1+g)^{T-2}$	$\mathbb{E}[\widetilde{D}_T^1] = \mathbb{E}[\widetilde{D}_1^1] \cdot (1+g)^{T-1}$	
$\vdots$	$\vdots$		$\vdots$	$\vdots$	
$D_0^{T-1} = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_0^\ell]$	$D_1^{T-1} = D_0^{T-1} \cdot (1+g)$		$\mathbb{E}[\widetilde{D}_T^{T-1}] = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_{T-1}^\ell]$	$\mathbb{E}[\widetilde{D}_T^{T-1}] = \mathbb{E}[\widetilde{D}_1^{T-1}] \cdot (1+g)$	

Figure 3.3: Discontinuous financing with T debt categories.

$\eta^{(T)} := \lim_{k\to\infty} \eta_k^{(T)}$ (see Lemma 3). For the limit holds

$$\eta^{(T)} := 1 - \tau \cdot r \cdot \theta^{(T)} \cdot \sum_{t=0}^{T-1} \frac{(1+g)^t}{(1+r)^{t+1} \cdot (\eta^{(T)})^t} \,. \tag{3.34}$$

We cannot derive a closed-form solution for this limit, but it can be computed using a spreadsheet software or other programs.

The general valuation formula for a perpetuity has the same structure as Eq. (3.24), where we assumed a perpetual annuity for two debt categories. If $g < k^*$, we can apply Lemma 2 and derive for an arbitrary number of debt categories

$$V_0^\ell = \frac{E[\widetilde{FCF_1}]}{k^* - g} + \sum_{s=1}^{T-1} \sum_{j=1}^{s} \frac{\tau \cdot r \cdot D_0^{T-j} \cdot (1+g)^{T-s-1}}{(1+r^*)^{T-s}} \,, \tag{3.35}$$

where $k^* := (1+\rho^u) \cdot \eta^{(T)} - 1$ and $r^* := (1+r) \cdot \eta^{(T)} - 1$. The expression D_0^{T-j}, $j \in \{1, \dots, T-1\}$, represents the amount of debt of category $T-j$ that should not be adjusted in period 0, but in period $T-j$. These debt levels have to be exceptionally adjusted simultaneously at the valuation date. They are risk-free and can be discounted at the risk-free interest rate. The value of the tax shield of these debt categories is computed in the second term. In period zero, these are $T-1$ categories; in period one, these are $T-2$ categories; until in period $T-1$, it is only one debt category. Thereafter, starting in period T, every debt category has been adjusted once according to the ratio $\theta^{(T)}$, see Fig. 3.3.

Since we exclude the link to an explicit planning of the debt levels, we adjust all debt categories according to the market value of the firm at the valuation date, that is, $D_0^j = \theta^{(T)} \cdot V_0^\ell$ for $j \in \{0, \dots, T-1\}$. Plugging this in, we can simplify valuation Eq. (3.35) as we do in the following proposition.

Proposition 4. *If $g < k^*$, the market value of the firm for T debt categories can be computed by*

$$V_0^\ell = \frac{\mathbb{E}[\widetilde{FCF_1}]}{k^* - g} \cdot \left(1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} \cdot \frac{(1+x)^t - Tx - 1}{x^2}\right)^{-1} , \tag{3.36}$$

where

$$x = \frac{1+g}{(1+r) \cdot \eta^{(T)}} - 1 \, . \tag{3.37}$$

As in the case of two debt categories, the FCFs are discounted at the adjusted cost of capital k^*. Compared to Eq. (3.35), we inserted the relation $D_0^j = \theta^{(T)} \cdot V_0^\ell$ and solved the circularity problems to deduce a circularity-free valuation formula. Multiplying the market value of the firm by the debt-to-market value ratio yields the equity value. For detailed calculations, see the Proof of Proposition 4 in the Appendix.

To be able to apply the FCF approach in conjunction with the Gordon–Shapiro formula, see Eq. (3.25), we need to determine the levered cost of equity and the WACC. The formulas are captured in the following proposition.

Proposition 5. *Under the assumption of T debt categories, the levered cost of equity can be obtained by*

$$\rho^\ell = \rho^u + (\rho^u - r) \cdot \left(1 - \tau \cdot r \cdot \sum_{s=0}^{T-1} \left(1 - \frac{s}{T}\right) \cdot \frac{(1+g)^s}{(1+r)^{s+1} \cdot (\eta^{(T)})^s}\right) \cdot L \tag{3.38}$$

$$= \rho^u + (\rho^u - r) \cdot \left(1 - \frac{\tau \cdot r}{1+r} \cdot \frac{(1+x)^{T+1} - (T+1) \cdot x - 1}{T \cdot x^2}\right) \cdot L \, . \tag{3.39}$$

Furthermore, the WACC is given by

$$k^\tau = \rho^u - (\rho^u - r) \cdot \frac{\tau \cdot r}{1+r} \cdot \frac{(1+x)^{T+1} - (T+1) \cdot x - 1}{T \cdot x^2} \cdot \theta - r \cdot \tau \cdot \theta \, . \tag{3.40}$$

See the Appendix for a proof. While Eq. (3.39) can be used for computation, we can obtain a better interpretation from Eq. (3.38): We can again divide the derivation of the levered cost of equity into the operational risk ρ^u and a risk premium that depends on the difference of the unlevered cost of equity and the risk-free interest rate. The factor that incorporates the financial risk due to all debt categories can be interpreted as follows. Compared to active debt management according to HP, we have a reduced risk. The first term of the sum in Eq. (3.39) is

$\frac{1}{1+r}$ and reflects that the entire debt is certain in the subsequent period. The second term is $\left(1-\frac{1}{T}\right)\cdot\frac{1+g}{(1+r)^2\cdot\eta^{(T)}}$ and represents that the entire debt, except the part that is adjusted in the next period (i.e., a share of $1-\frac{1}{T}$ of the overall debt), is certain for two periods. This continues until, in the last term, only $\frac{1}{T}$ of the overall debt is considered, which is the debt category that is defined in the current period, and is, therefore, certain for T periods.

To compare these equations to passive debt management, we compute the risk-free part of the value of the tax shield. It is

$$\begin{aligned} VTS_0^R &= \tau\cdot r\cdot\theta\cdot V_0^\ell\cdot\sum_{s=0}^{T-1}\left(1-\frac{s}{T}\right)\cdot\frac{(1+g)^s}{(1+r)^{s+1}\cdot(\eta^{(T)})^s} \\ &= \tau\cdot r\cdot\theta\cdot V_0^\ell\cdot\frac{(1+x)^{T+1}-(T+1)\cdot x-1}{T\cdot x^2\cdot(1+r)}\ . \end{aligned} \tag{3.41}$$

Compared to passive debt management, the risk-free part of the tax shield is reduced. As explained above, only parts of the overall debt level are deterministic, which is expressed in the sum. Inserting Eq. (3.41) into Eq. (3.13) and (3.14) is an alternative way to derive the adjustment formula for the cost of equity and the WACC, respectively.

With these findings, it becomes clear that the expected financial risk is constant. In each period, a proportion of $\frac{1}{T}$ of the overall debt is certain for T periods, another proportion is certain for $T-1$ periods, and so on. It follows that it is possible to use Eq. (3.25) to calculate the market value of the firm. Therefore, we have determined a formula for the levered cost of equity and the WACC, which is easy to apply to calculate the market value of a firm with an arbitrary number of debt levels with the Gordon–Shapiro formula. Note that, for $T=2$, the formula simplifies to the above derived formulas for two debt categories (see Proposition 3). Furthermore, $T=1$ complies with active debt management according to ME since the firm has only one debt category that is adjusted every period.

Compared to standard discontinuous financing, we considered the maturity of debt and constructed debt categories in a way that yields a partial adjustment of the debt level. Thus, discontinuous financing with T debt categories might be more suitable to model the financing behavior of a firm. For more than two debt categories, the limit η has to be computed numerically. However, with the help of a spreadsheet software or other programs, this does not pose a problem. Overall, the results allow for a deep theoretical understanding not only of this new financing strategy but also of standard discontinuous financing. The following subsection illustrates the approach of discontinuous financing with debt categories using an example.

3.3.3 Illustration and comparison with other financing policies

We use the above example, see Section 3.2.3, to illustrate discontinuous financing with debt categories and to compare it with other financing strategies. We consider a firm with five debt categories and the same input parameters as above. To compute the market value of the firm under this assumption of five debt categories, we use the definition of $\eta^{(T)}$ from Eq. (3.34) and numerically obtain $\eta^{(5)} \approx 0.993$. Furthermore, we assume that all debt categories D_0^j, $j \in \{1, \ldots, 4\}$, are adjusted at the valuation date by multiplying the debt ratio $\theta^{(5)}$ by the market value of the firm, that is, $D_0^j = \theta^{(T)} \cdot V_0^\ell = 12\% \cdot V_0^\ell$. Therefore, we can apply the FCF approach and compute the WACC according to Eq. (3.40). To do so, we calculate $x = -0.017$, see Eq. (3.37), and obtain for the WACC

$$\begin{aligned} k^\tau &= 0.1 - (0.1 - 0.04) \cdot \frac{0.3 \cdot 0.04}{1 + 0.04} \cdot \frac{(1 - 0.017)^6 - 6 \cdot 0.017 - 1}{5 \cdot 0.017^2} \cdot 0.6 - 0.04 \cdot 0.3 \cdot 0.6 \\ &= 9.16\% \, . \end{aligned} \tag{3.42}$$

For the value of the levered firm, we can use the Gordon–Shapiro formula, see Eq. (3.25), and obtain

$$V_0^{\ell,\mathrm{DC}} = \frac{1{,}000}{9.16\% - 1.5\%} = 13{,}057.81 \, , \tag{3.43}$$

with a total amount of debt of

$$D_0^{\mathrm{DC}} = \theta \cdot V_0^{\ell,\mathrm{DC}} = 60\% \cdot 13{,}057.81 = 7{,}834.69 \, ,$$

where DC denotes the assumption of debt categories. For the equity value follows

$$E_0^{\ell,\mathrm{DC}} = V_0^{\ell,\mathrm{DC}} \cdot (1 - \theta) = 13{,}057.81 \cdot 40\% = 5{,}223.13 \, .$$

Since the firm is in a steady state, the expected market value of the firm, equity value, and debt levels grow at the constant growth rate g (see Tables 3.3, 3.4, and 3.5, respectively). Moreover, we can derive the levered cost of equity according to Eq. (3.39), which amounts to $\rho^\ell = 18.70\%$ and is constant in every period (see Table 3.6).

The cost of equity can alternatively be derived by computing the value of the risk-free part of the tax shields according to Eq. (3.41) and applying Eq. (3.13). We included the value of the risk-free part of the tax shields in Table 3.7.

Financing strategy	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Unlevered firm	11,764.71	11,941.18	12,120.29	12,302.10	12,486.63	12,673.93
Debt categories	13,057.81	13,253.68	13,452.49	13,654.27	13,859.09	14,066.97
Discontinuous financing	13,066.70	13,253.43	13,447.02	13,648.20	13,857.75	14,076.55
Active debt management	12,922.47	13,166.30	13,313.05	13,512.74	13,715.43	13,921.17
Passive debt management	16,523.46	16,771.32	17,022.88	17,278.23	17,537.40	17,800.46

Table 3.3: Market values of the firm under different financing strategies.

Financing strategy	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Unlevered firm	11,764.71	11,941.18	12,120.29	12,302.10	12,486.63	12,673.93
Debt categories	5,223.13	5,301.47	5,380.99	5,461.71	5,543.64	5,626.79
Discontinuous financing	5,226.68	5,295.81	5,370.04	5,450.06	5,536.64	5,630.62
Active debt management	5,168.99	5,246.52	5,325.22	5,405.10	5,486.17	5,568.47
Passive debt management	6,609.39	6,708.53	6,809.15	6,911.29	7,014.96	7,120.19

Table 3.4: Equity values under different financing strategies.

Financing strategy	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Unlevered firm	0.00	0.00	0.00	0.00	0.00	0.00
Debt categories	7,834.69	7,952.21	8,071.49	8,192.56	8,315.45	8,440.18
Discontinuous financing	7,840.02	7,957.62	8,076.99	8,198.14	8,321.11	8,445.93
Active debt management	7,753.48	7,869.78	7,987.83	8,107.65	8,229.26	8,352.70
Passive debt management	9,914.08	10,062.79	10,213.73	10,366.94	10,522.44	10,680.28

Table 3.5: Debt levels under different financing strategies.

Financing strategy	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Unlevered firm	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Debt categories	18.70%	18.70%	18.70%	18.70%	18.70%	18.70%
Discontinuous financing	18.51%	18.61%	18.72%	18.82%	18.91%	18.51%
Active debt management	18.90%	18.90%	18.90%	18.90%	18.90%	18.90%
Passive debt management	14.68%	14.68%	14.68%	14.68%	14.68%	14.68%

Table 3.6: Levered costs of equity under different financing strategies.

Financing strategy	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Unlevered firm	0.00	0.00	0.00	0.00	0.00	0.00
Debt categories	264.97	268.94	272.97	277.07	281.23	285.44
Discontinuous financing	431.08	354.24	272.92	186.91	96.01	464.40
Active debt managements	89.46	90.81	92.17	93.55	94.95	96.38
Passive debt managements	4,758.76	4,830.14	4,902.59	4,976.13	5,050.77	5,126.53

Table 3.7: Value of the risk-free part of the tax shield under different financing strategies.

In the following, we compare the equity value in the case of discontinuous financing with debt categories to that under the assumption of other financing strategies. In Section 3.2.3, we have already calculated the equity value in the case of standard discontinuous financing and obtained

$$E_0^{\ell,\mathrm{DF}} = 5{,}226.68\ ,$$

where DF denotes discontinuous financing. This value is slightly higher than the market value of the firm in the case of debt categories. We derive a negligible deviation of

$$\frac{E_0^{\ell,\mathrm{DF}} - E_0^{\ell,\mathrm{DC}}}{E_0^{\ell,\mathrm{DC}}} = \frac{5{,}226.68 - 5{,}223.13}{5{,}223.13} = 0.07\%\ .$$

The deviation occurs because, under standard discontinuous financing, at the valuation date, the tax shield is certain for five periods. Under debt categories, at the valuation date, only one fifth of the tax shield is certain for five periods. However, under discontinuous financing, in periods $1, 2, 3$, and 4, the tax shield is only certain for $4, 3, 2$, and 1 periods, respectively, which yields a value advantage of debt categories (see Table 3.3). This characteristic can also be observed for the costs of equity. Due to the lower financial risk, the cost of equity under standard discontinuous financing is lower than the cost of equity under debt categories at the valuation date, but this relation changes in periods 2, 3, and 4. Looking at a complete planning phase, the differences almost cancel each other out.

To compare the equity value under debt categories with that under the assumption of pure financing strategies, we first use the FCF approach to calculate the market value of the firm in the case of active debt management according to ME, which yields a levered cost of equity of

$\rho^{\ell} = 18.90\%$. We deduce $k^{\tau} = 9.24\%$ and calculate thereby

$$E_0^{\ell,\text{ADM}} = \frac{\mathbb{E}[\widetilde{FCF_1}]}{k^{\tau} - g} \cdot (1-\theta) = \frac{1{,}000}{9.24\% - 1.5\%} \cdot 40\% = 5.168{,}99\ , \tag{3.44}$$

where ADM denotes active debt management. The equity value is lower than in the case of discontinuous financing and debt categories, since the tax shields are only certain in the period of their occurrence and uncertain in all preceding periods. This can also be observed in the higher levered cost of equity, which implies that financial risk is higher. We deduce a deviation of the equity value of 1.05% and 1.12% for debt categories and standard discontinuous financing, respectively. For a higher T, the differences will become larger. In period 4, the levered cost of equity under standard discontinuous financing is nearly the same as under active debt management. In this period, the tax shield under discontinuous financing is also only certain for one period. However, the value remains slightly higher for discontinuous financing since the expected debt-to-market value ratio is 60.05% (see Table 3.2), which is higher than the debt-to-market value ratio of 60% under active debt management.

For passive debt management we also assume a debt-to-market value ratio of 60%, which yields a WACC of 7.55%. For the equity value, we obtain

$$E_0^{\ell,\text{PDM}} = \frac{E[\widetilde{FCF_1}]}{k^{\tau} - g} \cdot (1-\theta) = \frac{1{,}000}{7.55\% - 1.5\%} \cdot 40\% = 6{,}609.39\ , \tag{3.45}$$

where PDM denotes passive debt management. Unsurprisingly, the equity value under passive debt management is considerably higher than the other equity values. We derive deviations to discontinuous financing and debt categories of more than 25% because, for passive debt management, all tax shields are certain and can be discounted at the risk-free interest rate r, rather than the unlevered cost of equity ρ^u. Hence, the levered cost of equity is also lower. For a higher T, the deviations decrease since the planning phases, and, therefore, the maturity of debt becomes longer.

For every financing strategy, a comparison of Table 3.7 and 3.8 illustrates that for active debt management, discontinuous financing, and debt categories, only a small portion of the value of the tax shield comes from risk-free debt. For passive debt management, all future debt levels are risk-free such that the value of the risk-free part of the tax shields coincides with the value of the tax shield.

Financing strategy	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Unlevered firm	0.00	0.00	0.00	0.00	0.00	0.00
Debt categories	1,293.11	1,312.51	1,332.19	1,352.18	1,372.46	1,393.04
Discontinuous financing	1,302.00	1,312.25	1,326.73	1,346.10	1,371.12	1,402.62
Active debt managements	1,157.76	1,175.13	1,192.75	1,210.64	1,228.80	1,247.24
Passive debt managements	4,758.76	4,830.14	4,902.59	4,976.13	5,050.77	5,126.53

Table 3.8: Value of the tax shield under different financing strategies.

From period 5 onward, the structure is repeated. For discontinuous financing, the next planning phase starts, which implies a debt-to-market value ratio of 60% in period 5. For every other financing strategy, the equity value, market value of the firm, debt levels, value of the tax shield, and value of the risk-free part of the tax shields continue to grow at the growth rate g.

In this example, we always defined the debt-to-market value ratio deterministically since we excluded an explicit planning of debt levels. Thereby, we followed the line of the example in Clubb and Doran (1995, pp. 690–692). One could also define coinciding debt levels for every financing strategy and include an explicit forecast phase. Consequently, the debt-to-market value ratios would vary, but interpretations would be similar.

We conclude that the financing behavior of a firm should be carefully analyzed for choosing the most suitable financing strategy. The terminal value calculation with standard discontinuous financing shows shortcomings that can be rectified by debt categories. However, in the presented example, the value differences between standard discontinuous financing and discontinuous financing with debt categories are negligible. For other input parameters, similar results can be expected. An advantage of standard discontinuous financing is that the market value of the firm and, therefore, the equity value at the valuation date can be calculated without circularity problems. For debt categories, this is only possible for the special case of two debt categories. For more debt categories, the limit η has to be computed numerically. However, to calculate market values within a planning phase, the application of standard discontinuous financing also involves circularity problems. Since a spreadsheet software is usually used for valuation, the application of both financing strategies can be conducted easily. Overall, the assumption of debt categories seems more realistic since the shares of the overall debt are adjusted successively, instead of all at once every T periods. If standard discontinuous financing is used, it can be interpreted as an approximation of debt categories.

3.3.4 Application of debt categories in a two-phase model

In the previous subsections, we excluded an explicit forecast phase and specified the entire debt level according to the debt-to-market value ratio at the valuation date. In this section, we analyze the possibilities of a link to a detailed planning of debt levels. This explicit planning is typically based on deterministic debt levels such that we assume passive debt management in the explicit forecast phase. Thereafter, we mix active and passive debt management by applying discontinuous financing with T debt categories in the steady-state phase. We assume that the explicit forecast phase consists of S periods. As for every other combination of financing strategies in a two-phase model, it can be distinguished between an abrupt and a successive transition from the explicit forecast phase to the steady state (Koller et al. 2020, pp. 259–260).

In case of an abrupt change of financing strategies, all debt categories are adjusted according to $\theta^{(T)}$ at the beginning of the steady-state phase; that is, the debt level of period S is computed as $\widetilde{D}_S = \theta \cdot \widetilde{V}^{\ell}_S$. Thus, at the beginning of the steady-state phase, exceptionally, all debt categories are adjusted in the same period according to $\theta^{(T)}$, see Fig. 3.4. This approach does not directly consider the debt level D_{S-1} that results from the last period of the explicit forecast phase, but yields a refinancing. However, the debt ratio and the associated refinancing is planned together with the explicit debt levels and, therefore, does not constitute a problem. To calculate the market value of the firm at the beginning of the steady-state phase, the adjustment formulas from Proposition 5 can be used. This approach of an abrupt change of financing strategies is very similar to the assumption of a steady state with active debt management, which is a common approach in corporate valuation practice. When active debt management is used for a steady state after an explicit forecast phase with passive debt management, the firm has to refinance according to the specified debt ratio (see also studies on hybrid financing, e.g., Kruschwitz et al. 2007; Dierkes and Gröger 2010; Dierkes and de Maeyer 2020).

It is also possible to adjust the debt-to-market value ratio θ such that the expected debt level at the end of the explicit forecast phase, $\mathbb{E}[\widetilde{D}_S] = \theta \cdot \mathbb{E}[\widetilde{V}^{\ell}_S]$, coincides with an explicitly planned debt level D_S. This can, for example, be conducted by using a spreadsheet software. The debt levels are then adjusted exactly as in the first approach: In the first period of the steady state, all debt categories are adjusted according to the specified debt ratio. Whether D_S or $\mathbb{E}[\widetilde{D}_S]$ is used has no effect on the market value of the firm at the beginning of the steady-state phase (for a more detailed analysis, see Dierkes and de Maeyer 2020) such that it can again be computed with the help of Proposition 5.

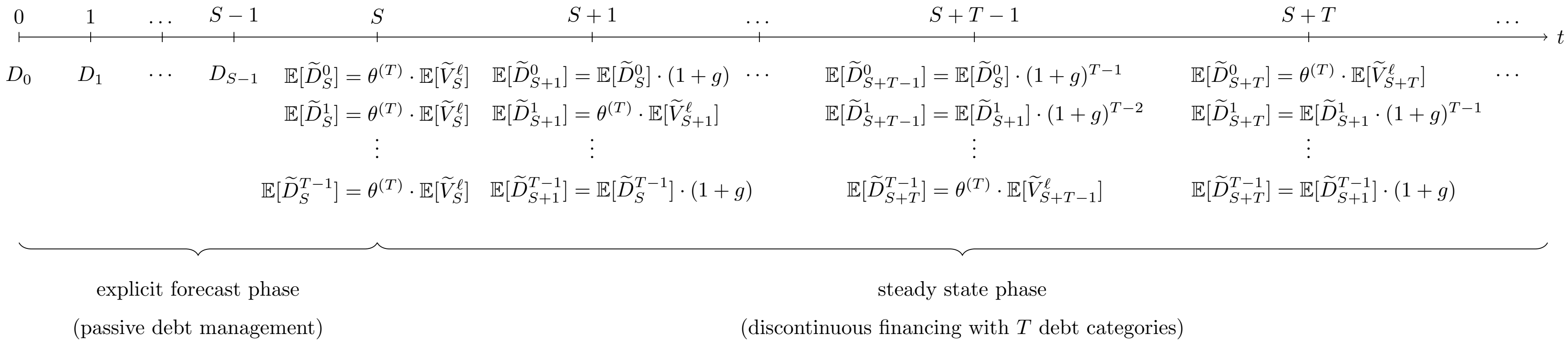

Figure 3.4: Two-phase model with an abrupt change of financing strategies.

Alternatively, a successive transition from passive debt management to debt categories can be assumed by including a convergence phase. To apply such an approach, the debt level D_S^* of period S must be explicitly planned. However, the resulting debt level $\widetilde{D}_S$ of this period S does not coincide with D_S^* since debt category 0 is adjusted according to the debt-to-market value ratio. All other debt categories are defined as a fraction of $\frac{1}{T}$ of the fixed level D_S^*, see Fig. 3.5. In period $S+1$, category 1 is adjusted according to the defined debt-to-market value ratio $\theta^{(T)}$. All other categories grow at the specified growth rate. In the second period after the end of the explicit planning, category 2 is adjusted according to this ratio and so on. In period $S+T-1$, all categories have been adjusted once according to $\theta^{(T)}$ such that this corresponds to the first period of the steady-state phase. It follows that the convergence phase consists of T periods, after which the steady state with debt categories begins. To calculate the market value of the firm $\widetilde{V}_S^{\ell}$ at the of the explicit forecast phase, Eq. (3.35) can be used. For the debt levels D_S^{T-j}, the debt levels that result from an explicit planning have to be inserted. The example from Section 3.3.3 can be adjusted accordingly.

3.3.5 Risk of default for discontinuous financing and debt categories

In the above analysis, we concentrate on the derivation of the valuation equations and adjustment formulas, as well as their consequences for a steady state. We abstained from the integration of the risk of default to keep this focus. This is a common procedure when analyzing new financing strategies. However, the risk of default and the potential losses in value due to costs of financial distress should be taken into account (see, e.g., Almeida and Philippon 2007; Korteweg 2007; Lahmann et al. 2018). These costs consist of, for example, legal fees, costs due to customer losses and qualified employees leaving the firm in a crisis (Korteweg 2007, footnote 3; Lahmann et al. 2018, pp. 80–81). The consideration of the risk of default in corporate valuation has already been extensively analyzed, but is still intensively discussed (see, e.g., Sick 1990; Kruschwitz et al. 2005; Friedrich 2016; Lahmann et al. 2018). The most pragmatic solution, which is often applied, is to use a risk-adjusted cost of capital, rather than the risk-free interest rate for calculating the tax shields and discounting the risk-free part of the tax shields. This was, for example, performed in the analysis of active and passive debt management in Inselbag and Kaufold (1997), and the implementation of discontinuous financing of Clubb and Doran (1995) and Arnold et al. (2018). Accordingly, our analysis can be easily adapted by inserting the cost of debt r_D for the risk-free interest rate r. The discount rate r_D depends on assumptions regarding the tax treatment (for different possibilities of considering taxes in the case of default and the cost of debt, see, e.g., Sick 1990; Kruschwitz et al. 2005; Rapp 2006; Krause and Lahmann 2016; Baule 2019).

0	1	$\dots$	$S-1$	S	$S+1$	$\dots$	$S+T-1$	$S+T$	$\dots$ t
D_0	D_1	$\cdots$	D_{S-1}	$\mathbb{E}[\widetilde{D}_S^0] = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_S^\ell]$	$\mathbb{E}[\widetilde{D}_{S+1}^0] = \mathbb{E}[\widetilde{D}_S^0] \cdot (1+g)$	$\cdots$	$\mathbb{E}[\widetilde{D}_{S+T-1}^0] = \mathbb{E}[\widetilde{D}_S^0] \cdot (1+g)^{T-1}$	$\mathbb{E}[\widetilde{D}_{S+T}^0] = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_{S+T}^\ell]$	$\cdots$
				$D_S^1 = \frac{1}{T} \cdot D_S^*$	$\mathbb{E}[\widetilde{D}_{S+1}^1] = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_{S+1}^\ell]$		$\mathbb{E}[\widetilde{D}_{S+T-1}^1] = \mathbb{E}[\widetilde{D}_{S+1}^1] \cdot (1+g)^{T-2}$	$\mathbb{E}[\widetilde{D}_{S+T}^1] = \mathbb{E}[\widetilde{D}_{S+1}^1] \cdot (1+g)^{T-1}$	
				$\vdots$	$\vdots$		$\vdots$	$\vdots$	
				$D_S^{T-1} = \frac{1}{T} \cdot D_S^*$	$D_{S+1}^{T-1} = D_S^{T-1} \cdot (1+g)$		$\mathbb{E}[\widetilde{D}_{S+T-1}^{T-1}] = \theta^{(T)} \cdot \mathbb{E}[\widetilde{V}_{S+T-1}^\ell]$	$\mathbb{E}[\widetilde{D}_{S+T}^{T-1}] = \mathbb{E}[\widetilde{D}_{S+T-1}^{T-1}] \cdot (1+g)$	

explicit forecast phase (passive debt management) — convergence phase — steady state phase (discontinuous financing with T debt categories)

Figure 3.5: Two-phase model with a successive change of financing strategies.

For a more explicit analysis of the risk of default in the case of discontinuous financing, we refer to Lahmann et al. (2018) and Arnold et al. (2019). They analyze the connection between the probability of default and the length of the planning phases T (Arnold et al. 2019, pp. 356–358). In particular, they consider a continuous time model with a geometric Brownian motion with drift-rate μ and volatility σ for the changes in the value of the unlevered firm and analyze the probability of default at refinancing dates. Lahmann et al. (2018) distinguish between an endogenous and an exogenous insolvency trigger (Lahmann et al. 2018, pp.87–88). They outline, for example, that the longer the planning phases (the higher T) and the higher the debt-to-market value ratio, the higher the probability of default (Lahmann et al. 2018, pp.109–110). Furthermore, Arnold et al. (2019) discuss other possibilities of considering the risk of default in the case of discontinuous financing. Overall, they develop a framework that displays well the consequences of the integration of the risk of default for discontinuous financing.

The argumentation of Lahmann et al. (2018) and Arnold et al. (2019) can be transferred to debt categories. Since only parts of the overall debt level are adjusted according to a debt-to-market value ratio, the geometric Brownian Motion may drive down the value of the unlevered firm $\widetilde{V}_t^u$ at some period t so low that it is less than the debt level $\widetilde{D}_t$ in this period. The corresponding probability might be very low, but this should trigger a default. Furthermore, the fact that the probability of default increases for a higher T and a higher debt-to-market value ratio, as is the case with standard discontinuous financing, is also true for discontinuous financing with debt categories.

These fundamental considerations form an overview of the integration of the risk of default in the case of debt categories. However, analyses on how exactly the risk of default can be integrated into the valuation equations are beyond the scope of this study. We laid the groundwork for an investigation of financing with debt categories by obtaining valuation equations under the assumption of risk-free debt and leave additional analyses for further research. For the application of this financing strategy in corporate valuation practice, we recommend the common approach to use of the cost of debt, instead of the risk-free interest rate, which can be easily implemented. After all, the sound integration of the risk of default is not a specific problem of discontinuous financing with debt categories but applies to all financing strategies.

3.4 Conclusions

The choice of a financing strategy is a central issue for terminal value calculation. Since the terminal value accounts for a large part of the equity value, the financing strategy should accurately reflect the real financing behavior of a firm. We addressed this problem and introduced debt categories as a suitable financing strategy in a steady state. Under this assumption, different layers of debt were adjusted successively. We obtained valuation equations and an adjustment formula for the cost of equity.

The foundation for this new financing strategy is standard discontinuous financing. As a mix of active and passive debt management, it provides the opportunity to depict a broad range of firm financing strategies with a slow adjustment of debt levels toward a fixed debt ratio. In this study, we clarified the differences between the approaches of discontinuous financing of Clubb and Doran (1995), Arnold et al. (2018, 2019), and Dierkes and de Maeyer (2020). We followed the approach of a perpetuity with growth of Arnold et al. (2019) and Dierkes and de Maeyer (2020), and showed that it results in an inconstant financial risk and, thus, an inconstant levered cost of equity. Moreover, we derived an adjustment formula for the period-specific levered cost of equity. The adjustment formula has a similar form to those for active and passive debt management. The knowledge of the adjustment formula offers the possibility to unlever and relever beta factors, as well as to calculate the market value of the firm using the FCF or FtE approach with period-specific costs of capital. Since the sole adjustment of the entire debt level after a planning phase and the associated consequences for financial risk might still not be close to the real financing behavior of firms, we introduced debt categories.

For discontinuous financing with debt categories, we assumed that a specified share of the overall debt is adapted in every period. The resulting debt categories were successively adjusted while considering the maturity of debt. First, we derived a valuation formula for two debt categories and an adjustment formula for the levered cost of equity. We showed that discontinuous financing with debt categories results in a constant expected financial risk and a constant levered cost of equity. Second, we extended the approach to an arbitrary number of debt categories. Independent of the number of debt categories, we obtain a financing policy for the steady state with the property of constant financial risk. Consequently, the Gordon-Shapiro formula can be applied.

Additionally, we presented an example to illustrate the theoretical findings and analyzed the value effects of the different financing strategies. When comparing standard discontinuous financing to active debt management, we obtained a small deviation. The difference is much larger when standard discontinuous financing is compared to passive debt management. The same results hold for discontinuous financing with debt categories. Moreover, we found that the deviation between the market value of the firm under standard discontinuous financing and discontinuous financing with debt categories is small. Consequently, despite the advantage of discontinuous financing with debt categories of depicting a broader range of real financing strategies of firms, valuation with standard discontinuous financing can still be applied. The latter can be interpreted as an approximation for the assumption of discontinuous financing with debt categories.

Our analysis focused on the consequences for a steady state. In particular, in the main part, we excluded an explicit forecast phase. Thereafter, we discussed possibilities of linking the assumption of a steady state with debt categories to a detailed planning of debt levels. The derived valuation equations can be easily adjusted to one of these models. This enables the application of debt categories in a two-phase model. Furthermore, we excluded the risk of default and the costs of financial distress in our basic analysis in order to derive the valuation equations und adjustment formulas on a clear theoretical basis for our new financing policy, as it is common in comparable analyses for other financing policies. Nevertheless, it is important to analyze the additional incorporation of risk of default, such that we discussed limitations and possible solutions of this assumption afterwards. We pointed out that the application of the cost of debt can be easily implemented and laid out fundamental characteristics of an explicit analysis, but left a more detailed analysis to further research. Overall, by introducing discontinuous financing with debt categories, we presented a new possibility to depict the financing behavior of firms in a steady state and contribute to the ongoing discussion on terminal value calculation.

A Appendix

Proof of Proposition 1. First, we deduce the formula for the WACC. To do so, we introduce additional notation to simplify the calculations. Let

$$G = 1 + g, \qquad R = 1 + r, \qquad K = 1 + \rho^u .$$

Since the function PVA does always depend on g in our setting, we denote it as a function of C and s, where $C \in \{R, K\}$ and $s \in \mathbb{N}$. It is

$$f(C, s) = \mathrm{PVA}(C - 1, G - 1, s) = \frac{1}{C - G} \cdot \left(1 - \frac{G^s}{C^s}\right) .$$

We use this new defined notation to rewrite the definition of the cost of capital, see Eq. (3.10), and the claim

$$\begin{aligned} 1 + k_t^\tau :=& \frac{\mathbb{E}[\widetilde{FCF}_1] \cdot G^t + \mathbb{E}[\widetilde{V}_{t+1}^\ell]}{\mathbb{E}[\widetilde{V}_t^\ell]} \\ =& (1 - \tau \cdot r \cdot \theta_t \cdot f(R, T - t)) \cdot K - \tau \cdot r \cdot \theta_t \cdot (1 - R \cdot f(R, T - t)) . \end{aligned} \tag{3.46}$$

Furthermore, we can rewrite the computation of the market value of the firm at time t, see Eq. (3.5), as

$$\mathbb{E}[\widetilde{V}_t^\ell] = \mathbb{E}[\widetilde{FCF}_1] \cdot G^t \cdot f(K, T - t) + \tau \cdot r \cdot \theta \cdot V_0^\ell \cdot G^t \cdot f(R, T - t) + V_0^\ell \cdot \frac{G^T}{K^{T-t}} . \tag{3.47}$$

To prove the claim, we note that

$$f(C, s - 1) = f(C, s) - \frac{G^{s-1}}{C^s} \tag{3.48}$$

and

$$\left(1 - \frac{G^s}{C^s}\right) = f(C, s) \cdot (C - G) . \tag{3.49}$$

By using these relations, we can rearrange the numerator of Eq. (3.46) to obtain

$$\begin{aligned}
\mathbb{E}[\widetilde{FCF}_1] \cdot G^t + \mathbb{E}[\widetilde{V}^\ell_{t+1}] &\overset{(3.47)}{=} \mathbb{E}[\widetilde{FCF}_1] \cdot G^t \cdot (1 + G \cdot f(K, T-t-1)) \\
&\quad + \tau \cdot r \cdot \theta \cdot V^\ell_0 \cdot G^t \cdot G \cdot f(R, T-t-1) \\
&\quad + K \cdot V^\ell_0 \cdot \frac{G^T}{K^{T-t}} \\
&\overset{(3.48)}{=} \mathbb{E}[\widetilde{FCF}_1] \cdot G^t \cdot \left(1 + G \cdot f(K, T-t) - \frac{G^{T-t}}{K^{T-t}}\right) \\
&\quad + \tau \cdot r \cdot \theta \cdot V^\ell_0 \cdot G^t \cdot \left(G \cdot f(R, T-t) - \frac{G^{T-t}}{R^{T-t}}\right) \\
&\quad + K \cdot V^\ell_0 \cdot \frac{G^T}{K^{T-t}} \\
&\overset{(3.49)}{=} \mathbb{E}[\widetilde{FCF}_1] \cdot G^t \cdot K \cdot f(K, T-t) \\
&\quad + \tau \cdot r \cdot \theta \cdot V^\ell_0 \cdot G^t \cdot (R \cdot f(R, T-t) - 1) \\
&\quad + K \cdot V^\ell_0 \cdot \frac{G^T}{K^{T-t}} \ .
\end{aligned}$$

We add and subtract $K \cdot \tau \cdot r \cdot \theta \cdot V^\ell_0 \cdot G^t \cdot f(R, T-t)$ and use again Eq. (3.47), which yields

$$\mathbb{E}[\widetilde{FCF}_1] \cdot G^t + \mathbb{E}[\widetilde{V}^\ell_{t+1}] = K \cdot \mathbb{E}[\widetilde{V}^\ell_t] + \tau \cdot r \cdot \theta \cdot V^\ell_0 \cdot G^t \cdot ((R-K) \cdot f(R, T-t) - 1) \ . \tag{3.50}$$

Since the debt-to-market value ratio is inconstant, we need to calculate the debt-to-market value ratio of period t. It is

$$\frac{\theta \cdot V^\ell_0 \cdot G^t}{\mathbb{E}[\widetilde{V}^\ell_t]} = \frac{D_0 \cdot G^t}{\mathbb{E}[\widetilde{V}^\ell_t]} = \frac{D_t}{\mathbb{E}[\widetilde{V}^\ell_t]} = \theta_t \ .$$

We can use this relation and the modified expression of the numerator, see Eq. (3.50), to compute

$$\frac{\mathbb{E}[\widetilde{FCF}_1] \cdot G^t + \mathbb{E}[\widetilde{V}^\ell_{t+1}]}{\mathbb{E}[\widetilde{V}^\ell_t]} = K + \tau \cdot r \cdot \theta_t \cdot ((R-K) \cdot f(R, T-t) - 1) \ .$$

Rearranging and inserting the definition of k^τ_t, see Eq. (3.46), yields

$$1 + k^\tau_t = K \cdot (1 - \tau \cdot r \cdot \theta_t \cdot f(R, T-t)) - \tau \cdot r \cdot \theta_t \cdot (1 - R \cdot f(R, T-t)) \ .$$

By inserting the original notation, we obtain

$$
\begin{aligned}
1 + k_t^\tau = \rho^u \cdot (1 - \tau \cdot r \cdot \theta_t \cdot \mathrm{PVA}(r, g, T - t)) - \tau \cdot r \cdot \theta_t \cdot (1 - r \cdot \mathrm{PVA}(r, g, T - t)) \\
+ 1 - \tau \cdot r \cdot \theta_t \cdot \mathrm{PVA}(r, g, T - t) + \tau \cdot r \cdot \theta_t \cdot \cdot \mathrm{PVA}(r, g, T - t) \ ,
\end{aligned}
$$

which yields Eq. (3.9).

It remains to deduce the formula for the cost of equity. The computation is straightforward. By using the definition of the WACC, $k_t^\tau = \rho^\ell \cdot (1 - \theta_t) + r \cdot (1 - \tau) \cdot \theta_t$, and equating it to Eq. (3.9), we obtain

$$
\begin{aligned}
\rho^\ell \cdot (1 - \theta_t) &= k_t^\tau - r \cdot (1 - \tau) \cdot \theta_t \\
&= \rho^u - (\rho^u - r) \cdot \tau \cdot r \cdot \theta_t \cdot \mathrm{PVA}(r, g, T - t) - \tau \cdot r \cdot \theta_t - r \cdot (1 - \tau) \cdot \theta_t \\
&= \rho^u \cdot (1 - \theta_t) + (\rho^u - r) \cdot (1 - \tau \cdot r \cdot \mathrm{PVA}(r, g, T - t)) \cdot \theta_t \ .
\end{aligned}
$$

Dividing both sides by $(1 - \theta_t)$ and defining $L_t := \frac{\theta_t}{1-\theta_t}$ yields the levered cost of equity. □

Proof of Proposition 2. From the results in Section 3.3.1, we deduce

$$
V_0^\ell = \lim_{T \to \infty} \sum_{t=1}^{T} \frac{E[\widetilde{FCF}_t]}{\prod_{s=1}^{t} (1 + \rho^u) \cdot \eta_{T-s+1}^{(2)}} + \frac{\tau \cdot r \cdot D_0^1}{(1 + r) \cdot \eta_T^{(2)}} \ . \tag{3.51}
$$

To show that the first sum converges, we want to apply Lemma 2. To do so, we define

$$
a_k := \eta_k \cdot \frac{1 + \rho^u}{1 + g} \ .
$$

For $k \to \infty$, follows $a_k \searrow a$ with

$$
a := \eta \cdot \frac{1 + \rho^u}{1 + g} \ ,
$$

since $\eta_k \searrow \eta$, see the proof of Lemma 1. By assumption holds $a > 1$. It follows that, by applying Lemma 2, Eq. (3.51) simplifies to

$$
V_0^\ell = \frac{\mathbb{E}[\widetilde{FCF}_1]}{k^* - g} + \frac{\tau \cdot r \cdot D_0^1}{1 + r^*} = \frac{\mathbb{E}[\widetilde{FCF}_1]}{k^* - g} + \frac{\tau \cdot r \cdot \theta^{(2)} \cdot V_0^\ell}{1 + r^*} \ .
$$

Solving the circularity problem, that is, solving for the market value of the firm, yields the claim. □

Proof of Proposition 3. In this proof, we forgo the labeling of the case of two debt categories in the exponent of the adjustment sequence and write η instead of $\eta^{(2)}$. The formula for the computation of the value of the levered firm in Eq. (3.24) must be equal to Eq. (3.25). Solving for the WACC yields

$$k^{\tau} = k^* - (k^* - g) \cdot \frac{\tau \cdot r \cdot \theta^{(2)}}{1 + r^*} \; . \tag{3.52}$$

Applying the relation $k^{\tau} = \rho^{\ell} \cdot (1 - \theta) + r \cdot (1 - \tau) \cdot \theta$, equating Eq. (3.52) and this expression, and solving for $\rho^{\ell} \cdot (1 - \theta)$ yields

$$\rho^{\ell} \cdot (1 - \theta) = k^* - (k^* - g) \cdot \frac{\tau \cdot r \cdot \theta^{(2)}}{1 + r^*} - r \cdot \theta + \tau \cdot r \cdot \theta \; .$$

By plugging in the definition of k^* and r^*, we obtain

$$\begin{aligned} \rho^{\ell} \cdot (1 - \theta) &= (1 + \rho^u) \cdot \eta - 1 - \tau \cdot r \cdot \theta^{(2)} \cdot \frac{(1 + \rho^u) \cdot \eta - 1 - g}{(1 + r) \cdot \eta} - r \cdot \theta + \tau \cdot r \cdot \theta \\ &= (1 + \rho^u) \cdot \eta - 1 - \tau \cdot r \cdot \theta^{(2)} \cdot \left(\frac{1 + \rho^u}{1 + r} - \frac{1 + g}{(1 + r) \cdot \eta} \right) - r \cdot \theta + \tau \cdot r \cdot \theta \; . \end{aligned}$$

By using that η is a fixed point of the sequence $(\eta_k)_{k \in \mathbb{N}}$, see Eq. (3.23), we obtain

$$\begin{aligned} \rho^{\ell} \cdot (1 - \theta) &= (1 + \rho^u) \cdot \left(1 - \frac{\tau \cdot r \cdot \theta^{(2)}}{1 + r} - \frac{\tau \cdot r \cdot \theta^{(2)} \cdot (1 + g)}{(1 + r)^2 \cdot \eta} \right) - 1 \\ &\quad - \tau \cdot r \cdot \theta^{(2)} \cdot \left(\frac{1 + \rho^u}{1 + r} - \frac{1 + g}{(1 + r) \cdot \eta} \right) - r \cdot \theta + \tau \cdot r \cdot \theta \\ &= (1 + \rho^u) \cdot \left(1 - 2 \cdot \frac{\tau \cdot r \cdot \theta^{(2)}}{1 + r} - \frac{\tau \cdot r \cdot \theta^{(2)} \cdot (1 + g)}{(1 + r)^2 \cdot \eta} \right) - 1 \\ &\quad + (1 + r) \cdot \left(\frac{\tau \cdot r \cdot \theta}{1 + r} + \frac{\tau \cdot r \cdot \theta^{(2)} \cdot (1 + g)}{(1 + r)^2 \cdot \eta} \right) - r \cdot \theta \; . \end{aligned}$$

Adding and subtracting $\rho^u \cdot \theta$ results in

$$\rho^{\ell} \cdot (1 - \theta) = (\rho^u - r) \cdot \left(\theta - \frac{\tau \cdot r \cdot \theta}{1 + r} - \frac{1}{2} \cdot \frac{\tau \cdot r \cdot \theta \cdot (1 + g)}{(1 + r)^2 \cdot \eta} \right) + \rho^u \cdot (1 - \theta) \; .$$

Dividing both sides by $(1 - \theta)$ and defining $L := \frac{\theta}{1 - \theta}$ as the leverage yields the cost of equity.

The expression of k^τ can be derived by applying the definition of the WACC and inserting ρ^ℓ. We obtain

$$\begin{aligned}
k^\tau &= (1-\theta)\cdot\rho^\ell + r\cdot(1-\tau)\cdot\theta \\
&= (1-\theta)\cdot\rho^u + (\rho^u - r)\cdot\left(1 - \frac{\tau\cdot r}{1+r} - \frac{1}{2}\cdot\frac{\tau\cdot r\cdot(1+g)}{(1+r)^2\cdot\eta}\right)\cdot\theta + r\cdot\theta - r\cdot\tau\cdot\theta \\
&= \rho^u - (\rho^u - r)\cdot\left(\frac{\tau\cdot r}{1+r} - \frac{1}{2}\cdot\frac{\tau\cdot r\cdot(1+g)}{(1+r)^2\cdot\eta}\right)\cdot\theta - (1+r)\cdot\frac{r\cdot\tau\cdot\theta}{1+r} \\
&= \rho^u - \tau\cdot r\cdot\theta\cdot\frac{(1+\rho^u)}{1+r} - (\rho^u - r)\cdot\frac{1}{2}\cdot\frac{\tau\cdot r\cdot\theta\cdot(1+g)}{(1+r)^2\cdot\eta}\ ,
\end{aligned}$$

which proves the claim. □

Proof of Proposition 4. Note that

$$\sum_{s=1}^{T-1}\sum_{t=1}^{s}\frac{(1+g)^{T-s-1}}{(1+r^*)^{T-s}} = \sum_{s=1}^{T-1}\frac{(T-s)\cdot(1+g)^{s-1}}{(1+r^*)^s}\ .$$

Using this result, inserting $D_0^t = \theta^{(T)}\cdot V_0^\ell$, and solving the circularity problem in Eq. (3.35) yields

$$V_0^\ell = \frac{\mathbb{E}[\widetilde{FCF}_1]}{k^* - g}\cdot\left(1 - \tau\cdot r\cdot\theta^{(T)}\cdot\sum_{s=1}^{T-1}\frac{(T-s)\cdot(1+g)^{s-1}}{(1+r^*)^s}\right)^{-1}\ . \tag{3.53}$$

We can simplify this expression using Lemma 4 to

$$V_0^\ell = \frac{\mathbb{E}[\widetilde{FCF}_1]}{k^* - g}\cdot\left(1 - \frac{\tau\cdot r\cdot\theta^{(T)}}{1+r}\cdot\frac{(1+x)^T - Tx - 1}{x^2}\right)^{-1}\ ,$$

where x is defined as in Eq. (3.37), which shows the claim. □

Proof of Proposition 5. The proof of the adjustment formula has the same structure as the proof of Proposition 3. The formula for the computation of the value of the levered firm, see Eq. (3.53) must be equal to Eq. (3.25). Solving for the WACC yields

$$\begin{aligned}
k^\tau &= (k^* - g)\cdot\left(1 - \tau\cdot r\cdot\theta^{(T)}\cdot\sum_{s=1}^{T-1}\frac{(T-s)\cdot(1+g)^{s-1}}{(1+r^*)^s}\right) + g \\
&= k^* - (k^* - g)\cdot\tau\cdot r\cdot\theta^{(T)}\cdot\sum_{s=1}^{T-1}\frac{(T-s)\cdot(1+g)^{s-1}}{(1+r^*)^s}\ .
\end{aligned}$$

Note that we again forgo the labeling of the case of T debt categories and write η instead of $\eta^{(T)}$ in the following. Applying the relation $k^\tau = \rho^\ell\cdot(1-\theta) + r\cdot(1-\tau)\cdot\theta$, equating it to the above equation, and solving for $\rho^\ell \cdot (1-\theta)$ yields

$$\begin{aligned}\rho^\ell \cdot (1-\theta) &= k^* - (k^* - g) \cdot \tau \cdot r \cdot \theta^{(T)} \cdot \sum_{s=1}^{T-1} \frac{(T-s)\cdot(1+g)^{s-1}}{(1+r^*)^s} - r \cdot (1-\tau)\cdot\theta \\ &= (1+\rho^u)\cdot\eta - 1 - ((1+\rho^u)\cdot\eta - (1+g))\cdot\tau\cdot r\cdot\theta^{(T)} \\ &\quad \cdot \sum_{s=1}^{T-1} \frac{(T-s)\cdot(1+g)^{s-1}}{(1+r^*)^s} - r\cdot(1-\tau)\cdot\theta \; .\end{aligned}$$

Using that η is a fixed point of the sequence $(\eta_k)_{k\in\mathbb{N}}$, see Eq. (3.33), results in

$$\begin{aligned}\rho^\ell \cdot (1-\theta) = (1+\rho^u)\cdot\left(1 - \frac{\tau\cdot r\cdot\theta^{(T)}}{1+r} - \sum_{t=1}^{T-1}\frac{\tau\cdot r\cdot\theta^{(T)}\cdot(1+g)^t}{(1+r)^{t+1}\cdot\eta^t}\right) - 1 \\ - (1+\rho^u)\cdot\tau\cdot r\cdot\theta^{(T)}\cdot\sum_{s=1}^{T-1}\frac{(T-s)\cdot(1+g)^{s-1}}{(1+r)^s\cdot\eta^{s-1}} \\ + (1+r)\cdot\tau\cdot r\cdot\theta^{(T)}\cdot\sum_{s=1}^{T-1}\frac{(T-s)\cdot(1+g)^s}{(1+r)^{s+1}\cdot\eta^s} - r\cdot\theta \\ + (1+r)\cdot\frac{\tau\cdot r\cdot\theta^{(T)}\cdot T}{1+r} \; ,\end{aligned}$$

which can be written as

$$\begin{aligned}\rho^\ell \cdot (1-\theta) &= (1+\rho^u)\cdot\left(1 - \tau\cdot r\cdot\theta^{(T)}\cdot\left(\sum_{t=0}^{T-1}\frac{(1+g)^t}{(1+r)^{t+1}\cdot\eta^t} - 1\right.\right. \\ &\quad \left.\left. + \sum_{s=0}^{T-1}\frac{(T-s-1)\cdot(1+g)^s}{(1+r)^{s+1}\cdot\eta^s}\right)\right) \\ &\quad + (1+r)\cdot\tau\cdot r\cdot\theta^{(T)}\cdot\sum_{s=0}^{T-1}\frac{(T-s)\cdot(1+g)^s}{(1+r)^{s+1}\cdot\eta^s} \\ &\quad - r\cdot\theta + \rho^u\cdot\theta - \rho^u\cdot\theta \\ &= (\rho^u - r)\cdot\left(\theta - \tau\cdot r\cdot\theta^{(T)}\cdot\sum_{s=0}^{T-1}\frac{(T-s)\cdot(1+g)^s}{(1+r)^{s+1}\cdot\eta^s}\right) + \rho^u\cdot(1-\theta) \; .\end{aligned}$$

Dividing both sides by $(1-\theta)$ yields

$$\begin{aligned}\rho^u &= \rho^u + (\rho^u - r)\cdot\left(1 - \tau\cdot r\cdot\frac{1}{T}\cdot\sum_{s=0}^{T-1}\frac{(T-s)\cdot(1+g)^s}{(1+r)^{s+1}\cdot\eta^s}\right)\cdot\frac{\theta}{1-\theta} \\ &= \rho^u + (\rho^u - r)\cdot\left(1 - \tau\cdot r\cdot\sum_{s=0}^{T-1}\left(1 - \frac{s}{T}\right)\cdot\frac{(1+g)^s}{(1+r)^{s+1}\cdot\eta^s}\right)\cdot\frac{\theta}{1-\theta} \; .\end{aligned}$$

Inserting the leverage $L = \frac{\theta}{1-\theta}$ yields Eq. (3.38). As in the Proof of Proposition 4, we can simplify the sum using Lemma 4, which results in

$$\rho^\ell = \rho^u + (\rho^u - r) \cdot \left(1 - \frac{\tau \cdot r}{1+r} \cdot \frac{(1+x)^{T+1} - (T+1) \cdot x - 1}{Tx^2}\right) \cdot \frac{\theta}{1-\theta} ,$$

where x is defined as in Eq. (3.37). This proves Eq. (3.39).

Inserting this expression for ρ^ℓ in the definition of the WACC yields

$$\begin{aligned} k^\tau &= \rho^u \cdot (1-\theta) + (\rho^u - r) \cdot \left(1 - \frac{\tau \cdot r}{1+r} \cdot \frac{(1+x)^{T+1} - (T+1) \cdot x - 1}{Tx^2}\right) \cdot \theta + r \cdot (1-\tau) \cdot \theta \\ &= \rho^u - (\rho^u - r) \cdot \frac{\tau \cdot r}{1+r} \cdot \frac{(1+x)^{T+1} - (T+1) \cdot x - 1}{Tx^2} \cdot \theta - r \cdot \tau \cdot \theta , \end{aligned}$$

which shows the claim. □

Lemma 1. *Let $a > 0$, $b > 0$, and $a^2 > b$. We define $(\gamma_k)_{k \in \mathbb{N}}$ as a recursive sequence, with $\gamma_1 := 2a$ and*

$$\gamma_k := 2a - \frac{b}{\gamma_{k-1}}$$

for $k > 1$. Then, this sequence converges to

$$\gamma = a + \sqrt{a^2 - b} .$$

Proof. We show that $(\gamma_k)_{k \in \mathbb{N}}$ is monotonously decreasing and bounded from below. We can show the former by induction. Since $a > 0$ and $b > 0$, it follows that $\gamma_2 < \gamma_1$. For $k \in \mathbb{N}$, we assume that $\gamma_k < \gamma_{k-1}$ and conclude

$$\gamma_{k+1} = 2a - \frac{b}{\gamma_k} < 2a - \frac{b}{\gamma_{k-1}} = \gamma_k .$$

Next, we show that it is bounded from below by

$$\gamma := a + \sqrt{a^2 - b} .$$

If $\gamma_k = \gamma + \varepsilon$ for $\varepsilon > 0$, that is, γ_k is greater than γ, we show that the next term is also greater than γ: Since $\gamma > 0$, we have

$$\gamma_{k+1} = 2a - \frac{b}{\gamma_k} = 2a - \frac{b}{\gamma+\varepsilon} = 2a - \frac{b}{\gamma} + \frac{b}{\gamma} - \frac{b}{\gamma+\varepsilon} = \gamma + \left(\frac{b}{\gamma} - \frac{b}{\gamma+\varepsilon}\right) > \gamma \, .$$

Additionally, since $a > 0$ and $b > 0$, it follows that $\gamma_1 = 2a > \gamma$. Hence, the sequence $(\gamma_n)_{n\in\mathbb{N}}$ is monotonously decreasing and bounded from below by γ which implies that $(\gamma_n)_{n\in\mathbb{N}}$ converges. The limit follows from

$$\lim_{k\to\infty} \gamma_{k+1} = 2a - \frac{b}{\lim_{k\to\infty} \gamma_k} \, ,$$

which yields

$$\lim_{k\to\infty} \gamma_k = a + \sqrt{a^2 - b} = \gamma \, .$$

□

Corollary 1. *Let* $\tau, r, \theta \in [0,1], \theta^{(2)} := \frac{1}{2}\theta$, *and*

$$-1 < g < \frac{1}{4} \cdot \frac{(1 + r - \tau \cdot r \cdot \theta^{(2)})^2}{\tau \cdot r \cdot \theta^{(2)}} - 1 \, .$$

In the case $T = 2$, *that is, for two debt categories, the sequence* $(\eta_k^{(2)})_{k\in\mathbb{N}}$ *with* $\eta_1^{(2)} := 1 - \frac{\tau \cdot r \cdot \theta^{(2)}}{1+r}$ *and*

$$\eta_k^{(2)} = 1 - \frac{\tau \cdot r \cdot \theta^{(2)}}{1+r} - \frac{\tau \cdot r \cdot \theta^{(2)} \cdot (1+g)}{(1+r)^2 \cdot \eta}$$

converges to $\eta^{(2)}$ *for* $k \to \infty$, *where*

$$\eta^{(2)} = \frac{1 + r - \tau \cdot r \cdot \theta^{(2)}}{2 \cdot (1+r)} + \frac{1}{2 \cdot (1+r)} \cdot \sqrt{(1+r)^2 - 2 \cdot (1+r) \cdot \tau \cdot r \cdot \theta^{(2)} + (\tau \cdot r \cdot \theta^{(2)} - 4 \cdot (1+g)) \cdot \tau \cdot r \cdot \theta^{(2)}} \, .$$

Proof. We want to apply Lemma 1. To do so, we define

$$a := \frac{1}{2} \cdot \left(1 - \frac{\tau \cdot r \cdot \theta^{(2)}}{1+r}\right) \quad \text{and} \quad b := \frac{\tau \cdot r \cdot \theta^{(2)} \cdot (1+g)}{(1+r)^2} \, .$$

It remains to show that the assumptions of Proposition 1 are valid. Since $\tau \cdot r \cdot \theta^{(2)} < 1$ and $1 + r > 1$, it follows $a > 0$. Moreover, $b > 0$ holds by assumption. To check $a^2 > b$, note that

$$\begin{aligned} g &< \frac{1}{4} \cdot \frac{(1 + r - \tau \cdot r \cdot \theta^{(2)})^2}{\tau \cdot r \cdot \theta^{(2)}} - 1 \\ \Leftrightarrow \quad \frac{1}{4} \cdot \left(1 - \frac{\tau \cdot r \cdot \theta^{(2)}}{1 + r}\right)^2 &> \frac{\tau \cdot r \cdot \theta^{(2)} \cdot (1 + g)}{(1 + r)^2} \; . \end{aligned}$$

□

Lemma 2. *Let $(a_n)_{n \in \mathbb{N}}$ be a sequence that converges from above to $a > 1$ for $n \to \infty$. It follows*

$$\lim_{T \to \infty} \sum_{t=1}^{T} \frac{1}{\prod_{s=1}^{t} a_{T-s+1}} = \sum_{t=1}^{\infty} \frac{1}{a^t} = \frac{1}{a - 1} \; .$$

Proof. We define

$$S_T := \sum_{t=1}^{T} \frac{1}{\prod_{s=1}^{t} a_{T-s+1}} \; .$$

By assumption holds $a_t \geq a > 1$, which implies $S_T \leq \frac{1}{a-1}$. Furthermore, by definition, we have

$$S_{T+1} = \frac{1}{a_{T+1}} \cdot (1 + S_T) \; .$$

It follows $S_{T+1} \geq S_T$. Thus, the sequence $(S_T)_{T \in \mathbb{N}}$ is monotonously increasing and bounded from above. Hence, the sequence converges. For the limit holds

$$\lim_{T \to \infty} S_{T+1} = \frac{1}{\lim_{T \to \infty} a_{T+1}} \cdot \left(1 + \lim_{T \to \infty} S_T\right) \; .$$

Since $\lim_{T \to \infty} a_t = a$, the claim follows. □

Lemma 3. *Let $\tau, r, \theta \in [0,1]$, and $\theta^{(T)} := \frac{1}{T}\theta$. For an arbitrary number of T debt categories, the sequence $(\eta_k^{(T)})_{k \in \mathbb{N}}$ with $\eta_1^{(T)} := 1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r}$ and*

$$\eta_k^{(T)} := 1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1 + r} - \sum_{t=1}^{\min\{k,T\}-1} \frac{\tau \cdot r \cdot \theta^{(T)} \cdot (1 + g)^t}{(1 + r)^{t+1} \cdot \prod_{s=1}^{t} \eta_{k-s}^{(T)}}$$

converges to $\eta^{(T)}$ for $k \to \infty$ if T is even,

$$-1 \leq g \leq (1 + r) \cdot \left(\frac{1}{T^2} \cdot \frac{1 + r}{\tau \cdot r \cdot \theta^{(T)}}\right)^{\frac{1}{n-1}} \cdot \frac{T - 1}{T} - 1 \; ,$$

and

$$\frac{\tau \cdot r \cdot \theta}{1+r} \leq \frac{1}{T} \ ,$$

or if $g > -1$ *and* T *is uneven.*

Proof. The proof is similarly structured to the proof of Lemma 1. First, we need to show that the function

$$f(x) = x^T - x^{T-1} + \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} \cdot \sum_{j=0}^{T-1} \frac{(1+g)^j}{(1+r)^j} \cdot x^{T-1-j}$$

has at least one real root. For T odd, this is a well-known result (see e.g., Kriz and Pultr 2013, p. 9). For T even, note that $f(x) > 0$ for $x \in \{0, 1\}$. If we can show that there exists an $x \in (0,1)$ with $f(x) < 0$, it follows that f has a root $x \in (0,1)$. We want to show

$$f\left(\frac{T-1}{T}\right) < 0 \ . \tag{3.54}$$

From the assumptions follows

$$\frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} \cdot T \cdot \left(\max\left\{\frac{1+g}{1+r}, x\right\}\right)^{T-1} \leq x^{T-1} \cdot (1-x) \ ,$$

for $x = \frac{T-1}{T}$. We obtain

$$\frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} \cdot \sum_{j=0}^{T-1} \frac{(1+g)^j}{(1+r)^j} \cdot x^{T-1-j} \leq x^{T-1} \cdot (1-x) \ ,$$

which implies that Eq. (3.54) is true. We conclude that $\eta_k^{(T)}$ has at least one fixed point $\eta \in \mathbb{R}$.

Let η^* be the largest real fixed point. We want to show that the sequence converges to η^*. To do so, we show that $(\eta_k^{(T)})_{k \in \mathbb{N}}$ is monotonously decreasing and bounded from below. The former assumption is proven by induction. Note that $\eta_1^{(T)} \geq \eta_2^{(T)} \geq \cdots \geq \eta_T^{(T)}$ holds. We now assume that $\eta^{(T)}_{k-T+1} \geq \eta^{(T)}_{k-T+2} \geq \cdots \geq \eta^{(T)}_{k}$ for $k \geq T$ and conclude

$$\begin{aligned}
\eta^{(T)}_{k+1} &= 1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} - \sum_{t=1}^{T-1} \frac{\tau \cdot r \cdot \theta^{(T)} \cdot (1+g)^t}{(1+r)^{t+1} \cdot \prod_{s=1}^{t} \eta^{(T)}_{k+1-s}} \\
&\leq 1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} - \sum_{t=1}^{T-1} \frac{\tau \cdot r \cdot \theta^{(T)} \cdot (1+g)^t}{(1+r)^{t+1} \cdot \prod_{s=1}^{t} \eta^{(T)}_{k-s}} \\
&= \eta^{(T)}_{k} \; .
\end{aligned}$$

It remains to show that $(\eta^{(T)}_{k})_{k\in\mathbb{N}}$ is bounded from below by η^*. We assume that $\eta^{(T)}_{k} \geq \eta^*$ and show $\eta^{(T)}_{k+1} \geq \eta^*$. The inequality $\eta^{(T)}_{k} \geq \eta^*$ implies $\eta^{(T)}_{k-1}, \ldots, \eta^{(T)}_{k+2-T} \geq \eta^*$, which yields

$$\begin{aligned}
\eta^{(T)}_{k+1} &= 1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} - \sum_{t=1}^{T-1} \frac{\tau \cdot r \cdot \theta^{(T)} \cdot (1+g)^t}{(1+r)^{t+1} \cdot \prod_{s=1}^{t} \eta^{(T)}_{k+1-s}} \\
&\geq 1 - \frac{\tau \cdot r \cdot \theta^{(T)}}{1+r} - \sum_{t=1}^{T-1} \frac{\tau \cdot r \cdot \theta^{(T)} \cdot (1+g)^t}{(1+r)^{t+1} \cdot (\eta^*)^t} \\
&= \eta^* \; .
\end{aligned}$$

We conclude that $(\eta^{(T)}_{t})_{k\in\mathbb{N}}$ converges to the largest real fixed point η^*. By defining $\eta^{(T)} := \eta^*$, the claim follows. □

Lemma 4. *For $T \in \mathbb{N}$ and $x > 0$ holds*

$$\sum_{s=0}^{T-1} (T-s) \cdot (1+x)^s = \frac{(1+x)^{T+1} - (T+1) \cdot x - 1}{x^2} \; .$$

Proof. For a proof see Gradshteyn and Ryzhik (2007, Eq. 0.113). □

4 Cross-border discounted cash flow valuation

Joint work with Stefan Dierkes.

Abstract

Cross-border discounted cash flow (DCF) valuation is an indispensable tool for valuing international firms. Whereas it is indicated in the literature that the value of a firm is independent of the used currency, many details of the valuation process are not clear: If quantities are converted to another currency, it is ambiguous whether spot or forward exchange rates should be used and how corresponding costs of capital are determined. In addition, debt financing and the implementation of a two-phase model are mostly not considered. Therefore, we develop a consistent framework for cross-border DCF valuation. Based on a sound analysis of the multi-period global capital asset pricing model, we derive valuation approaches in both foreign and home currency, and clarify the differences between the application of spot and forward exchange rates. Furthermore, we include the possible correlation between exchange rate risk and business risk of a firm, deduce corresponding costs of capital, and incorporate our findings into a two-phase model. Thereby, we establish a framework in which all approaches yield the same result at the valuation date. As debt financing has an important impact on the value of a firm, we analyze the characteristics of active and passive debt management in cross-border DCF valuation. Finally, we discuss the implications of the different valuation approaches from a theoretical and practical perspective.

4.1 Introduction

Over the last decades, the relevance of cross-border investments steadily increased and many international companies emerged. Alongside this development, cross-border valuation has become an important tool for analyzing investments in foreign countries. However, the literature on discounted cash flow (DCF) valuation (Berk and DeMarzo 2020; Brealey et al. 2020; Koller et al. 2020; Kruschwitz and Löffler 2020) is often restricted to national valuation. If different currencies occur, it is indicated in the literature that the market value should be independent of the currency in which the valuation is conducted (see, for example, Koller et al. 2020, p. 507). However, important details are not explained: If quantities are converted to another currency, it is ambiguous whether spot or forward exchange rates should be used for the conversion and how costs of capital are derived and adjusted accordingly. Furthermore, the consideration of a correlation between exchange rate risk and business risk of a firm, and the implementation of a two-phase model is not clear. Moreover, existing literature does not elaborate on the impact of debt financing and corresponding adjustments to the valuation equations.

In this study, we derive a consistent cross-border DCF valuation model. We examine the global capital asset pricing model (GCAPM) to deduce conditions for its application. Based on this, we derive valuation approaches in home currency (HC), where we distinguish between the conversion of cash flows with spot and forward exchange rates. Thereby, we establish a framework in which the HC-valuation approaches yield the same result at the valuation date as the valuation approach in foreign currency (FC). Furthermore, we incorporate our findings into a two-phase model and demonstrate the characteristics of debt financing. Additionally, we compare the different valuation approaches and assess them in terms of practicability.

Complications involving different currencies arise when a company generates cash flows in FC and managers or shareholders are interested in the HC value of these cash flows (Berk and DeMarzo 2020, p. 1099). To determine this value, an FC- or HC-valuation approach can be conducted. In the former, cash flows are discounted at a cost of capital based on the foreign capital market and the valuation result is converted to HC at the current spot exchange rate. In the latter, cash flows are converted to HC first, and then discounted at a cost of capital expressed in HC (Koller et al. 2020, p. 508). It is expected that when applied correctly, the FC- and HC-valuation approaches are equivalent and yield the same result (Koller et al. 2020, p. 507; for further analysis, see Butler et al. 2013). However, several details for the application of HC-valuation approaches are indistinct. Based on existing literature, HC-valuation approaches can be distinguished into whether spot or forward exchange rates are used for the conversion of cash flows. However, the literature mostly focuses on the HC-valuation approach with forward exchange rates (Erasmus and Ernst 2014; Berk and DeMarzo 2020, Chapter 31; Brealey et al. 2020, Chapter 27; Holthausen and Zmijewski 2020, Chapter 17; Koller et al. 2020, Chapter 27, Appendix G), whereas the use of expected spot exchange rates for the conversion of cash flows (Bekaert and Hodrick 2018, Section 15–16, O'Brien 2022) is rarely discussed. The connection between these approaches, the derivation of consistent discount rates, and other conditions for their application are ambiguous. Therefore, the main goal of our study is the consistent implementation of DCF valuation with HC-valuation approaches. Thereby, we aim to establish conditions under which HC-valuation approaches with spot and forward exchange rates, respectively, yield the same market value at the valuation date as the FC-valuation approach. Furthermore, we clarify consequences for future market values derived with different approaches. We identify five components to achieve this goal, as follows.

The first component is the derivation of consistent discount rates. For national valuation, the cost of capital can be derived from the capital asset pricing model (CAPM). For international valuation, the literature rarely elaborates on the derivation of the cost of capital. O'Brien (2022) derives a discount rate that is consistent with the international capital asset pricing model (ICAPM), which considers the integration of capital markets (for literature on the ICAPM, see Solnik 1974; Sercu 1980; Stulz 1981; Adler and Dumas 1983; Sercu 2009). A simplified model is the GCAPM (O'Brien 1999; Schramm and Wang 1999; Stulz 1999), where it is assumed that the purchasing power parity (PPP) holds (Schramm and Wang 1999, p. 65; Koller et al. 2020, pp. 512–516; Holthausen and Zmijewski 2020, p. 865). While Fama (1977) derived conditions for the application of the multi-period local CAPM (LCAPM), corresponding conditions for deriving a deterministic cost of capital based on the multi-period GCAPM are not explicitly analyzed in the literature. Furthermore, it is unclear how the cost of capital differs depending on whether forward or spot exchange rates are applied.

The second component is analyzing the integration of a correlation between exchange rate risk and business risk of a firm. Beyond the classification based on spot or forward exchange rates, HC-valuation approaches can be further distinguished into whether a correlation between exchange rate risk and business risk of a firm is considered. The literature assumes this correlation to be zero in large parts of the analysis (Berk and DeMarzo 2020), excludes it (Bekaert and Hodrick 2018), or does not mention it (Holthausen and Zmijewski 2020; Koller et al. 2020). It is included in the analysis of O'Brien (2022) but only for a constant perpetual growth model. Regarding the inclusion of the correlation, Bekaert and Hodrick (2018) point out that, "As a practical matter, no one does this." (Bekaert and Hodrick 2018, p. 723). Berk and DeMarzo

(2020) critically assess their assumption of zero correlation and state, "Such an assumption often makes sense if the firm operates as a local firm in the foreign market [...]. However, many firms use imported inputs in their production processes or export some of their output to foreign countries." (Berk and DeMarzo 2020, p. 1110). Following this argument, for international companies, the correlation between exchange rate risk and business risk of a firm should be considered. However, a consistent integration of this correlation has not been theoretically analyzed. In particular, effects of neglecting the correlation in the conversion of cash flows on the costs of capital have not been addressed.

The third component is the incorporation of the different valuation approaches into a two-phase model. The projection of cash flows is usually conducted using such a two-phase model that includes an explicit forecast phase and a steady state phase (see, for example, Brealey et al. 2020, pp. 95–99). Whereas in the explicit forecast phase, detailed planning is possible, afterwards, it is assumed that the firm has reached a steady state. In this steady state phase, the expectation of all input parameters grows at a constant rate, which implies that the expected market value also grows at this rate (Koller et al. 2020, pp. 259–260). If quantities occur in FC and grow at a constant rate, it is unclear as to how this growth can be transferred to HC.

The fourth component displays the integration of debt financing. Since interest on debt is deductible from corporate income, debt financing has an immediate impact on the market value of a firm (Kruschwitz and Löffler 2020, p. 73). In the process of DCF valuation, active (Miles and Ezzell 1980, 1985; Harris and Pringle 1985) or passive debt management (Modigliani and Miller 1958; Ashton and Atkins 1978) is usually assumed. However, in cross-border DCF valuation, the effects of debt financing are not clear. The cited literature either assumes the firm to be equity-financed or does not explicitly consider debt financing. For active debt management, free cash flow (FCF) or flow to equity (FtE) methods are popular. For passive debt management, adjusted present value (APV) or FtE methods are often applied (Inselbag and Kaufold 1997; Berk and DeMarzo 2020, pp. 680–701). All these methods have not been theoretically analyzed in cross-border valuation.

The fifth component is to deduce implications of our results for corporate valuation theory and practice. Thus far, there does neither exist a comparison between different cross-border valuation approaches, nor are there recommendations on which approach should be applied. At first, the FC-valuation approach may seem more practical since it is similar to national valuation (for an overview on national valuation, see, for example, Berk and DeMarzo 2020; Koller et al. 2020;

Kruschwitz and Löffler 2020). However, the FC-valuation approach requires knowledge about the foreign capital market. In particular, "One problem with this approach is that it is sometimes difficult to determine the appropriate foreign currency discount rate." (Bekaert and Hodrick 2018, p. 722). Furthermore, the FC-valuation approach does not incorporate possible misvaluation of exchange rates (for a debate of the FC- versus the HC-valuation approach regarding exchange rate misvaluation, see O'Brien 2017, pp. 86-87). Koller et al. (2020) recommend, "Use a domestic-capital WACC if the cross-border business is financed and taxed at domestic interest and tax rates. As international companies tend to borrow in their parent country at parent company currencies, this is the most common approach." (Koller et al. 2020, p. 516). This highlights the importance of HC-valuation approaches but further theoretical analyses and recommendations for corporate valuation practice are not made.

Other general problems of cross-border valuation include the forecasting of cash flows, debt and interdependencies between subsidiaries in several countries, as well as the integration of corresponding country risks, legal requirements, differences in international accounting standards, and international taxation (for an overview of problems in international valuation, see, for example, Erasmus and Ernst 2014, pp. 16–25). An overview of existing approaches to integrate political risk including a new proposal can be found in Bekaert et al. (2016). For elaborations on special characteristics of international taxation, we refer to Berk and DeMarzo (2020, pp. 1103–1106) and Holthausen and Zmijewski (2020, pp. 871–881). Overall, an analysis of all aspects of cross-border valuation is beyond the scope of this study. Therefore, we concentrate on DCF valuation, with a focus on the valuation of a foreign firm from the perspective of a domestic investor. For this setting, the FC-valuation approach is usually suitable, but the described gaps in the literature on HC-valuation approaches already apply to this case of cross-border DCF valuation. Thus, we analyze such a basic setting to derive a sound framework that can then be transferred to a more general setting.

Schüler (2021) has addressed some of the described problems for the valuation of a foreign firm. The author started with a single-period framework, in which he analyzed the GCAPM and derived valuation equations for an unlevered firm using spot and forward exchange rates, respectively. In the multi-period case, Schüler stated valuation equations for the HC-valuation approach with spot exchange rates, including correlation of exchange rate and business risks, and the HC-valuation approach with forward exchange rates, where correlation is excluded. Furthermore, corresponding growth rates are derived and the model is extended to passive debt management. Despite the important contribution to a consistent cross-border DCF framework, several aspects still need further investigation. The multi-period GCAPM and corresponding conditions are not analyzed. The HC-valuation approach with spot exchange rates and a neglection of covariances is not deduced in the multi-period case. Consequences and limitations of these approaches are not addressed, and active debt management is not covered. Moreover, respective advantages and disadvantages are not discussed.

In this study, we derive conditions and valuation equations for a consistent framework for cross-border DCF valuation of a foreign firm. We analyze the multi-period GCAPM and elaborate on necessary conditions for its application. Based on that, using the FC-valuation approach as a starting point, we develop valuation equations for different HC-valuation approaches. In particular, we analyze HC-valuation approaches with spot and forward exchange rates, with and without explicitly considering correlations between exchange rate risk and business risk of a firm, respectively. We thereby clarify the effects of such a correlation and provide a justification for neglecting the correlation in the approaches discussed in the literature. In particular, we show conditions in which all approaches yield the same result at the valuation date. Furthermore, we clarify that correct future market values can only be derived if covariances are not neglected. Moreover, we extend our model to active and passive debt management. All results are embedded into a two-phase model and analyzed for practicability.

This study is organized as follows. In the following Section 4.2, we present assumptions and analyze the GCAPM in a multi-period setting. In Section 4.3, we present valuation equations for an unlevered firm based on our results on the GCAPM. The valuation of a levered firm with active and passive debt management, respectively, is presented in Section 4.4. Afterwards, in Section 4.5, we discuss the theoretical and practical implications of our study for international valuation. Finally, we summarize the results in Section 4.6.

4.2 Capital market model

In corporate valuation practice, the application of the LCAPM is a popular model to estimate the cost of equity. However, the LCAPM is based on segmented markets (Sercu 2009, p. 679). In a cross-border valuation with integrated markets, the cost of equity should be estimated based on the GCAPM (Krapl and O'Brien 2016; Koller et al. 2020, p. 512). Several studies have examined the GCAPM (O'Brien 1999; Schramm and Wang 1999; Stulz 1999) and the influence of the choice of model on the cost of capital (Koedijk et al. 2002; Koedijk and van Dijk 2004a,b; Ejara et al. 2020), but conditions for the application in a multi-period context have not yet been analyzed. Fama (1977) conducted a study on the multi-period LCAPM. In particular, Fama excludes diversification options across periods; that is, the portfolio decision of an investor at some time t cannot be used to hedge against uncertainty at time $t + 1$. To do so, it is assumed that all input variables are deterministic at the valuation date. It follows that expectations and covariances do not depend on available information (Fama 1977). To provide a sound basis for cross-border DCF valuation, we transfer the results of Fama (1977) and examine the application of the GCAPM in a multi-period context.

We examine two countries with HC and FC, respectively. We assume that the capital market participants of both countries are risk-averse and have homogeneous expectations. Furthermore, the capital markets of both countries are assumed to be perfect, complete, and fully integrated. This implies that there are no restrictions or barriers and that the markets are free of arbitrage opportunities (Sercu 2009, p. 667). The global capital market results from the two perfectly correlated local capital markets. The derived relations also hold if more countries are considered. The global market then comprises more countries, but the relations obtained in this section remain valid.

Alternatively, one could start with two capital markets that are not perfectly correlated. An example of such a setting can be found in Sercu (2009, pp. 668-669). This model considers that the development of the spot exchange rate may depend on the development of an economy. When the FC is expensive, a recession is more probable than a boom, as this expensive currency means that the foreign economy is not very competitive and vice versa (Sercu 2009, pp. 668–669). However, the assumption of fully integrated markets is widely accepted and is a condition for the application of the GCAPM (Stulz 1999; Koedijk et al. 2002; Sercu 2009, Chapter 19; Krapl and O'Brien 2016; Holthausen and Zmijewski 2020, Chapter 17; Koller et al. 2020, Chapter 27), such that we use it for the analysis in our study.

For exchange rates, we use the direct quotation, which means that the cost of one unit of FC is given in units of HC (Sercu 2009, p. 72). Furthermore, we assume the Fisher hypothesis to hold (Bekaert and Hodrick 2018, pp. 395–396) and the nominal risk-free interest rates to be deterministic for all periods.[1] We require the covered interest parity (CIP) (Sercu 2009, pp. 123–125) but not the uncovered interest parity (Sercu 2009, p. 430; Holthausen and Zmijewski

[1]The implications of Siegel's paradox (Siegel 1972; Solnik 1993) are not discussed.

2020, pp. 865–867) to hold.[2] The CIP is given by (Koller et al. 2020, p. 511)

$$F_t = S_0 \cdot \frac{\prod_{k=1}^{t}(1+i_k^{\mathrm{HC}})}{\prod_{k=1}^{t}(1+i_k^{\mathrm{FC}})} \,, \tag{4.1}$$

where S_0 is the current spot exchange rate and F_t is the forward exchange rate of period t, which is the rate used in a contract made today to exchange a fixed amount in period t. Furthermore, i_k^{FC} and i_k^{HC} depict the risk-free interest rates of period k in FC and HC, respectively. Superscript FC (HC) always refers to a quantity expressed in FC (HC). Eq. (4.1) states that it is irrelevant whether an amount is invested at the risk-free interest rate in FC for t periods and converted at the forward exchange rate, or if the same amount is exchanged at the current spot exchange rate and invested at the risk-free interest rate in HC for t periods.

To rule out arbitrage, we require further for $r \leq t$ (Sercu 2009, pp. 123–125)

$$\widetilde{F}_{r,t} = \widetilde{S}_r \cdot \frac{\prod_{k=r+1}^{t}(1+i_k^{\mathrm{HC}})}{\prod_{k=r+1}^{t}(1+i_k^{\mathrm{FC}})} \,, \tag{4.2}$$

where $\widetilde{F}_{r,t}$ is the forward exchange rate that is used in contracts made in period r to exchange a fixed amount in period t, and $\widetilde{S}_r$ is the spot exchange rate of period r. These quantities are uncertain at the valuation date but certain from the view of period r. Note that we always label uncertain variables by a tilde. Thus, the condition of Eq. (4.2) states the CIP from the view of period r. These assumptions imply that the forward exchange rate of period t does not have to be equal to the expected spot exchange rate of this period, but the forward exchange rate is the certainty equivalent of the spot exchange rate (Sercu 2009, p. 135).

We also assume the purchasing power parity (PPP) to hold (Schramm and Wang 1999, p. 65; Koller et al. 2020, Appendix G; Brealey et al. 2020, pp. 725–726; Holthausen and Zmijewski 2020, p. 865). We do not consider personal taxes. These assumptions are in line with the assumptions of the GCAPM (O'Brien 1999, p. 74, Koller et al. 2020, 827–829) and coincide with those of Schüler (2021, pp. 619–621).

In the GCAPM, the global market portfolio replaces the local market portfolio of the LCAPM (O'Brien 1999, p. 74) and contains securities from the capital markets of both countries (Koller et al. 2020, p. 513). Since we assume PPP to hold, all investors hold the same global market portfolio (Koller et al. 2020, p. 828). However, the rate of return of the global market portfolio

[2]Implications for the case that the uncovered interest parity holds are discussed in Schüler 2021.

depends on the currency it is expressed in. We start by analyzing the properties of the GCAPM in HC before we further examine the relations of future spot exchange rates, and thereby, derive the GCAPM in FC.

To transfer the results of Fama (1977), we require the expectation of the return of the global market portfolio expressed in HC of some period t to be independent of the available information before time t. Therefore, the return $\tilde{r}_t^{M,\mathrm{HC}}$ is independent of the σ-algebra $\mathcal{F}_{t-1}$, where $(\mathcal{F}_t)_{t\in\mathbb{N}}$ is the natural filtration of the process $(\tilde{r}_t^{M,\mathrm{HC}})_{t\in\mathbb{N}}$. This implies

$$\mathbb{E}_\theta[\tilde{r}_t^{M,\mathrm{HC}}] = \mathbb{E}[\tilde{r}_t^{M,\mathrm{HC}}] \quad \text{for } \theta < t \ ,$$

where $\mathbb{E}_\theta[\cdot]$ denotes the expectation conditioned on $\mathcal{F}_\theta$. This assumption is the first requirement for deterministic input parameters.

The same assumption is required for the return of a security k. The expected return of the security in some period t must be independent of the information available before time t, which implies $\mathbb{E}_\theta[\tilde{r}_t^{k,\mathrm{HC}}] = \mathbb{E}[\tilde{r}_t^{k,\mathrm{HC}}]$ for $\theta < t$. It follows that the covariance of the return r_t^k and the return of the global market portfolio is also independent of the available information. Consequently, we obtain for the global beta

$$\frac{\mathrm{Cov}_\theta[\tilde{r}_t^{k,\mathrm{HC}}, \tilde{r}_t^{M,\mathrm{HC}}]}{\mathrm{Var}_\theta[\tilde{r}_t^{M,\mathrm{HC}}]} = \frac{\mathrm{Cov}[\tilde{r}_t^{k,\mathrm{HC}}, \tilde{r}_t^{M,\mathrm{HC}}]}{\mathrm{Var}[\tilde{r}_t^{M,\mathrm{HC}}]} = \beta_t^{k,\mathrm{HC}} \ ,$$

for $\theta < t$, where Cov denotes the covariance and Var the variance. It follows that all input parameters of the security market line are deterministic, which yields

$$\mathbb{E}[\tilde{r}_t^{k,\mathrm{HC}}] = i_t^{\mathrm{HC}} + (\mathbb{E}[\tilde{r}_t^{M,\mathrm{HC}}] - i_t^{\mathrm{HC}}) \cdot \beta_t^{k,\mathrm{HC}} = i_t^{\mathrm{HC}} + MRP_t^{\mathrm{HC}} \cdot \beta_t^{k,\mathrm{HC}} \ ,$$

where MRP denotes the global market risk premium.

To convert the quantities from HC to FC, we analyze the characteristics of the exchange rates. The change of future spot exchange rates is defined as

$$\tilde{\delta}_t^S = \frac{\widetilde{S}_t}{\widetilde{S}_{t-1}} - 1 \ .$$

To obtain deterministic input parameters, we require further the expected change in spot exchange rates of period t to be independent of the occurred states before this time. It implies that, although the spot exchange rates of period $t-1$ and t are random variables, the ratio $\tilde{\delta}_t^S$ is independent of the information available before time t and thus,[3]

$$\mathbb{E}_\theta[\tilde{\delta}_t^S] = \mathbb{E}[\tilde{\delta}_t^S] \quad \text{for } \theta < t \, . \tag{4.3}$$

Since the two capital markets are fully integrated, all properties of the capital market in HC transfer to that in FC. Again, the expected return of some period t is assumed to be independent of the available information before time t, which yields

$$\mathbb{E}_\theta[\tilde{r}_t^{M,\mathrm{FC}}] = \mathbb{E}[\tilde{r}_t^{M,\mathrm{FC}}] \quad \text{for } \theta < t \, .$$

Furthermore, the expected return of some security k expressed in FC does not depend on available information and follows the security market line

$$\mathbb{E}[\tilde{r}_t^{k,\mathrm{FC}}] = i_t^{\mathrm{FC}} + (\mathbb{E}[\tilde{r}_t^{M,\mathrm{FC}}] - i_t^{\mathrm{FC}}) \cdot \beta_t^{k,\mathrm{FC}} = i_t^{\mathrm{FC}} + MRP_t^{\mathrm{FC}} \cdot \beta_t^{k,\mathrm{FC}} \, .$$

To convert the expected return of the global market portfolio from HC to FC, we compute

$$\begin{aligned} 1 + \mathbb{E}[\tilde{r}_t^{M,\mathrm{FC}}] &= \mathbb{E}\left[(1 + \tilde{r}_t^{M,\mathrm{HC}}) \cdot \frac{\widetilde{S}_{t-1}}{\widetilde{S}_t}\right] \\ &= \left(1 + \mathbb{E}[\tilde{r}_t^{M,\mathrm{HC}}]\right) \cdot \mathbb{E}\left[\frac{\widetilde{S}_{t-1}}{\widetilde{S}_t}\right] + \mathrm{Cov}\left[\tilde{r}_t^{M,\mathrm{HC}}, \frac{\widetilde{S}_{t-1}}{\widetilde{S}_t}\right] \, . \end{aligned} \tag{4.4}$$

The expected return in HC is multiplied by the reciprocal of the change in spot exchange rates. Since these quantities can interact, a covariance is added when considering a product of expectations, see the second line of Eq. (4.4). Analogously, we can convert the expected return from FC to HC by

$$\begin{aligned} \mathbb{E}[\tilde{r}_t^{M,\mathrm{HC}}] &= \left(1 + \mathbb{E}[\tilde{r}_t^{M,\mathrm{FC}}]\right) \cdot \left(1 + \mathbb{E}[\tilde{\delta}_t^S]\right) + \mathrm{Cov}\left[\tilde{r}_t^{M,\mathrm{FC}}, \tilde{\delta}_t^S\right] - 1 \\ &= \mathbb{E}[\tilde{r}_t^{M,\mathrm{FC}}] + \mathbb{E}[\tilde{\delta}_t^S] + \mathbb{E}\left[\tilde{r}_t^{M,\mathrm{FC}} \cdot \tilde{\delta}_t^S\right] \, . \end{aligned} \tag{4.5}$$

Here, the return in FC is multiplied by the change in spot exchange rates, and a covariance is added. Alternatively, in the second line of Eq. (4.5), the return of the global market portfolio expressed in HC can be computed by adding the expected return of the market portfolio expressed in FC to the expected change of spot exchange rates. Moreover, the expectation of the product

[3] Eq. (4.3) is equivalent to the process $(\tilde{S}_t/\mathbb{E}[\tilde{S}_t])_{t\in\mathbb{N}}$ being a martingale, see Appendix B.1 for further elaborations.

has to be added to consider interaction between those quantities.

Overall, we have derived conditions for deterministic input parameters at the valuation date by transferring the results of Fama (1977). Furthermore, we have derived a link between the capital markets in HC and FC via spot exchange rates.

4.3 Valuation of an unlevered firm

4.3.1 Assumptions

We consider an unlevered foreign company that generates cash flows in FC. This company is to be valued from the perspective of a domestic investor such that the market value of the firm needs to be expressed in HC. It follows that one has to decide on the currency of the valuation: Either the cash flow is first converted into HC and then discounted in HC (HC-valuation approach), or the cash flow is discounted in FC and then converted to HC (FC-valuation approach) (Schramm and Wang 1999, p. 64; Koller et al. 2020, p. 508).

If the HC-valuation approach is used, another distinction can be made. The conversion of cash flows can either be done using spot or forward exchange rates. We distinguish the HC-valuation approaches further into whether a correlation between exchange rate risk and business risk of the firm is explicitly considered, which yields four possible HC-valuation approaches. If we do not explicitly consider the correlation in the conversion of cash flows, we label the approach as a *modified* approach. Note that we do not assume the correlation to be zero but analyze the effects of neglecting it in the conversion of cash flows. Since it may be difficult to derive information on covariances in corporate valuation practice, this is a useful concept (Bekaert and Hodrick 2018, p. 723). We derive a consistent framework in which the FC-valuation approach and all HC-valuation approaches yield the same market value at the valuation date. For the derivation of costs of capital, we apply our findings on the GCAPM.

In this section, we derive valuation equations for an unlevered firm. We assume that future free cash flows (FCF) expressed in FC of the foreign company are given, where the FCF always describes the cash flow of an unlevered firm. The projection of the cash flows is based on a two-phase model: For the explicit forecast phase, which comprises T periods, a detailed business plan is developed. Based on that, period-specific cash flows are computed. After the explicit forecast phase, it is assumed that the firm has reached a steady state, in which all input parameters grow at the state-specific rate $\tilde{g}^{\mathrm{FC}}$. It follows that the expectation of all input parameters grows at the constant rate $\mathbb{E}[\tilde{g}^{\mathrm{FC}}]$, which does not depend on the information available at time t. In particular, for the steady state, it is assumed that the expected FCF grows at this rate. We analyze the effects of this steady state on the HC-valuation approaches by deriving corresponding growth rates in HC. In the following, we always start with the FC-valuation approach and then derive valuation equations for the HC-valuation approaches.

4.3.2 FC-valuation approach

We use the FC-approach as a starting point since it is well-known from national valuation. Only multiplication of the market value at the valuation date by the current spot exchange rate constitutes an additional step. The expected market value in FC of some period t is

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{FC}}] = \frac{\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{FC}}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\mathrm{FC}}]}{1+\rho_{t+1}^{u,\mathrm{FC}}} \ ,$$

where $\widetilde{FCF}_t^{\mathrm{FC}}$ is the FCF and $\widetilde{V}_t^u$ is the market value of the unlevered firm of period t. If the unlevered firm is considered, we denote it by a superscript u. The discount rate $\rho^{u,\mathrm{FC}}$ displays the unlevered cost of equity, which follows from applying the GCAPM in FC as

$$\rho_t^{u,\mathrm{FC}} = \mathbb{E}[\tilde{r}_t^{u,\mathrm{FC}}] = i_t^{\mathrm{FC}} + (\mathbb{E}[\tilde{r}_t^{M,\mathrm{FC}}] - i_t^{\mathrm{FC}}) \cdot \beta_t^{u,\mathrm{FC}} \ . \tag{4.6}$$

Multiplying the market value in FC at the valuation date by the current spot exchange rate yields the market value in HC as

$$V_0^{u,\mathrm{HC}} = \left(\sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}]}{\prod_{k=1}^{t}(1+\rho_k^{u,\mathrm{FC}})} + \frac{\mathbb{E}[\widetilde{V}_T^{u,\mathrm{FC}}]}{\prod_{k=1}^{T}(1+\rho_k^{u,\mathrm{FC}})} \right) \cdot S_0 \ . \tag{4.7}$$

To derive the expected market value at the beginning of the steady state phase, we apply the Gordon-Shapiro formula (Gordon and Shapiro 1956) and derive

$$\mathbb{E}[\widetilde{V}_T^{u,\mathrm{FC}}] = \frac{\mathbb{E}[\widetilde{FCF}_{T+1}^{\mathrm{FC}}]}{\rho^{u,\mathrm{FC}} - \mathbb{E}[\tilde{g}^{\mathrm{FC}}]} \ .$$

Note that we assume the business risk, and therefore the unlevered cost of equity, of the foreign company to be constant in the steady state phase.

4.3.3 HC-valuation approach with spot exchange rates

In this approach, we analyze the conversion of cash flows from FC to HC using spot exchange rates. For the expectation of the FCF of period t follows

$$\begin{aligned}\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},S}] &= \mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}} \cdot \widetilde{S}_t] \\ &= \mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] + \mathrm{Cov}[\widetilde{FCF}_t^{\mathrm{FC}}, \widetilde{S}_t] \ . \end{aligned} \tag{4.8}$$

The conversion of the cash flow results in an expectation of the product of the FCF and the spot exchange rate. When using a product of expectations, the covariance of these quantities must be added, see the second line of Eq. (4.8). Superscript S indicates that we exchange the FCF at the respective spot exchange rate using this pattern.

We can convert the expected market value from FC to HC following the same pattern, that is,

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},S}] = \mathbb{E}[\widetilde{V}_t^{u,\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] + \mathrm{Cov}[\widetilde{V}_t^{u,\mathrm{FC}}, \widetilde{S}_t] \ . \tag{4.9}$$

To derive a recursive valuation formula, the converted cash flows need to be discounted at a domestic cost of capital. Thus, we use the cost of equity that is estimated with the security market line of the GCAPM in HC. The expected market value of some period t can be derived as

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},S}] = \frac{\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{HC},S}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\mathrm{HC},S}]}{1+\rho_{t+1}^{u,\mathrm{HC},S}} \ ,$$

where

$$\rho_t^{u,\mathrm{HC},S} = \mathbb{E}[\tilde{r}_t^{u,\mathrm{HC}}] = i_t^{\mathrm{HC}} + (\mathbb{E}[\tilde{r}_t^{M,\mathrm{HC}}] - i_t^{\mathrm{HC}}) \cdot \beta_t^{u,\mathrm{HC}} \ .$$

Thus, the sum of the FCF and the market value of period $t+1$ is discounted at the unlevered cost of equity. All quantities are expressed in HC.

To establish a connection between the discount rates in FC and HC, we compare this approach to the FC-valuation approach and use our results on the multi-period GCAPM to obtain (Schüler 2021, Eq. (42); O'Brien 2022, Eq. (10))

$$\rho_t^{u,\mathrm{HC},S} = (1+\rho_t^{u,\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) + \mathrm{Cov}[\tilde{r}_t^{u,FC}, \tilde{\delta}_t^S] - 1 \ , \tag{4.10}$$

see Appendix B.2.1. Thus, to obtain the unlevered cost of equity in HC, the cost of equity in FC is converted to HC by multiplying it by the expected change in spot exchange rates, and the covariance of the return of the unlevered firm and the change in spot exchange rates is added. The derived cost of equity is deterministic since the distribution of the return of the unlevered firm and the change in spot exchange rates does not depend on future information, see Section 4.2. This assumption and thus our extensive analysis of the GCAPM is essential. Naturally, we have obtained the analogous link for the costs of equity in HC and FC as we did for the return of the global market portfolio, see Eq (4.5). Rearranging Eq. (4.10) yields

$$\rho_t^{u,\mathrm{HC},S} = \rho_t^{u,\mathrm{FC}} + \mathbb{E}[\tilde{\delta}_t^S] + \mathbb{E}[\tilde{r}_t^{u,FC} \cdot \tilde{\delta}_t^S] \,.$$

Thus, the unlevered cost of equity in HC is composed of the business risk in FC, exchange rate risk, and an interaction term between those risks.

To obtain a growth rate in HC, we compute the change of the expected market values from period t to $t+1$ conditioned on all available information at time t, see Appendix B.2.2, and conclude (Schüler 2021, Eq. (39))

$$\mathbb{E}[\tilde{g}^{\mathrm{HC},S}] = (1 + \mathbb{E}[\tilde{g}^{\mathrm{FC}}]) \cdot (1 + \mathbb{E}[\tilde{\delta}^S]) + \mathrm{Cov}[\tilde{g}^{\mathrm{FC}}, \tilde{\delta}^S] - 1 \,. \tag{4.11}$$

To obtain this growth rate, the growth rate in FC is multiplied by the change in spot exchange rates and a covariance is added. The computation follows the same structure as the computation of the unlevered cost of equity in HC, see Eq. (4.10). Again, the derived conditions for the application of the GCAPM are essential. Only with this, we can show that the expected growth rate in HC is deterministic at the valuation date.[4]

For the market value at the valuation date follows

$$V_0^{u,\mathrm{HC}} = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},S}]}{\prod_{k=1}^{t}(1+\rho_k^{u,\mathrm{HC},S})} + \frac{\mathbb{E}[\tilde{V}_T^{u,\mathrm{HC},S}]}{\prod_{k=1}^{T}(1+\rho_k^{u,\mathrm{HC},S})} \,,$$

where the Gordon-Shapiro formula can be used to calculate the market value of period T. As the current spot exchange rate is deterministic, for $t = 0$, the covariance disappears in Eq. (4.9) and this market value coincides with the market value derived by the FC-valuation approach, see Eq. (4.7).

[4]In the steady state phase, we assume a constant change of exchange rates. It follows that the cost of equity and the growth rate in HC are constant. If this was not assumed, both quantities would be time-dependent.

We conclude that if the correlation between exchange rate risk and business risk is explicitly considered in the conversion of cash flows, it also has to be considered in the cost of capital. This happens automatically if the cost of equity based on the domestic capital market is applied. Furthermore, the correlation also has to be explicitly considered for the conversion of the growth rate.[5]

4.3.4 Modified HC-valuation approach with spot exchange rates

In this approach, we analyze consequences of omitting the covariance in the conversion of the FCF. We specify

$$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},S^*}] = \mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] \, . \tag{4.12}$$

This expectation is based on the constructed cash flow

$$\widetilde{FCF}_t^{\mathrm{HC},S^*} = \widetilde{FCF}_t^{\mathrm{FC}} \cdot \widetilde{S}_t - \mathrm{Cov}[\widetilde{FCF}_t^{\mathrm{FC}}, \widetilde{S}_t] \, .$$

Since the covariance is deterministic, it disappears when taking expectations. Thus, the expectation of the FCF in HC of Eq. (4.12) does not coincide with the expectation from Eq. (4.8). The fact that the underlying definition of the FCF does not depict a real but a modified cash flow on the capital market in HC is denoted by the superscript star S^*.

The conversion of the expected market value applying this pattern results in

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},S^*}] = \mathbb{E}[\widetilde{V}_t^{u,\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] \, . \tag{4.13}$$

This is a modified market value, in analogy to the modified cash flow: Compared to the HC-valuation approach with spot exchange rates, the covariance is omitted. To obtain a recursive valuation equation using the modified cash flow and the modified market value, we adjust the discount rate accordingly. We use the FC-valuation approach and expand the valuation equation with future expected spot exchange rates, see Appendix B.2.3. For the modified expected market

[5] The valuation equation and the formula for the growth rate coincide with Schüler (2021, Eq. (39), (42)). However, the author does not elaborate on underlying conditions. Furthermore, in Schüler (2021, Eq. (42)), it is not labeled how the expected market value of period one is converted to HC but it is left up to the reader to differentiate. In particular, the expectation "$\mathbb{E}[V_{U,\mathrm{HC},1}]$" appears in both parts of Eq. (42). However, these terms do not coincide since it makes a difference whether forward or spot exchange rates are used for the conversion of future quantities to HC (see also Section 4.3.7). We clarify this by distinguishing between different expectations and by labeling how a certain value is converted.

value at period t follows

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},S^*}] = \frac{\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{HC},S^*}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\mathrm{HC},S^*}]}{1+\rho_{t+1}^{u,\mathrm{HC},S^*}} \;, \tag{4.14}$$

where

$$\rho_t^{u,\mathrm{HC},S^*} = (1+\rho_t^{u,\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) - 1 \;. \tag{4.15}$$

To compute the corresponding discount rate for this approach, we multiply the cost of equity in FC by the change in spot exchange rates. Thus, if covariances are neglected in the numerator, they also have to be neglected in the denominator to obtain the correct valuation result at the valuation date (see Eq. (4.10) for a comparison of discount rates). Due to our assumptions from Section 4.2, the discount rate from Eq. (4.15) is deterministic. However, it cannot be derived by applying the GCAPM in HC. Instead, the discount rate is based on the cost of equity in FC. In particular, it cannot be referred to as a cost of capital. Therefore, we refer to it as a modified discount rate. Overall, we call this a *modified* approach.

Rearranging Eq. (4.15) yields

$$\rho_t^{u,\mathrm{HC},S^*} = \rho_t^{u,\mathrm{FC}} + \mathbb{E}[\tilde{\delta}_t^S] + \rho^{u,\mathrm{FC}} \cdot \mathbb{E}[\delta_t^S] \;.$$

Thus, the modified discount rate also consists of the business risk expressed in FC, the exchange rate risk, and an interaction term. However, because of the neglection of the covariance in the numerator, the interaction term is a product of expectations instead of an expectation of products.

To apply the approach with expected spot exchange rates for the steady state phase, we need to convert the growth rate in FC to a modified growth rate in HC. To do so, we compute the change in market values of some period t to $t+1$ and obtain

$$\mathbb{E}[\tilde{g}^{\mathrm{HC},S^*}] = (1+E[\tilde{g}^{\mathrm{FC}}]) \cdot (1+\mathbb{E}[\tilde{\delta}^S]) - 1 \;. \tag{4.16}$$

As for the conversion of other quantities in this approach, the computation of the growth rate is conducted without an explicit consideration of covariances.

At the valuation date, the covariance of the current market value and current spot exchange rate amounts to zero, such that Eqs. (4.9) and (4.13) coincide. Thus, we obtain the same market value as with the previous approaches, that is,

$$V_0^{u,\mathrm{HC}} = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},S^*}]}{\prod_{k=1}^{t}(1+\rho_k^{u,\mathrm{HC},S^*})} + \frac{\mathbb{E}[\widetilde{V}_T^{u,\mathrm{HC},S^*}]}{\prod_{k=1}^{T}(1+\rho_k^{u,\mathrm{HC},S^*})} \ ,$$

where the market value of period T can again be derived with the Gordon-Shapiro formula. We conclude that if the correlation between exchange rate risk and business risk is neglected in the conversion of cash flows, it must also be neglected in the cost of capital. Consequently, the discount rate cannot be derived from the domestic capital market but is based on the foreign cost of equity. Furthermore, the correlation must also be neglected in the conversion of the growth rate.

4.3.5 HC-valuation approach with forward exchange rates

When using forward exchange rates for the conversion of the FCF to HC, we agree today on a contract to exchange the expected FCF of period t at the forward exchange rate F_t (Bekaert and Hodrick 2018, p. 106). At that time, the FCF usually does not coincide with this expectation. Depending on the development of the FCF, we have either exchanged too much or too little at the forward exchange rate F_t. Consequently, we have to exchange the outstanding amount at the uncertain spot exchange rate $\widetilde{S}_t$ (Berk and DeMarzo 2020, p. 1098). For the converted FCF in HC follows

$$\widetilde{FCF}_t^{\mathrm{HC},F} = \underbrace{\mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}] \cdot F_t}_{\text{forward transaction}} + \underbrace{(\widetilde{FCF}_t^{\mathrm{FC}} - \mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}]) \cdot \widetilde{S}_t}_{\text{spot transaction}} \ .$$

Taking expectations yields

$$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},F}] = \mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}] \cdot F_t + \mathrm{Cov}[\widetilde{FCF}_t^{\mathrm{FC}}, \widetilde{S}_t] \ . \tag{4.17}$$

Since this approach combines spot and forward exchange rates in the conversion of cash flows, we also need a combination of discount rates, using both spot and forward exchange rates, to obtain the correct result at the valuation date (for more details consider Appendix B.3). This is very complex and therefore, not practical. Consequently, we will not consider the HC-valuation approach with forward exchange rates further.

4.3.6 Modified HC-valuation approach with forward exchange rates

By considering the spot transaction in the HC-valuation approach with forward exchange rates, the conversion of the FCF results in a complex combination of spot and forward exchange rates. Consequently, it may be useful to leave out this spot transaction and thus, only consider the forward transaction. We define this adjusted expectation regarding forward exchange rates as

$$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},F^*}] = \mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}] \cdot F_t \; . \tag{4.18}$$

Compared to Eq. (4.17), the covariance is neglected. Thus, it is a modified cash flow, which we denote by superscript F^*.

The conversion of the market value from FC to HC following this pattern results in a modified expected market value, that is,

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},F^*}] = \mathbb{E}[\widetilde{V}_t^{u,\mathrm{FC}}] \cdot F_t \; . \tag{4.19}$$

The derivation of a recursive valuation equation is analogous to the modified HC-valuation approach with spot exchange rates, see Appendix B.2.4. For the expected market value of period t follows

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},F^*}] = \frac{\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{HC},F^*}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\mathrm{HC},F^*}]}{1+\rho_{t+1}^{u,\mathrm{HC},F^*}} \; , \tag{4.20}$$

where

$$\rho_t^{u,\mathrm{HC},F^*} = (1+\rho_t^{u,\mathrm{FC}}) \cdot (1+\delta_t^F) - 1 \; , \tag{4.21}$$

and

$$\delta_t^F = \frac{F_t}{F_{t-1}} - 1 = \frac{1+i_t^{\mathrm{HC}}}{1+i_t^{\mathrm{FC}}} - 1 \; .$$

The corresponding discount rate for this approach is computed by multiplying the cost of equity in FC by the change in forward exchange rates. This discount rate cannot be derived with the GCAPM in HC. It is a modified discount rate based on the foreign cost of equity. Thus, we do not refer to it as a cost of capital. Rearranging Eq. (4.21) yields

$$\rho_t^{u,\mathrm{HC},F^*} = \rho_t^{u,\mathrm{FC}} + \delta_t^F + \rho_t^{u,\mathrm{FC}} \cdot \delta_t^F \; .$$

Compared with the modified HC-approach with spot exchange rates, the exchange rate risk is considered through the change in forward exchange rates.

The modified expected growth rate expressed in HC using forward exchange rates follows as

$$\mathbb{E}[\tilde{g}^{\mathrm{HC},F^*}] = (1 + \mathbb{E}[\tilde{g}^{\mathrm{FC}}]) \cdot (1 + \delta^F) - 1 \,. \tag{4.22}$$

At the valuation date, the forward exchange rate is equal to the current spot exchange rate, such that Eq. (4.19) coincides with Eq. (4.9). In particular, at the valuation date, the market value derived with this approach equals the market values derived with the other approaches. We conclude

$$V_0^{u,\mathrm{HC}} = \sum_{t=1}^{T} \frac{\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},F^*}]}{\prod_{k=1}^{t}(1 + \rho_k^{u,\mathrm{HC},F^*})} + \frac{\mathbb{E}[\widetilde{V}_T^{u,\mathrm{HC},F^*}]}{\prod_{k=1}^{T}(1 + \rho_k^{u,\mathrm{HC},F^*})} \,.$$

We conclude that to obtain the correct valuation result at the valuation date, the covariance is neglected in the conversion of cash flows, the discount rate and the growth rate.[6] Consequently, as outlaid above, the discount rate cannot be referred to as a cost of capital but constitutes a modified discount rate.

4.3.7 Interim summary of the valuation of an unlevered firm

We started with the FC-valuation approach for the valuation of an unlevered foreign firm, which is well-known from national valuation. Afterwards, we have presented HC-valuation approaches with spot and forward exchange rates. We have distinguished the HC-valuation approaches further whether the correlation between exchange rate risk and business risk of the firm is explicitly considered in the conversion of cash flows. If we neglect covariances, we call the approach a *modified* approach. For each approach, we derive corresponding discount rates. For the HC-valuation approach with spot exchange rates, the discount rate is based on the domestic capital market. In contrast, the modified HC-valuation approaches require modified discount rates, which use the foreign cost of capital. They are converted to HC by multiplying them by the change in exchange rates. Since the resulting discount rates are not directly based on the GCAPM, we do not refer to them as a "cost of capital". Furthermore, to apply the HC-valuation approaches in a two-phase model, we convert the growth rate from FC to HC. Depending on the valuation approach, the conversion of the growth rate differs. Overall, if the correlation is

[6] The valuation equation and the formula for the growth rate coincide with Schüler (2021, Eq. (39), (42)). For a further comparison, see Footnote 5.

Parameter	Valuation approaches			
	FC	S	S^*	F^*
free cash flow	$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}]$	$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},S}]$	$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},S^*}]$	$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},F^*}]$
discount rate	$\rho_t^{u,\mathrm{FC}}$	$\rho_t^{u,\mathrm{HC},S}$	$\rho_t^{u,\mathrm{HC},S^*}$	$\rho_t^{u,\mathrm{HC},F^*}$
growth rate	$\mathbb{E}[\tilde{g}^{\mathrm{FC}}]$	$\mathbb{E}[\tilde{g}^{\mathrm{HC},S}]$	$\mathbb{E}[\tilde{g}^{\mathrm{HC},S^*}]$	$\mathbb{E}[\tilde{g}^{\mathrm{HC},F^*}]$
future market value	$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{FC}}]$	$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},S}]$	$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},S^*}]$	$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},F^*}]$
market value	$V_0^{u,\mathrm{HC}}$			

Table 4.1: Input parameters and market values for the valuation of an unlevered firm, where FC denotes the FC-valuation approach; S denotes the HC-valuation approach with spot exchange rates; S^* denotes the modified HC-valuation approach with spot exchange rates; and F^* denotes the modified HC-valuation approach with forward exchange rates.

omitted in the conversion of cash flows, it also has to be neglected in the computation of discount rates and growth rates. An overview of input parameters can be found in Table 4.1.

We highlight that future market values derived by different HC-valuation approaches do not coincide: The correct future market value can only be derived with the HC-valuation approach with spot exchange rates. The application of a modified HC-valuation approach results in modified market values, see Eqs. (4.13) and (4.19). Furthermore, with the FC-valuation approach, it is not intended to derive future market values in HC. However, at the valuation date, the current spot exchange rate is deterministic such that it is equal to the current forward exchange rate, and the covariance of the current market value and current spot exchange rate amounts to zero. Thus, Eqs. (4.7), (4.9), (4.13) and (4.19) coincide for $t = 0$. Consequently, at the valuation date, all valuation approaches yield the same result.

We conclude that for the valuation of an unlevered firm, all approaches can be applied to obtain the correct market value at the valuation date, but it is essential to use consistent assumptions. Since the HC-valuation approach with forward exchange rates was impractical, we did not consider it further. In the following, we transfer our results on the HC-valuation approach with spot exchange rates and the modified HC-valuation approaches to the valuation of a levered firm.

4.4 Valuation of a levered firm

4.4.1 Assumptions and overview of financing policies

Since interest on debt is deductible from corporate income, debt financing has a direct impact on the market value of a firm. However, it also adds another risk component. In addition to business risk and exchange rate risk that we considered in the valuation of an unlevered firm, financial risk has to be taken into account (Copeland et al. 2014, p. 535, Schüler 2021, Section 4). To derive valuation equations on a clear theoretical basis and focus on the effects of foreign currency debt, we do not include the costs of financial distress and possibility of default, which implies that debt is risk-free. This strong assumption is common in other fundamental analyses (see, for example, Miles and Ezzell 1985) and can be relaxed by using the cost of debt instead of the risk-free interest rate.

We continue to value a foreign company from the view of a domestic investor. This foreign company issues debt in FC. When passive debt management is used, the amount of debt D_t^{FC} is defined deterministically at the valuation date for all periods $t \in \mathbb{N}$ (Modigliani and Miller 1958, 1963). When active debt management is used, the debt-to-market value ratio θ_t is defined deterministically at the valuation date. It follows that the amount of debt $\widetilde{D}_t^{\mathrm{FC}} = \theta_t \cdot \widetilde{V}_t^{\ell,\mathrm{FC}}$ is a random variable (Miles and Ezzell 1980, 1985; Harris and Pringle 1985), where $\widetilde{V}_t^{\ell,\mathrm{FC}}$ denotes the market value of the levered company. Note that if the levered firm is considered, we denote it by superscript ℓ. As before, we also label in the exponent whether a quantity is expressed in FC or HC and how it is converted. The conversions follow the same pattern as in Section 4.3. We consider these two financing strategies and derive valuation equations, starting with active debt management.

4.4.2 Valuation approaches for active debt management

In this section, we assume active debt management according to Miles and Ezzell (Miles and Ezzell 1980, 1985).[7] Thus, tax shields in FC are certain in the period of their occurrence but uncertain from the view of all previous periods. For national valuation, Miles and Ezzell (1980) have shown that the unlevered cost of equity is the appropriate discount rate for the uncertain tax shields (for DCF methods with active debt management in national valuation, see, for example, Kruschwitz and Löffler 2020, pp. 96–114). We transfer their results to the valuation of a foreign company. We do not explicitly state formulas for the application of a two-phase model since the results from Section 4.3 still hold. In particular, the derived formulas for growth rates, see Eqs.

[7] A short discussion on active debt management according to Harris and Pringle can be found in Appendix B.4.

(4.11), (4.16), and (4.22), in conjunction with the Gordon-Shapiro formula, can also be applied for a levered firm with active debt management.

We start by deducing the FCF method since its application is circularity-free for active debt management. Afterwards, we use the obtained results to present the FtE method. The APV method for active debt management can be found in Appendix B.5.

4.4.2.1 FC-valuation approach

To apply the FCF method, the weighted average cost of capital (WACC) is required. For the FC-valuation approach, it can be computed as

$$\rho_t^{\tau,\mathrm{FC}} = \rho_t^{\ell,\mathrm{FC}} \cdot (1-\theta_{t-1}) + i_t^{\mathrm{FC}} \cdot (1-\tau^{\mathrm{FC}}) \cdot \theta_{t-1} \ , \tag{4.23}$$

where τ^{FC} is the foreign corporate tax rate and $\rho^{\ell,\mathrm{FC}}$ denotes the levered cost of equity under active debt management in FC. The levered cost of equity can be derived by applying the GCAPM in FC with the levered beta, that is,

$$\rho_t^{\ell,\mathrm{FC}} = \mathbb{E}[\tilde{r}_t^{\ell,\mathrm{FC}}] = i_t^{\mathrm{FC}} + (\mathbb{E}[\tilde{r}_t^{M,\mathrm{FC}}] - i_t^{\mathrm{FC}}) \cdot \beta_t^{\ell,\mathrm{FC}} \ .$$

Alternatively, the levered cost of equity can be derived by applying the adjustment formula for active debt management to the unlevered cost of equity (Miles and Ezzell 1985) as

$$\rho_t^{\ell,\mathrm{FC}} = \rho_t^{u,\mathrm{FC}} + (\rho_t^{u,\mathrm{FC}} - i_t^{\mathrm{FC}}) \cdot \left(1 - \frac{\tau^{\mathrm{FC}} \cdot i_t^{\mathrm{FC}}}{1 + i_t^{\mathrm{FC}}}\right) \cdot L_{t-1} \ , \tag{4.24}$$

where $L_{t-1} = \frac{\theta_{t-1}}{1-\theta_{t-1}}$ denotes the leverage at time $t-1$. The adjustment formula considers that tax shields are certain in the period of their occurrence and uncertain from the view of all previous periods. The expected market value in FC of period t follows as

$$\mathbb{E}[\widetilde{V}_t^{\ell,\mathrm{FC}}] = \frac{\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{FC}}] + \mathbb{E}[\widetilde{V}_{t+1}^{\ell,\mathrm{FC}}]}{1 + \rho_{t+1}^{\tau,\mathrm{FC}}} \ . \tag{4.25}$$

To derive the equity value at the valuation date, the debt-to-market value ratio is used to obtain $E_0^{\ell,\mathrm{FC}} = V_0^{\ell,\mathrm{FC}} \cdot (1-\theta_0)$.

The equity value can be alternatively derived with the FtE method. The FtE depicts the payments to the shareholders of a levered firm. Since the computation of the FtE requires information about future debt levels, a circularity problem arises, which can be solved by using a spreadsheet software. This is well-known from national valuation. The expected FtE in FC of period t is

$$\mathbb{E}[\widetilde{FtE}_t^{\mathrm{FC}}] = \mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}] + \mathbb{E}[\widetilde{TS}_t^{\mathrm{FC}}] - \mathbb{E}[\widetilde{I}_t^{\mathrm{FC}}] + \mathbb{E}[\Delta\widetilde{D}_t^{\mathrm{FC}}] \ ,$$

where $\widetilde{D}_t^{\mathrm{FC}} = \theta_t \cdot \widetilde{V}_t^{\ell,\mathrm{FC}}$ is the debt level, $\widetilde{I}_t^{\mathrm{FC}} = i_t^{\mathrm{FC}} \cdot \widetilde{D}_t$ are the interest expenses, $\widetilde{TS}_t^{\mathrm{FC}} = \tau^{\mathrm{FC}} \cdot \widetilde{I}_t^{\mathrm{FC}}$ is the tax shield, and $\mathbb{E}[\Delta\widetilde{D}_t^{\mathrm{FC}}] = \mathbb{E}[\widetilde{D}_t^{\mathrm{FC}}] - \mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{FC}}]$ is the difference in debt levels of period t. To derive the expected equity value in FC of period t, we discount the sum of the expected FtE and equity value of period $t+1$ at the levered cost of equity, that is,

$$\mathbb{E}[\widetilde{E}_t^{\ell,\mathrm{FC}}] = \frac{\mathbb{E}[\widetilde{FtE}_{t+1}^{\mathrm{FC}}] + \mathbb{E}[\widetilde{E}_{t+1}^{\ell,\mathrm{FC}}]}{1 + \rho_{t+1}^{\ell,\mathrm{FC}}} \ . \tag{4.26}$$

The equity value in HC can be derived by multiplying the equity value in FC at the current spot exchange rate, that is, $E_0^{\ell,\mathrm{HC}} = E_0^{\ell,\mathrm{FC}} \cdot S_0$. The conversion of the market value and other quantities at the valuation date to HC follow analogously.

4.4.2.2 HC-valuation approaches

The FCF-method

We continue to derive valuation equations for the different HC-valuation approaches, that is, the HC-valuation approach with spot exchange rates and the modified HC-valuation approach with spot and forward exchange rates, respectively. As before, the correlation between exchange rate risk and business risk of the firm is only explicitly considered in the HC-valuation approach with spot exchange rates. In the modified HC-valuation approaches, covariances are neglected. The conversion of the FCF has been described in Section 4.3, see Eqs. (4.8), (4.12) and (4.18). The conversion of the market value of the levered firm for the HC-valuation approach with spot exchange rates and the modified HC-valuation approaches follows the same pattern as the conversion of the market value of the unlevered firm, see Eqs. (4.9), (4.13) and (4.19). To derive a recursive valuation equation, we discount the sum of the expected FCF and the expected market value of the levered firm at the WACC. For the (modified) expected market value of period t follows

$$\mathbb{E}[\widetilde{V}_t^{\ell,HC,j}] = \frac{\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{HC},j}] + \mathbb{E}[\widetilde{V}_{t+1}^{\ell,\mathrm{HC},j}]}{1 + \rho_{t+1}^{\tau,\mathrm{HC},j}} \ , \tag{4.27}$$

where $j \in \{S, S^*, F^*\}$. The corresponding WACC for the three HC-valuation approaches now needs to be specified. We derive a link between the WACC in HC ($\rho^{\tau,\mathrm{HC},j}$) and the WACC in

FC ($\rho^{\tau,\mathrm{FC}}$), by comparing Eq. (4.27) to Eq. (4.25) for $j \in \{S, S^*, F^*\}$, respectively, and obtain

$$\rho_t^{\tau,\mathrm{HC},S} = (1+\rho_t^{\tau,\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) + (1-\theta_{t-1}) \cdot \mathrm{Cov}[\tilde{r}_t^{\ell,\mathrm{FC}}, \tilde{\delta}_t^S] - 1 \tag{4.28}$$

$$\rho_t^{\tau,\mathrm{HC},S^*} = (1+\rho_t^{\tau,\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) - 1 \tag{4.29}$$

$$\rho_t^{\tau,\mathrm{HC},F^*} = (1+\rho_t^{\tau,\mathrm{FC}}) \cdot (1+\delta_t^F) - 1 \; . \tag{4.30}$$

For a detailed computation of Eq. (4.28) see Appendix B.6.2. Eqs. (4.29) and (4.30) follow analogously. We obtain similar relations as in the valuation of an unlevered firm. In the HC-valuation approach with spot exchange rates, see Eq. (4.28), the covariance enters only with the factor $1-\theta_{t-1}$. Since the risk-free interest rate is deterministic, the corresponding covariance amounts to zero. For the modified HC-valuation approaches, see Eqs. (4.29) and (4.30), covariances are neglected. Thus, as before, if the correlation between exchange rate risk and business risk of the firm is explicitly considered in the conversion of cash flows, it also must be considered in the cost of capital, and vice versa.

Inserting the formula for the WACC in FC, see Eq. (4.23), into Eqs. (4.28), (4.29) and (4.30), respectively, yields

$$\rho_t^{\tau,\mathrm{HC},j} = \rho_t^{\ell,\mathrm{HC},j} \cdot (1-\theta_{t-1}) + \eta_t^{\tau,\mathrm{HC},j} \cdot \theta_{t-1} \; , \tag{4.31}$$

where $j \in \{S, S^*, F^*\}$ and

$$\eta_t^{\tau,\mathrm{HC},S} = \eta^{\tau,\mathrm{HC},S^*} = (1+i_t^{\mathrm{FC}} \cdot (1-\tau^{\mathrm{FC}})) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) - 1 \tag{4.32}$$

$$\eta_t^{\tau,\mathrm{HC},F^*} = (1+i_t^{\mathrm{FC}} \cdot (1-\tau^{\mathrm{FC}})) \cdot (1+\delta_t^F) - 1 \; , \tag{4.33}$$

see Appendix B.6.3. The structure of the WACC, see Eq. (4.31), does not change. It is the sum of the levered cost of equity, which is weighted with the equity-to-market value ratio, and the cost of capital $\eta^{\tau,\mathrm{HC}}$, which is weighted by the debt-to-market value ratio. The deductibility of interest on debt is considered in $\eta^{\tau,\mathrm{HC}}$: Since the tax shield appears in FC, the risk-free interest rate in FC is multiplied by the factor $1-\tau$ and is converted to HC only afterwards. The risk-free interest rate in FC is deterministic such that the computation of $\eta^{\tau,\mathrm{HC}}$ does not involve a covariance. In particular, $\eta^{\tau,\mathrm{HC},S}$ and $\eta^{\tau,\mathrm{HC},S^*}$ coincide, see Eq. (4.32).

Note, that the foreign debt-to-market value ratio θ is the same from an HC-perspective such that it is used in the computation of the WACC in HC. In particular, for the HC-valuation approach with spot exchange rates, we obtain

$$\tilde{\theta}_t^{\mathrm{HC},S} = \frac{\widetilde{D}_t^{\mathrm{HC},S}}{\widetilde{V}_t^{\mathrm{HC},S}} = \frac{\widetilde{D}_t^{\mathrm{FC}} \cdot \widetilde{S}_t}{\widetilde{V}_t^{\mathrm{FC}} \cdot \widetilde{S}_t} = \frac{\widetilde{D}_t^{\mathrm{FC}}}{\widetilde{V}_t^{\mathrm{FC}}} = \theta_t^{\mathrm{FC}} = \theta_t \ .$$

It follows that the debt-to-market value ratio in HC is deterministic and equal to the defined debt-to-market value ratio in FC. The modified HC-valuation approach with spot exchange rates does not use state-dependent quantities but modified expectations, such that we obtain

$$\theta_t^{\mathrm{HC},S^*} = \frac{\mathbb{E}[\widetilde{D}_t^{\mathrm{HC},S^*}]}{\mathbb{E}[\widetilde{V}_t^{\mathrm{HC},S^*}]} = \frac{\mathbb{E}[\widetilde{D}_t^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t]}{\mathbb{E}[\widetilde{V}_t^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t]} = \frac{\mathbb{E}[\widetilde{D}_t^{\mathrm{FC}}]}{\mathbb{E}[\widetilde{V}_t^{\mathrm{FC}}]} = \theta_t^{\mathrm{FC}} = \theta_t \ .$$

Again, the debt-to-market value ratio in HC coincides with the specified ratio θ. The modified HC-valuation approach with forward exchange rates follows analogously.

Next, we derive the levered cost of equity in HC, $\rho^{\ell,\mathrm{HC},j}$ for $j \in \{S, S^*, F^*\}$. The levered cost of equity for the HC-valuation approach with spot exchange rates, that is $j = S$, can be derived from the GCAPM in HC as

$$\rho_t^{\ell,\mathrm{HC},S} = \mathbb{E}[\tilde{r}_t^{\ell,\mathrm{HC}}] = i_t^{\mathrm{HC}} + (\mathbb{E}[\tilde{r}_t^{M,\mathrm{HC}}] - i_t^{\mathrm{HC}}) \cdot \beta_t^{\ell,\mathrm{HC}} \ .$$

Compared with the derivation of the unlevered cost of equity, the levered beta instead of the unlevered beta is used. Again, the modified HC-valuation approaches are not based on the domestic capital market such that the GCAPM in HC cannot be applied. Nevertheless, adjustment formulas can be used. In Appendix B.6.4, for the (modified) HC-valuation approach with spot exchange rates, we obtain,

$$\rho_t^{\ell,\mathrm{HC},j} = \rho_t^{u,\mathrm{HC},j} + \left(\rho_t^{u,\mathrm{HC},j} - \eta_t^{\mathrm{HC},S}\right) \cdot \left(1 - \frac{\tau^{\mathrm{FC}} \cdot i_t^{\mathrm{FC}}}{1 + i_t^{\mathrm{FC}}}\right) \cdot L_{t-1} \ , \tag{4.34}$$

where $j \in \{S, S^*\}$ and

$$\eta_t^{\mathrm{HC},S} = (1 + i_t^{\mathrm{FC}}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) - 1 \ ;$$

and for the modified HC-valuation approach with forward exchange rates

$$\rho_t^{\ell,\mathrm{HC},F^*} = \rho_t^{u,\mathrm{HC},F^*} + (\rho_t^{u,\mathrm{HC},F^*} - i_t^{\mathrm{HC}}) \cdot \left(1 - \frac{\tau \cdot i_t^{\mathrm{FC}}}{1 + i_t^{\mathrm{FC}}}\right) \cdot L_{t-1} \ . \tag{4.35}$$

The overall structure of the adjustment formulas remains unchanged: To the unlevered cost of

Parameter	**Valuation approaches**			
	FC	S	S^*	F^*
free cash flow	$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}]$	$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},S}]$	$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},S^*}]$	$\mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},F^*}]$
flow to equity	$\mathbb{E}[\widetilde{FtE}_t^{\mathrm{FC}}]$	$\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},S}]$	$\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},S^*}]$	$\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},F^*}]$
levered discount rate	$\rho_t^{\ell,\mathrm{FC}}$	$\rho_t^{\ell,\mathrm{HC},S}$	$\rho_t^{\ell,\mathrm{HC},S^*}$	$\rho_t^{\ell,\mathrm{HC},F^*}$
WACC	$\rho_t^{\tau,\mathrm{FC}}$	$\rho_t^{\tau,\mathrm{HC},S}$	$\rho_t^{\tau,\mathrm{HC},S^*}$	$\rho_t^{\tau,\mathrm{HC},F^*}$
growth rate	$\mathbb{E}[\tilde{g}^{\mathrm{FC}}]$	$\mathbb{E}[\tilde{g}^{\mathrm{HC},S}]$	$\mathbb{E}[\tilde{g}^{\mathrm{HC},S^*}]$	$\mathbb{E}[\tilde{g}^{\mathrm{HC},F^*}]$
future market value	$\mathbb{E}[\widetilde{V}_t^{\mathrm{FC}}]$	$\mathbb{E}[\widetilde{V}_t^{\mathrm{HC},S}]$	$\mathbb{E}[\widetilde{V}_t^{\mathrm{HC},S^*}]$	$\mathbb{E}[\widetilde{V}_t^{\mathrm{HC},F^*}]$
market value	$V_0^{\ell,\mathrm{HC}}$			
equity value	$E_0^{\ell,\mathrm{HC}}$			

Table 4.2: Input parameters and market values for the FCF and the FtE method under active debt management, where FC denotes the FC-valuation approach; S denotes the HC-valuation approach with spot exchange rates; S^* denotes the modified HC-valuation approach with spot exchange rates; and F^* denotes the modified HC-valuation approach with forward exchange rates.

equity in HC, a risk premium is added. If spot exchange rates are used, see Eq. (4.34), the risk premium depends on the difference of the unlevered cost of equity and $\eta^{\mathrm{HC},S}$. In the computation of $\eta^{\mathrm{HC},S}$, the risk-free interest rate in FC is multiplied by the change in spot exchange rates. We subtract $\eta^{\mathrm{HC},S}$ instead of the risk-free interest rate in HC to consider the exchange rate risk of the tax shields. Compared with $\eta^{\tau,\mathrm{HC},S}$, which was used in the computation of the WACC, the tax shield is not considered. Future forward exchange rates are certain such that the corresponding tax shields in HC do not contain exchange rate risk and the risk-free interest rate is used in Eq. (4.35). The third term of the adjustment formulas coincides for all approaches and considers that tax shields are certain in the period of their occurrence such that they can be discounted at the risk-free interest rate for one period. The involved quantities are expressed in FC since the tax shield appears in FC. Finally, the expression is multiplied by the leverage ratio. The formulas for unlevering and relevering beta follow accordingly.

Table 4.2 summarizes the input parameters for the different approaches. The equity value can be derived by using the debt-to-market value ratio, that is, $E_0^{\ell,\mathrm{HC}} = V_0^{\ell,\mathrm{HC}} \cdot (1-\theta_0)$. Under consistent assumptions, all approaches yield the same result at the valuation date. Furthermore, as for the unlevered firm, future market values differ.

The FtE method

The derivation of the levered cost of equity, which is needed for the FtE method, has already been demonstrated. It remains to deduce the FtE in HC for the different HC-valuation approaches. If the FtE in FC is available, we can use it to directly compute the FtE in HC. The conversion follows the same pattern as the conversion of the FCF. We obtain for the expected FtE in HC

$$\begin{aligned}
\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},S}] &= \mathbb{E}[\widetilde{FtE}_t^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] + \mathrm{Cov}[\widetilde{FtE}_t^{\mathrm{FC}}, \widetilde{S}_t] \\
\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},S^*}] &= \mathbb{E}[\widetilde{FtE}_t^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] \\
\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},F^*}] &= \mathbb{E}[\widetilde{FtE}_t^{\mathrm{FC}}] \cdot F_t \; .
\end{aligned}$$

Breaking the computation up into its components yields

$$\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},j}] = \mathbb{E}[\widetilde{FCF}_t^{\mathrm{HC},j}] + \mathbb{E}[\widetilde{TS}_t^{\mathrm{HC},j}] - \mathbb{E}[\widetilde{I}_t^{\mathrm{HC},j}] + \mathbb{E}[\Delta \widetilde{D}_t^{\mathrm{HC},j}] \; ,$$

where $j \in \{S, S^*, F^*\}$. However, note that for the computation of the tax shield, the interest expenses and difference in debt levels, exchange rates of period t have to be used. Although the tax shield and the interest expenses are based on the debt level of period $t-1$, they appear in period t such that this is the reference period for conversion. In the last term, by using an exchange rate of period t, the change in exchange rates between the period of borrowing and repaying debt is considered. It follows that $\mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{HC},j}]$ cannot be used in the computation of the FtE. Instead, if spot exchange rates are used, we obtain for the expected interest expenses

$$\begin{aligned}
\mathbb{E}[\widetilde{I}_t^{\mathrm{HC},S}] &= i_t^{\mathrm{FC}} \cdot \mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{FC}} \cdot \widetilde{S}_t] \\
&= i_t^{\mathrm{FC}} \cdot \Big(\mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] + \mathrm{Cov}[\widetilde{D}_{t-1}^{\mathrm{FC}}, \widetilde{S}_t] \Big) \; .
\end{aligned}$$

Thus, the expected debt level in FC of period $t-1$, $\mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{FC}}]$, is multiplied by the spot exchange rate of period t and a covariance is added. The computation of the expected tax shield and expected change in debt levels follow accordingly.

For the modified valuation approaches, we obtain the modified expected interest expenses as

$$\mathbb{E}[\widetilde{I}_t^{\mathrm{HC},S^*}] = i_t^{\mathrm{FC}} \cdot \mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] = i_t^{\mathrm{FC}} \cdot \mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{HC},S^*}] \cdot (1 + \mathbb{E}[\delta_t^S]) \tag{4.36}$$

$$\mathbb{E}[\widetilde{I}_t^{\mathrm{HC},F^*}] = i_t^{\mathrm{FC}} \cdot \mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{FC}}] \cdot F_t = i_t^{\mathrm{FC}} \cdot \mathbb{E}[\widetilde{D}_{t-1}^{\mathrm{HC},F^*}] \cdot (1 + \delta_t^F) \; . \tag{4.37}$$

Again, the computation of the modified expected tax shield and modified expected change in debt levels follow accordingly. As neither the debt level in FC nor the debt level in HC is known at the valuation date, the computation of the FtE contains a circularity problem.

The conversion of the equity value also follows the same pattern as the conversion of the unlevered firm, see Eqs. (4.9), (4.13) and (4.19). To derive a recursive valuation equation, we discount the sum of the expected FtE and equity value at the levered cost of equity. For the (modified) expected equity value of period t follows

$$\mathbb{E}[\widetilde{E}_t^{\ell,\mathrm{HC},j}] = \frac{\mathbb{E}[\widetilde{FtE}_{t+1}^{\mathrm{HC},j}] + \mathbb{E}[\widetilde{E}_{t+1}^{\ell,\mathrm{HC},j}]}{1+\rho_{t+1}^{\ell,\mathrm{HC},j}} \,. \tag{4.38}$$

where $j \in \{S, S^*, F^*\}$. The overview in Table 4.2 also shows the input parameters for the FtE method. In particular, all DCF methods are equivalent and at the valuation date, the FC-valuation approach and all HC-valuation approaches coincide.[8]

4.4.3 Valuation approaches for passive debt management

In this section, we assume passive debt management (for DCF methods with passive debt management in national valuation, see, for example, Kruschwitz and Löffler 2020, pp. 90–95). Thus, all tax shields in FC are certain. We start with the APV method since it can be applied without circularity problems. It uses the value-additivity principle by computing the value of the unlevered firm and the value of the tax shield separately. Valuation equations for the unlevered firm are derived in Section 4.3. It remains to derive valuation equations for the value of the tax shield in FC and HC, respectively. Afterwards, we deduce the FtE method for passive debt management because it is widely used in corporate valuation practice.

4.4.3.1 FC-valuation approach

Since future debt levels are certain under the assumption of passive debt management, the tax shield in FC of some period t is certain. We obtain

$$TS_t^{\mathrm{FC}} = \tau^{\mathrm{FC}} \cdot i_t^{\mathrm{FC}} \cdot D_{t-1}^{\mathrm{FC}} \,. \tag{4.39}$$

[8] Note that there is another possibility of deriving the FtE method in HC. Alternatively, it is possible to use the values that are derived with the FCF method in conjunction with the HC-valuation approach with spot exchange rates. All resulting values are expressed in HC such that for the computation of the equity value the same adjustment formulas can be used as for national valuation. However, the resulting discount rate differs from the cost of capital derived from the GCAPM in HC.

The value of the tax shield in FC can be computed by discounting the tax shields at the risk-free interest rate. For the value of the tax shield of period t, $V_t^{TS,\mathrm{FC}}$, we obtain

$$V_t^{TS,\mathrm{FC}} = \frac{TS_{t+1}^{\mathrm{FC}} + V_{t+1}^{TS,\mathrm{FC}}}{1 + i_{t+1}^{\mathrm{FC}}} \ . \tag{4.40}$$

The equity value follows as $\mathbb{E}[\widetilde{E}_t^{\ell,\mathrm{FC}}] = \mathbb{E}[\widetilde{V}_t^{u,\mathrm{FC}}] + V_t^{TS,\mathrm{FC}} - D_t^{\mathrm{FC}}$.

Alternatively, the equity value can be derived with the FtE method. The FtE in FC is calculated as

$$\mathbb{E}[\widetilde{FtE}_t^{\mathrm{FC}}] = \mathbb{E}[\widetilde{FCF}_t^{\mathrm{FC}}] + TS_t^{\mathrm{FC}} - I_t^{\mathrm{FC}} + \widetilde{D}_t^{\mathrm{FC}} \ .$$

Compared with active debt management, the tax shield, interest expenses, and difference in debt levels are deterministic. The equity value of period t follows from

$$\mathbb{E}[\widetilde{E}_t^{\ell,\mathrm{FC}}] = \frac{\mathbb{E}[\widetilde{FtE}_{t+1}^{\mathrm{FC}}] + \mathbb{E}[\widetilde{E}_{t+1}^{\ell,\mathrm{FC}}]}{1 + \rho_{t+1}^{\ell,\mathrm{FC}}} \ , \tag{4.41}$$

where $\rho^{\ell,\mathrm{FC}}$ denotes the levered discount rate under passive debt management in FC. However, the GCAPM cannot be applied to derive this discount rate. For passive debt management, the distribution of the return of the levered firm, $\tilde{r}_t^{\ell,\mathrm{FC}}$, at some time t depends on the occurred states before this time, since financing risk differs between the states of each period. In particular, the expectation is dependent on the available information at some time $\theta < t$. This is not consistent with our assumptions of the multi-period GCAPM such that the levered discount rate does not constitute a cost of capital (see also Kruschwitz and Löffler 2020, pp. 107–109 on this characteristic in national valuation). Nevertheless, also for passive debt management, it is possible and widely used to derive a suitable discount rate $\rho^{\ell,\mathrm{FC}}$ to obtain a consistent valuation equation for the FtE method. Similar to the procedure in the modified HC-valuation approaches, it can be derived by applying the adjustment formula for passive debt management to the unlevered cost of equity (Inselbag and Kaufold 1997, Eq. (7))

$$\rho_t^{\ell,\mathrm{FC}} = \rho_t^{u,\mathrm{FC}} + (\rho_t^{u,\mathrm{FC}} - i_t^{\mathrm{FC}}) \cdot \frac{D_{t-1}^{\mathrm{FC}} - V_{t-1}^{TS,\mathrm{FC}}}{\mathbb{E}[\widetilde{E}_{t-1}^{\ell,\mathrm{FC}}]} \ . \tag{4.42}$$

This formula contains a circularity problem since the equity value is not known at the valuation date. This can be solved by applying a spreadsheet software. At the valuation date, we convert the equity value to HC using the current spot exchange rate, that is $E_0^{\ell,\mathrm{HC}} = E_0^{\ell,\mathrm{FC}} \cdot S_0$.

4.4.3.2 HC-valuation approaches

The APV method

As for active debt management, we derive valuation equations for the HC-valuation approach with spot exchange rates and the modified HC-valuation approach with spot and forward exchange rates, respectively. To derive the APV method, we convert the tax shield in FC, see Eq. (4.39), to HC. Since the tax shield is a deterministic quantity under the assumption of passive debt management, covariances amount to zero. In particular, both approaches using spot exchange rates coincide. We obtain

$$\begin{aligned}\mathbb{E}[\widetilde{TS}_t^{\mathrm{HC},S}] = \mathbb{E}[\widetilde{TS}_t^{\mathrm{HC},S^*}] &= TS_t^{\mathrm{FC}} \cdot \mathbb{E}[\widetilde{S}_t] \\ TS_t^{\mathrm{HC},F^*} &= TS_t^{\mathrm{FC}} \cdot F_t \ .\end{aligned}$$

For the (modified) value of the tax shield of period t follows

$$\begin{aligned}\mathbb{E}[\widetilde{V}_t^{TS,\mathrm{HC},S}] &= \frac{\mathbb{E}[\widetilde{TS}_{t+1}^{\mathrm{HC},S}] + \mathbb{E}[\widetilde{V}_{t+1}^{TS,\mathrm{HC},S}]}{1+\eta_{t+1}^{\mathrm{HC},S}} \\ V_t^{TS,\mathrm{HC},F^*} &= \frac{TS_{t+1}^{\mathrm{HC},F^*} + V_{t+1}^{TS,\mathrm{HC},F^*}}{1+i_{t+1}^{\mathrm{HC}}} \ ,\end{aligned}$$

where $\eta^{\mathrm{HC},S}$ is defined as in Section 4.4.2.2. If spot exchange rates are used for the conversion of the tax shield, the resulting amount is not risk-free anymore. In particular, exchange rate risk has to be considered such that $\eta^{\mathrm{HC},S}$ is applied. Since forward exchange rates are deterministic, the tax shields are certain in HC such that the risk-free interest rate in HC is the correct discount rate.

For passive debt management, in the steady state phase, the debt levels grow at expected rates instead of growing state-specific. Thus, for the HC-valuation approach with spot exchange rates, the expected growth rate amounts to

$$\mathbb{E}[\tilde{g}^{HC,S^*}] = (1 + E[\tilde{g}^{\mathrm{FC}}]) \cdot (1 + \mathbb{E}[\tilde{\delta}^S]) - 1 \ ,$$

where compared with before, the covariance disappears. In line with the calculation of the tax shield, the calculation of the growth rate coincides with the growth rate of the modified HC-valuation approach with spot exchange rates. Thus, the Gordon-Shapiro formula can be applied to calculate the value of the tax shield for passive debt management, but for the HC-valuation approach with spot exchange rates, the growth rate differs from the growth rate of the FCF.

Parameter	**Valuation approaches**			
	FC	S	S^*	F^*
free cash flow	$\mathbb{E}[\widetilde{FCF}_t^{\text{FC}}]$	$\mathbb{E}[\widetilde{FCF}_t^{\text{HC},S}]$	$\mathbb{E}[\widetilde{FCF}_t^{\text{HC},S^*}]$	$\mathbb{E}[\widetilde{FCF}_t^{\text{HC},F^*}]$
debt level	D_t^{FC}	$\mathbb{E}[\widetilde{D}_t^{\text{HC},S}]$	$\mathbb{E}[\widetilde{D}_t^{\text{HC},S}]$	$D_t^{\text{HC},F}$
tax shield	TS_t^{FC}	$\mathbb{E}[\widetilde{TS}_t^{\text{HC},S}]$	$\mathbb{E}[\widetilde{TS}_t^{\text{HC},S}]$	$TS_t^{\text{HC},F}$
flow to equity	$\mathbb{E}[\widetilde{FtE}_t^{\text{FC}}]$	$\mathbb{E}[\widetilde{FtE}_t^{\text{HC},S}]$	$\mathbb{E}[\widetilde{FtE}_t^{\text{HC},S^*}]$	$\mathbb{E}[\widetilde{FtE}_t^{\text{HC},F^*}]$
discount rate of tax shield	i_t^{FC}	$\eta_t^{\text{HC},S}$	$\eta_t^{\text{HC},S}$	i_t^{HC}
discount rate of FtE	$\rho^{\ell,\text{FC}}$	–	$\rho_t^{\ell,\text{HC},S}$	$\rho^{\ell,\text{HC},F^*}$
growth rate of the FCF	$\mathbb{E}[\tilde{g}^{\text{FC}}]$	$\mathbb{E}[\tilde{g}^{\text{HC},S}]$	$\mathbb{E}[\tilde{g}^{\text{HC},S^*}]$	$\mathbb{E}[\tilde{g}^{\text{HC},F^*}]$
growth rate of the VTS	$\mathbb{E}[\tilde{g}^{\text{FC}}]$	$\mathbb{E}[\tilde{g}^{\text{HC},S^*}]$	$\mathbb{E}[\tilde{g}^{\text{HC},S^*}]$	$\mathbb{E}[\tilde{g}^{\text{HC},F^*}]$
growth rate of the FtE	$\mathbb{E}[\tilde{g}^{\text{FC}}]$	–	$\mathbb{E}[\tilde{g}^{\text{HC},S^*}]$	$\mathbb{E}[\tilde{g}^{\text{HC},F^*}]$
future market value	$\mathbb{E}[\widetilde{V}_t^{\text{FC}}]$	$\mathbb{E}[\widetilde{V}_t^{\text{HC},S}]$	$\mathbb{E}[\widetilde{V}_t^{\text{HC},S^*}]$	$\mathbb{E}[\widetilde{V}_t^{\text{HC},F^*}]$
market value	$V_0^{\ell,\text{HC}}$			
equity value	$E_0^{\ell,\text{HC}}$			

Table 4.3: Input parameters and market values for the APV and the FtE method under passive debt management, where FC denotes the FC-valuation approach; S denotes the HC-valuation approach with spot exchange rates; S^* denotes the modified HC-valuation approach with spot exchange rates; and F^* denotes the modified HC-valuation approach with forward exchange rates.

This is also displayed in Table 4.3, which gives an overview of the input parameters and market values using different valuation approaches. As before, the value of the tax shield of period $t \neq 0$ differs between the approaches but coincides at the valuation date.

The FtE method

In the derivation of valuation equations for the HC-valuation approach with spot exchange rates, we always use that the covariance of the return of the firm and the change in spot exchange rates does not depend on future information. Since for passive debt management, financing risk differs between the states of some period, this assumption is not fulfilled. It follows that the shown relations no longer hold and the levered cost of equity cannot be derived ("–" in Table 4.3). In particular, it is not possible to apply the adjustment formula in a similar form as for active debt management.[9] Note also that in the steady state phase, the expected FtE does not increase at a constant growth rate. Since the value of the tax shield increases at a different growth rate than the FCF, the growth of the FtE is not constant (see Dierkes and Schäfer 2021, for how to incorporate different growth rates into terminal value calculation). Due to the dependency of covariances on future information, we do not further consider the FtE method for the HC-valuation approach with spot exchange rates.

For the modified HC-valuation approaches, we convert the FtE to HC following the same pattern as before. If the FtE in FC is available, we obtain

$$
\begin{aligned}
\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},S^*}] &= \mathbb{E}[\widetilde{FtE}_t^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] \\
\mathbb{E}[\widetilde{FtE}_t^{\mathrm{HC},F^*}] &= \mathbb{E}[\widetilde{FtE}_t^{\mathrm{FC}}] \cdot F_t \,.
\end{aligned}
$$

If the FtE is calculated by breaking it up into its components, we refer to the explanations for active debt management. In particular, the debt level of period $t-1$ is always converted using an exchange rate of period t, see also Eqs. (4.36) and (4.37). As debt levels are deterministic for passive debt management, the computation of the FtE does not involve a circularity problem.

The discount rates for the modified HC-valuation approaches use the discount rate in FC. As outlaid in Section 4.4.3.1, $\rho^{\ell,\mathrm{FC}}$ is not a cost of capital but we can use this discount rate to derive a modified adjustment formula. For both approaches, the relationship between the levered discount rate in FC and the levered discount rate in HC is the same as for active debt management, see Eqs. (4.55) and (4.56) in Appendix B.6.1. We insert the adjustment formula for $\rho^{\ell,\mathrm{FC}}$, see Eq. (4.42), to obtain

$$
\rho_t^{\ell,\mathrm{HC},S^*} = \rho_t^{u,\mathrm{HC},S^*} + \left(\rho_t^{u,\mathrm{HC},S^*} - \eta_t^{\mathrm{HC},S}\right) \cdot \frac{D_{t-1}^{\mathrm{FC}} - V_{t-1}^{TS,\mathrm{FC}}}{\mathbb{E}[\widetilde{E}_{t-1}^{\ell,\mathrm{FC}}]} \tag{4.43}
$$

$$
\rho_t^{\ell,\mathrm{HC},F^*} = \rho_t^{u,\mathrm{HC},F^*} + (\rho_t^{u,\mathrm{HC},F^*} - i_t^{\mathrm{HC}}) \cdot \frac{D_{t-1}^{\mathrm{FC}} - V_{t-1}^{TS,\mathrm{FC}}}{\mathbb{E}[\widetilde{E}_{t-1}^{\ell,\mathrm{FC}}]} \,. \tag{4.44}
$$

The derivation is analogous to Appendix B.6.4. Again, the overall structure of the adjustment formula remains unchanged. To consider exchange rate risk, the adjusted cost of capital $\eta^{\mathrm{HC},S}$ instead of the risk-free interest rate is used in the second term of Eq. (4.43). Future forward exchange rates are certain, such that we use the risk-free interest rate in HC in the second term

[9] Compared to the derivation of the levered cost of equity for active debt management in Appendix B.6, covariances depend on available information. However, based on our results on the APV method, it is possible to use the resulting numbers and apply the FtE method in the same way as for national valuation (see also Footnote 8).

of Eq. (4.44). In the third term, quantities in FC are used since tax shields appear in FC. The resulting discount rates are deterministic at the valuation date but not from the view of some period t.

The valuation equation follows as

$$\mathbb{E}[\widetilde{E}_t^{\ell,\mathrm{HC},j}] = \frac{\mathbb{E}[\widetilde{FtE}_{t+1}^{\mathrm{HC},j}] + \mathbb{E}[\widetilde{E}_{t+1}^{\ell,\mathrm{HC},j}]}{1+\rho_{t+1}^{\ell,\mathrm{HC},j}} ,$$

for $j \in \{S^*, F^*\}$. For the modified HC-valuation approaches, the Gordon-Shapiro formula can be applied with the growth rates from Section 4.3, see Eqs. (4.16) and (4.22). An overview of all input parameters can be found in Table 4.3.

4.5 Implications for international valuation in theory and practice

We have presented a consistent framework for the valuation of a foreign firm with the FC-valuation approach, the HC-valuation approach with spot exchange rates, and the modified HC-valuation approach with spot and forward exchange rates, respectively. A summary of our findings can be found in Table 4.4. We started with an unlevered firm and showed that the FC-valuation approach and the different HC-valuation approaches yield the same result at the valuation date ("+" in Table 4.4). This also holds for the valuation of a levered firm with either active or passive debt management. Thereby, we confirm existing results on the irrelevance of the chosen valuation approach. However, it has not been pointed out before that future market values do not coincide: The correct future market values can only be obtained with the HC-valuation approach with spot exchange rates. The modified HC-valuation approaches yield modified future market values ("$-$" in Table 4.4). Furthermore, in the FC-valuation approach, it is not intended to compute future market values in HC (blank space in Table 4.4). For value-based management, this can be a decisive disadvantage since future cash flows and market values can be relevant for decision-making (Rappaport 1998). However, if only the market value at the valuation date is of interest, from a theoretical perspective, each approach functions equally well.

As the first component of our goal of deriving a consistent framework for DCF valuation with HC-valuation approaches, we identified the derivation of consistent discount rates. Due to our analysis of the multi-period GCAPM and the conditions presented thereby, we show that the derived discount rates are deterministic. This is a prerequisite for our further analysis. However, the application of the GCAPM in HC is only possible for the HC-valuation approach with spot exchange rates, regardless of whether an unlevered or a levered firm is considered.

Characteristic	**unlevered firm**				**active debt man.**				**passive debt man.**			
	FC	S	S^*	F^*	FC	S	S^*	F^*	FC	S	S^*	F^*
valuation at $t = 0$	+	+	+	+	+	+	+	+	+	+	+	+
valuation at $t \geq 0$		+	−	−		+	−	−		+	−	−
cost of capital based on domestic market	−	+	−	−	−	+	−	−	−	−	−	−
consideration of covariance	−	+	−	−	−	+	−	−	−	+	−	−
consideration of covariance in TS					−	+	−	−	−	−	−	−
constant growth of the FCF	+	+	+	+	+	+	+	+	+	+	+	+
constant growth of VTS					+	+	+	+	+	+	+	+
constant growth of the FtE					+	+	+	+	+	−	+	+

Table 4.4: Characteristics of different valuation approaches, where FC denotes the FC-valuation approach; S denotes the HC-valuation approach with spot exchange rates; S^* denotes the modified HC-valuation approach with spot exchange rates; and F^* denotes the modified HC-valuation approach with forward exchange rates; and "+" ("−") means that a certain characteristic is (not) met, and a blank space means that a certain characteristic does not apply.

The computation of the corresponding cost of capital requires the estimation of the risk-free interest rate, market risk premium, and beta factor in HC. For the FC-valuation approach, the corresponding discount rate is based on the foreign capital market. Consequently, the foreign capital market has to be analyzed to derive estimates of the risk-free interest rate, market risk premium, and beta factor in FC. This information is also needed for the modified HC-valuation approaches: Since the modified discount rates are adjusted to the neglection of covariances in the conversion of cash flows, they are computed using the cost of equity in FC, which is based on the foreign capital market. Thus, to calculate a modified cost of capital, the estimation of the discount rate in FC is inevitable. Since the modified discount rates cannot be directly derived from the GCAPM, we do not refer to them as a "cost of capital". Overall, the analysis of the foreign capital market can only be avoided by applying the HC-valuation approach with spot exchange rates.

The second component was the correct integration of a correlation between exchange rate risk and business risk of the firm. We demonstrate that this correlation is explicitly considered in the HC-valuation approach with spot exchange rates by adding a covariance term in the conversion of cash flows, debt levels, costs of capital, and growth rates. If this covariance is neglected in the conversion of cash flows, it also has to be neglected in the computation of all other quantities to obtain the correct valuation result at the valuation date. This results in a modified valuation approach. An exception is the calculation of the tax shield for passive debt management: Since future debt levels are deterministic, covariances disappear. In particular, the computation of the value of the tax shield for passive debt management coincides for the HC-valuation approach with spot exchange rates and the modified HC-valuation approach with spot exchange rates.

The third component was the incorporation of the valuation approaches into a two-phase model. For each approach, we presented formulas for computation of growth rates. Thereby, in the steady state phase, we assume that the entire economy is in a steady state such that the expected change of exchange rates is constant. Consequently, business risk is also constant in HC such that for an unlevered firm, the Gordon-Shapiro formula can be applied for every HC-valuation approach. If this was not assumed, a constant business risk in FC would not transfer to HC since the expected change in exchange rates was not constant.

The fourth component was the integration of debt financing. We distinguish between active and passive debt management. For active debt management, we continue to apply the GCAPM and present adjustment formulas. Thereby, we show that the resulting levered cost of equity is deterministic. We demonstrate that the overall structure of adjustment formulas remains unchanged but adaptions have to be made to consider exchange rate risk. Furthermore, we show that the computed growth rates of the unlevered firm can be transferred to the valuation of a levered firm. The debt levels, and therefore the value of the tax shield and the FtE, grow at the same rate as the FCF. As for national valuation, only the FCF method can be applied circularity-free. The computation of the FtE contains a well-known circularity problem since future debt levels are not known at the valuation date. Another obstacle in the computation of the FtE in international valuation is the correct integration of the change in exchange rates. This happens automatically in the FCF method. Overall, for active debt management, the application of the FCF method is the most convenient choice but it is also possible to apply another method since all DCF methods are equivalent.

For passive debt management, the multi-period GCAPM cannot be applied to derive the levered cost of equity. This leads to problems regarding the discount rate in the HC-valuation approach with spot exchange rates. Moreover, in this approach, the growth rate of the value of the tax shield differs from that of the FCF, resulting in an inconstant growth of the FtE. Due to these irregularities, we do not recommend the application of the HC-valuation approach with spot exchange rates in conjunction with the FtE method. For the FtE method, valuation with a modified HC-valuation approach is more convenient: As for active debt management we show that the overall structure of adjustment formulas remains unchanged but adaptions have to be made to consider exchange rate risk. Moreover, in the modified HC-valuation approaches, the FtE grows at the same rate as the FCF. However, for every HC-valuation approach, the APV method can be applied circularity-free, such that this is the most convenient choice for passive debt management but again, all DCF methods are equivalent.

The fifth component was deducing practical implications. To do so, we need to distinguish between different occasions of cross-border valuation. In our study, we value a foreign company from the view of a domestic investor. Thereby, we deal with the simplest case of cross-border valuation. As all quantities appear in FC, the application of the FC-valuation approach is straightforward. Although this requires the analysis of the foreign capital market, the procedure is otherwise well-known from national valuation and does not involve estimating exchange rates. Furthermore, since the foreign company only operates in the foreign country, it can be difficult to identify a suitable peer group on the domestic capital market for estimating the beta factor. This can complicate the HC-valuation approach with spot exchange rates. Consequently, for the valuation of a foreign company, the FC-valuation approach is the most suitable approach.

If a company generates cash flows in both FC and HC and also issues debt in both FC and HC, the valuation becomes more extensive, but our results can be transferred (for characteristics on combining foreign and domestic debt, see Bekaert and Hodrick 2018, p 742; Schüler 2021, Section 5). If the HC-valuation approach with spot exchange rates is applied, all quantities and possible interdependencies can be expressed in HC. Furthermore, only the analysis of the domestic capital market is necessary. If more countries are considered, the benefits of this approach will increase further: It is still only necessary to analyze the domestic capital market. However, this approach comes with the disadvantage that the conversion of cash flows and other quantities requires information about covariances. Thus far, there does not exist a standardized approach to estimate these covariances.

As described in the introduction, in the literature of cross-border valuation, covariances are often neglected for practical reasons when converting cash flows. This suggests the application of a modified HC-valuation approach. If a modified HC-valuation approach is applied, one has to choose between forward and spot exchange rates. Whereas spot exchange rates have to be forecasted (for an overview of exchange rate forecasting, see, for example, Bekaert and Hodrick 2018, Chapter 10), forward exchange rates are known and therefore, easier to implement. Indeed, the literature often focuses on the approaches with forward exchange rates. Consequently, when choosing between the modified HC-valuation approaches, the modified HC-valuation approach with forward exchange rates can be more convenient. If either of the modified HC-valuation approaches is used for a more extensive cross-border valuation, all quantities are converted to HC. However, all involved foreign capital markets have to be analyzed to obtain the corresponding modified discount rates to discount the converted cash flows. Furthermore, for quantities that already occur in HC, a discount rate based on the domestic capital market has to be used, such that the domestic capital market also has to be analyzed. Alternatively, it is also possible to use the FC-valuation approach for each country individually and add up the resulting values; however also with this approach, one must analyze all foreign capital markets. Additionally, for both the modified HC-valuation approaches and the FC-valuation approach, it is not clear how interdependencies are considered. Therefore, we recommend to use the HC-valuation approach with spot exchange rates for more extensive cross-border valuations. This is in line with Koller et al. (2020) who recommend a domestic cost of capital for most cases of a valuation of an international company.

The use of a cost of capital derived from the domestic capital market in connection with a modified cash flow leads to deviating valuation results. O'Brien 2022 used a constant perpetual growth model with a cost of capital that is based on the ICAPM for his analysis. The author computed the deviation of market values that were deduced under different assumptions. In particular, the author compared a consistent application of an approach with spot exchange rates to an inconsistent approach where covariances are neglected in the conversion of cash flows, but a domestic discount rate is used. O'Brien shows that the deviation can be substantial (O'Brien 2022). Similar results can be expected for the application of the GCPAM. If forward exchange rates are used in connection with a domestic discount rate, we also expect similar results. Both cases can be subject for future research. Thereby, it is of particular interest whether modified cash flows computed with spot or forward exchange rates in combination with a domestic discount rate yield smaller deviations, and which input parameters influence these. Overall, to obtain the correct result at the valuation date, one needs to maintain consistent assumptions throughout the valuation process. To consistently apply the HC-valuation approach with spot exchange rates, it is subject to future research to develop a standardized method for estimating covariances.

4.6 Conclusion

In this study, we develop a consistent cross-border DCF valuation framework. We derive conditions for the application of the multi-period GCAPM, which serves as a basis for our analysis. We investigate four different possibilities of converting cash flows from FC to HC and thereby, present valuation approaches with spot and forward exchange rates with and without explicitly considering the correlation between exchange rate risk and business risk of a firm. For each method, we deduce corresponding costs of capital and present valuation equations. Furthermore, we incorporate our findings into a two-phase model, extend our model to active and passive debt management and derive implications for corporate valuation theory and practice.

We show that the FC-valuation approach and the different HC-valuation approaches yield the same result at the valuation date. Thereby, consistent assumptions have to be made regarding the use of spot or forward exchange rates, and the explicit consideration of the correlation between exchange rate risk and business risk of a firm. In particular, covariances are considered for the conversion of all quantities in the HC-valuation approach with spot exchange rates. In the modified HC-valuation approaches, covariances are consistently neglected. For future periods, we show that the valuation results differ since the application of a modified HC-valuation approach results in modified market values. The correct future market value can only be derived with the HC-valuation approach with spot exchange rates. This holds for an unlevered firm as well as for active and passive debt management.

We use the GCAPM as a basis to derive costs of capital. Depending on the conversion of cash flows, the discount rate has to be adjusted accordingly. We show that the HC-valuation approach with spot exchange rates is the only approach whose cost of capital is based on the domestic capital market. The discount rates of the modified HC-valuation approaches use the foreign cost of capital. For the valuation of a levered firm, we show that the overall structure of adjustment formulas for the levered cost of equity remains unchanged but adaptions have to be made to consider exchange rate risk. Futhermore, for each approach, we incorporate the valuation into a two-phase model and present formulas for the computation of growth rates. Thereby, we transfer the setting of the steady state from FC to HC. We show that the formulas that were presented for the unlevered firm can mostly also be used for the valuation of a levered firm.

We chose to analyze the valuation of a foreign company without debt in domestic currency and other interdependencies. This was necessary since even for this simple setting, the DCF framework of the HC-valuation approaches has not yet been clear. Although, the FC-valuation approach is usually the simplest choice for this occasion, our consistent implementation of HC-valuation approaches serves as an important basis. For the valuation of an international company, we recommend the application of the HC-valuation approach with spot exchange rates. The decisive advantage is that only the domestic capital market has to be analyzed to derive costs of capital. Other problems as the integration of country risks, legal requirements, differences in international accounting standards and international taxation are not considered in this study. However, also for these problems, we recommend applying the HC-valuation approach with spot exchange rates. Then all quantities are available in HC and adequate adjustments can be made. A more detailed elaboration is subject to further research. Overall, our analysis serves as a consistent framework to build upon.

B Appendix

B.1 Elaborations on the change in exchange rates

We define

$$\widetilde{M}_t := \frac{\widetilde{S}_t}{\mathbb{E}[\widetilde{S}_t]} \ .$$

In the following, we show that Eq. (4.3) is equivalent to the process $(\widetilde{M}_t)_{t\in\mathbb{N}}$ being a martingale: Let $(\mathcal{G}_t)_{t\in\mathbb{N}}$ be the natural filtration of $(\widetilde{S}_t)_{t\in\mathbb{N}}$. Thus, $(\widetilde{M}_t)_{t\in\mathbb{N}}$ is also adapted to this filtration. First, we show that Eq. (4.3) implies that the expected change in spot exchange rates is equal to the change of expected spot exchange rates: For $t \in \mathbb{N}$, we obtain

$$\widetilde{S}_{t-1} = \widetilde{S}_{t-1} \cdot \frac{\mathbb{E}_{t-1}[\widetilde{S}_t]}{\mathbb{E}_{t-1}[\widetilde{S}_t]} = \frac{\mathbb{E}_{t-1}[\widetilde{S}_t]}{\mathbb{E}_{t-1}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right]} = \frac{\mathbb{E}_{t-1}[\widetilde{S}_t]}{\mathbb{E}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right]} \ ,$$

where we used Eq. (4.3) in the last step. Taking expectations yields

$$\mathbb{E}[\widetilde{S}_{t-1}] = \frac{\mathbb{E}[\widetilde{S}_t]}{\mathbb{E}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right]} \quad \Leftrightarrow \quad \mathbb{E}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right] = \frac{\mathbb{E}[\widetilde{S}_t]}{\mathbb{E}[\widetilde{S}_{t-1}]} \ . \tag{4.45}$$

Now, we can show that Eq. (4.3) implies that $(\widetilde{M}_t)_{t\in\mathbb{N}}$ is a martingale. We have

$$\mathbb{E}_{t-1}[\widetilde{M}_t] = \mathbb{E}_{t-1}\left[\frac{\widetilde{S}_t}{\mathbb{E}[\widetilde{S}_t]}\right] = \frac{\widetilde{S}_{t-1}}{\mathbb{E}[\widetilde{S}_t]} \cdot \mathbb{E}_{t-1}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right] = \frac{\widetilde{S}_{t-1}}{\mathbb{E}[\widetilde{S}_t]} \cdot \frac{\mathbb{E}[\widetilde{S}_t]}{\mathbb{E}[\widetilde{S}_{t-1}]} = \widetilde{M}_{t-1} \ , \tag{4.46}$$

where we used Eqs. (4.3) and (4.45) in the third equation. It remains to show that $(\widetilde{M}_t)_{t\in\mathbb{N}}$ being a martingale implies Eq. (4.3). By dividing Eq. (4.46) on both sides by $\widetilde{S}_{t-1}$ and taking expectations, Eq. (4.45) follows. We use this to show

$$\mathbb{E}_{t-1}[\tilde{\delta}_t] = \frac{\mathbb{E}[\widetilde{S}_t]}{\widetilde{S}_{t-1}} \cdot \mathbb{E}_{t-1}[\widetilde{M}_t] = \frac{\mathbb{E}[\widetilde{S}_t]}{\widetilde{S}_{t-1}} \cdot \widetilde{M}_{t-1} = \frac{\mathbb{E}[\widetilde{S}_t]}{\mathbb{E}[\widetilde{S}_{t-1}]} = \mathbb{E}[\tilde{\delta}_t] \ .$$

With the law of total expectations, Eq. (4.3), and therefore the claim, follows.

As we assume PPP to hold, the expected change in spot exchange rates equals the expected change in inflation rates (Brealey et al. 2020, pp. 722–723; Koller et al. 2020, p. 828). It follows that by defining the process $(\widetilde{M}_t)_{t\in\mathbb{N}}$, we adjust the spot exchange rate for inflation effects. Since the resulting process is a martingale, Eq. (4.3) can be understood as the assumption that the process of the inflation-adjusted spot exchange rates $(\widetilde{M}_t)_{t\in\mathbb{N}}$ represents a fair betting game in the sense that for any given value $\widetilde{M}_{t-1}$, the inflation adjusted exchange rate on average does not change from period $t-1$ to t. In other words, the development of the conditional expected values of the process $(\widetilde{M}_t)_{t\in\mathbb{N}}$ follows the rules of rational expectation formation. This means that all the information available in some period t is correctly evaluated when forming these conditional expected values.

B.2 Derivation of equations for the valuation of an unlevered firm

B.2.1 Derivation of Eq. (4.10)

Note that

$$\mathrm{Corr}[\tilde{r}_{t+1}^{u,\mathrm{FC}}, \widetilde{FCF}_{t+1}^{\mathrm{FC}}] = 1 \ .$$

This correlation of 1 between the return of the unlevered firm and the FCF always holds true in our setting since it is a direct consequence of the characteristics of the GCAPM. It follows

$$\mathrm{Cov}_t[\tilde{r}_{t+1}^{u,\mathrm{FC}}, \tilde{\delta}_{t+1}^{S}] = \mathrm{Cov}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}} + \widetilde{V}_{t+1}^{u,\mathrm{FC}}, \widetilde{S}_{t+1}] \cdot \frac{1}{\widetilde{V}_t^{u,\mathrm{FC}} \cdot \widetilde{S}_t} \ . \tag{4.47}$$

The market value that is computed using the HC-valuation approach with spot exchange rates needs to coincide with the market value that is computed using the FC-valuation approach and multiplied by the spot exchange rate. We obtain for the market value in HC of some period t

$$\widetilde{V}_t^{u,\mathrm{FC}} \cdot \widetilde{S}_t \overset{!}{=} \frac{\mathbb{E}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}] \cdot \mathbb{E}_t[\widetilde{S}_{t+1}] + \mathrm{Cov}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}, \widetilde{S}_{t+1}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{HC},S}]}{1 + \rho_{t+1}^{u,\mathrm{HC},S}} \ .$$

Using

$$\mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{HC},S}] = \mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}}] \cdot \mathbb{E}_t[\widetilde{S}_{t+1}] + \mathrm{Cov}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}}, \widetilde{S}_{t+1}]$$

and solving for $1+\rho_{t+1}^{u,\mathrm{HC},S}$ yields

$$\begin{aligned}1+\rho_{t+1}^{u,\mathrm{HC},S} &= \frac{\left(\mathbb{E}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}]+\mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}}]\right)\cdot\mathbb{E}_t[\widetilde{S}_{t+1}]}{\widetilde{V}_t^{u,\mathrm{FC}}\cdot\widetilde{S}_t} \\ &\quad+\frac{\mathrm{Cov}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}},\widetilde{S}_{t+1}]+\mathrm{Cov}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}},\widetilde{S}_{t+1}]}{\widetilde{V}_t^{u,\mathrm{FC}}\cdot\widetilde{S}_t} \\ &= (1+\rho_{t+1}^{u,\mathrm{FC}})\cdot\frac{\mathbb{E}_t[\widetilde{S}_{t+1}]}{\widetilde{S}_t}+\mathrm{Cov}_t\left[\frac{\widetilde{FCF}_{t+1}^{\mathrm{FC}}+\widetilde{V}_{t+1}^{u,\mathrm{FC}}}{\widetilde{V}_t^{u,\mathrm{FC}}},\frac{\widetilde{S}_{t+1}}{\widetilde{S}_t}\right] \\ &= (1+\rho_{t+1}^{u,\mathrm{FC}})\cdot\mathbb{E}[\tilde{\delta}_{t+1}^{S}]+\mathrm{Cov}[\tilde{r}_{t+1}^{u,\mathrm{FC}},\tilde{\delta}_{t+1}^{S}]\,.\end{aligned}$$

In the second step we used Eq. (4.47). In the last step, we used that the return of the unlevered firm $\tilde{r}_{t+1}^{u,\mathrm{HC}}$ and the change of spot exchange rates $\tilde{\delta}_{t+1}^{S}$ is independent of the information available at time t, see Section 4.2.

B.2.2 Derivation of Eq. (4.11)

The change of the expected market values from period t to $t+1$ conditioned on information available at time t is computed as

$$\begin{aligned}1+\mathbb{E}_t[\tilde{g}_{t,t+1}^{HC,S}] &= \frac{\mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{HC},S}]}{\widetilde{V}_t^{u,\mathrm{HC},S}} \\ &= \frac{\mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}}]\cdot\mathbb{E}_t[\widetilde{S}_{t+1}]+\mathrm{Cov}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}},\widetilde{S}_{t+1}]}{\widetilde{V}_t^{u,\mathrm{FC}}\cdot\widetilde{S}_t} \\ &= (1+E[\tilde{g}^{\mathrm{FC}}])\cdot(1+\mathbb{E}_t[\tilde{\delta}_{t+1}^{S}])+\mathrm{Cov}_t[\tilde{g}^{\mathrm{FC}},\tilde{\delta}_{t+1}^{S}]\,,\end{aligned}$$

where we used Eq. (4.47). Since the change in spot exchange rates and the growth rate in FC does not depend on the available information at time t, the expected growth rate is deterministic and Eq. (4.11) follows.

B.2.3 Derivation of Eq. (4.14)

Converting the expected market value in FC of period t at the expected spot exchange rate of period t and expanding with $\mathbb{E}[\widetilde{S}_{t+1}]$ yields

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},S^*}]=\mathbb{E}[\widetilde{V}_t^{u,\mathrm{FC}}]\cdot\mathbb{E}[\widetilde{S}_t]=\frac{\left(\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{FC}}]+\mathbb{E}[\widetilde{V}_{t+1}^{u,\mathrm{FC}}]\right)\cdot\mathbb{E}[\widetilde{S}_{t+1}]}{(1+\rho_{t+1}^{u,\mathrm{FC}})\cdot(1+\mathbb{E}[\tilde{\delta}_{t+1}^{S}])}\,.$$

Note, that we used Eq. (4.45) in the last step. By defining

$$\rho_t^{u,\mathrm{HC},S^*} = (1+\rho_t^{u,\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) - 1 \ ,$$

we obtain

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},S^*}] = \frac{\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{HC},S^*}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\mathrm{HC},S^*}]}{(1+\rho_{t+1}^{u,\mathrm{HC},S^*})} \ .$$

B.2.4 Derivation of Eq. (4.20)

Converting the market value in FC of period t at the forward exchange rate F_t and expanding it with F_{t+1} yields the valuation equation at time t, that is,

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},F^*}] = \mathbb{E}[\widetilde{V}_t^{u,\mathrm{FC}}] \cdot F_t = \frac{\left(\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{FC}}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\mathrm{FC}}]\right) \cdot F_{t+1}}{(1+\rho_{t+1}^{u,\mathrm{FC}}) \cdot (1+\delta_{t+1}^F)} \ ,$$

where

$$\delta_{t+1}^F = \frac{F_{t+1}}{F_t} - 1 = \frac{1+i_{t+1}^{\mathrm{HC}}}{1+i_{t+1}^{\mathrm{FC}}} - 1 \ .$$

By defining

$$\rho_t^{u,\mathrm{HC},F^*} = (1+\rho_t^{u,\mathrm{FC}}) \cdot (1+\delta_t^F) - 1 = (1+\rho_t^{u,\mathrm{FC}}) \cdot \frac{1+i_t^{\mathrm{HC}}}{1+i_t^{\mathrm{FC}}} \ ,$$

we obtain

$$\mathbb{E}[\widetilde{V}_t^{u,\mathrm{HC},F^*}] = \frac{\mathbb{E}[\widetilde{FCF}_{t+1}^{\mathrm{HC},F^*}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\mathrm{HC},F^*}]}{(1+\rho_{t+1}^{u,\mathrm{HC},F^*})} \ .$$

B.3 Derivation of a valuation equation for the HC-valuation approach with forward exchange rates

To obtain a valuation equation for the HC-valuation approach with forward exchange rates, we need to derive the corresponding cost of capital to discount cash flows that are derived according to Eq. (4.17). To do so, we compare this approach to the market value derived by the FC-valuation approach and multiplied by the spot exchange rate. From the view of period t, we obtain

$$\widetilde{V}_t^{u,\mathrm{FC}} \cdot \widetilde{S}_t \stackrel{!}{=} \frac{\mathbb{E}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}] \cdot \widetilde{F}_{t,t+1} + \mathrm{Cov}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}, \widetilde{S}_{t+1}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{HC},F}]}{1+\rho_{t+1}^{u,\mathrm{HC},F}} , \tag{4.48}$$

with

$$\mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{HC},F}] = \mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}}] \cdot \widetilde{F}_{t,t+1} + \mathrm{Cov}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}}, \widetilde{S}_{t+1}] \ .$$

Solving for $1+\rho_{t+1}^{u,\mathrm{HC},F}$ yields

$$\begin{aligned}
1+\rho_{t,t+1}^{u,\mathrm{HC},F} &= \frac{\left(\mathbb{E}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}}]\right) \cdot \widetilde{F}_{t,t+1}}{\widetilde{V}_t^{u,\mathrm{FC}} \cdot \widetilde{S}_t} + \frac{\mathrm{Cov}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}, \widetilde{S}_{t+1}] + \mathrm{Cov}_t[\widetilde{V}_{t+1}^{u,\mathrm{FC}}, \widetilde{S}_{t+1}]}{\widetilde{V}_t^{u,\mathrm{FC}} \cdot \widetilde{S}_t} \\
&= (1+\rho_{t+1}^{u,\mathrm{FC}}) \cdot \frac{\widetilde{F}_{t,t+1}}{\widetilde{S}_t} + \mathrm{Cov}_t\left[\frac{\widetilde{FCF}_{t+1}^{\mathrm{FC}} + \widetilde{V}_{t+1}^{u,\mathrm{FC}}}{\widetilde{V}_t^{u,\mathrm{FC}}}, \frac{\widetilde{S}_{t+1}}{\widetilde{S}_t}\right] \\
&= (1+\rho_{t+1}^{u,\mathrm{FC}}) \cdot \frac{\widetilde{F}_{t,t+1}}{\widetilde{S}_t} + \mathrm{Cov}[\tilde{r}_{t+1}^{u,\mathrm{FC}}, \tilde{\delta}_{t+1}^{S}] \ ,
\end{aligned}$$

where we used Eq. (4.47) in the second step. Due to our assumptions of the multiperiod GCAPM, the covariance does not depend on the available information at time t. By applying the arbitrage condition, see Eq. (4.2), we obtain

$$\frac{\widetilde{F}_{t,t+1}}{\widetilde{S}_t} = \frac{\widetilde{S}_t}{\widetilde{S}_t} \cdot \frac{1+i_t^{\mathrm{HC}}}{1+i_t^{\mathrm{FC}}} = \frac{F_{t+1}}{F_t} \ ,$$

and conclude

$$\rho_{t+1}^{u,\mathrm{HC},F} = (1+\rho_{t+1}^{u,\mathrm{FC}}) \cdot \frac{F_{t+1}}{F_t} + \mathrm{Cov}\left[\tilde{r}_{t+1}^{u,FC}, \tilde{\delta}_{t+1}^{S}\right] - 1 \ .$$

Note that we use the uncertain forward exchange rate $\widetilde{F}_{t,t+1}$ for the conversion of the cash flow. Note also that we equate the computation of the market value using this approach with forward exchange rates to $\widetilde{V}_t^{u,\mathrm{FC}} \cdot \widetilde{S}_t$, see Eq. (4.48). Consequently, by using $\rho^{u,\mathrm{HC},F}$ as discount rate, we obtain a cash flow that is based on spot exchange rates. Thus, the resulting cash flow has to be discounted using $\rho^{u,\mathrm{HC},S}$.

B.4 Active debt management according to Harris and Pringle

Under the assumption of active debt management according to Harris and Pringle, debt levels are adjusted continuously. It follows that all tax shields are uncertain. The corresponding adjustment formulas for active debt management according to Harris and Pringle can be derived similarly as for active debt management according to Miles and Ezzel, see Appendix B.6. Compared to these adjustment formulas, see Eqs. (4.34) and (4.35), the term $\left(1 - \frac{\tau^{\text{FC}} \cdot i_t^{\text{FC}}}{1+i_t^{\text{FC}}}\right)$ is omitted since all tax shield are uncertain. Otherwise, the adjustment formulas remain the same. We obtain

$$\begin{aligned}
\rho_t^{\ell,\text{FC}} &= \rho_t^{u,\text{FC}} + (\rho_t^{u,\text{FC}} - i_t^{\text{FC}}) \cdot L_{t-1}^{\text{FC}} \\
\rho_t^{\ell,\text{HC},j} &= \rho_t^{u,\text{HC},j} + \left(\rho_t^{u,\text{HC},j} - \eta_t^{\text{HC},S}\right) \cdot L_{t-1}^{\text{FC}} \\
\rho_t^{\ell,\text{HC},F^*} &= \rho_t^{u,\text{HC},F^*} + (\rho_t^{u,\text{HC},F^*} - i_t^{\text{HC}}) \cdot L_{t-1}^{\text{FC}} \ ,
\end{aligned}$$

for $j \in \{S, S^*\}$. The FCF and the FtE method follow accordingly.

B.5 The APV method for active debt management

B.5.1 FC-valuation approach

The tax shield of some period t is calculated by multiplying the corporate tax rate in FC, τ^{FC}, by the interest expenses, which are composed of the risk-free interest rate times the amount of debt of the preceding period. For the expected tax shield of period t follows

$$\mathbb{E}[\widetilde{TS}_t^{\text{FC}}] = \tau^{\text{FC}} \cdot i_t^{\text{FC}} \cdot \theta_{t-1} \cdot \mathbb{E}[\widetilde{V}_{t-1}^{\ell,\text{FC}}] \ . \tag{4.49}$$

A recursive valuation equation for the expected value of the tax shields of period t in FC, $\widetilde{V}_t^{TS,\text{FC}}$, is given by

$$\mathbb{E}[\widetilde{V}_t^{TS,\text{FC}}] = \frac{\mathbb{E}[\widetilde{TS}_{t+1}^{\text{FC}}]}{1 + i_{t+1}^{\text{FC}}} + \frac{\mathbb{E}[\widetilde{V}_{t+1}^{TS,\text{FC}}]}{1 + \rho_{t+1}^{u,\text{FC}}} \ . \tag{4.50}$$

Since the debt level of period $t+1$ is already known in period t, the tax shield of period $t+1$ is certain and can be discounted at the risk-free interest rate in FC. The value of the tax shield of period $t+1$ consists of all future tax shields. In period t, it is uncertain since future debt levels are uncertain. Thus, it is discounted at the unlevered cost of equity (see, for example, Miles and Ezzell 1980, 1985; Kruschwitz and Löffler 2020, pp. 105–106).

To obtain the expected equity value, $\mathbb{E}[\widetilde{E}_t^{\ell,\text{FC}}]$, of period t, the debt level has to be subtracted from the total firm value

$$\mathbb{E}[\widetilde{E}_t^{\ell,\text{FC}}] = \mathbb{E}[\widetilde{V}_t^{u,\text{FC}}] + \mathbb{E}[\widetilde{V}_t^{TS,\text{FC}}] - \mathbb{E}[\widetilde{D}_t^{\text{FC}}] \ .$$

At the valuation date, to obtain the equity value in HC, we multiply the equity value in FC at the current spot exchange rate and obtain $E_0^{\ell,\text{HC}} = E_0^{\ell,\text{FC}} \cdot S_0$.

B.5.2 HC-valuation approaches

The tax shield in FC, see Eq. (4.49), can be converted to HC using spot or forward exchange rates. For the expected tax shield of some period t conditioned on information available at period $t-1$, we obtain using spot exchange rates

$$\mathbb{E}_{t-1}[\widetilde{TS}_t^{\mathrm{HC},S}] = \mathbb{E}_{t-1}[\widetilde{TS}_t^{\mathrm{FC}} \cdot \widetilde{S}_t] = \widetilde{TS}_t^{\mathrm{FC}} \cdot \mathbb{E}_{t-1}[\widetilde{S}_t] \ .$$

Note that the tax shield in FC of period t is certain from the view of period $t-1$. It follows that the covariance of the tax shield and the spot exchange rate of period t, conditioned on all information of period $t-1$, amounts to zero. However, if the expectation is computed at the valuation date, the tax shield of period t is not certain such that the covariance has to be included. We obtain

$$\mathbb{E}[\widetilde{TS}_t^{\mathrm{HC},S}] = \mathbb{E}[\widetilde{TS}_t^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] + \mathrm{Cov}[\widetilde{TS}_t^{\mathrm{HC},S}, \widetilde{S}_t] \ .$$

To derive a valuation equation for the value of the tax shield, we assume first that the firm is liquidated after T periods. For the value of the tax shield of period $T-1$ follows

$$\widetilde{V}_{T-1}^{TS,\mathrm{HC},S} = \widetilde{V}_{T-1}^{TS,\mathrm{FC}} \cdot \widetilde{S}_{T-1} = \frac{\widetilde{TS}_T^{\mathrm{FC}}}{1+i_T^{\mathrm{FC}}} \cdot \widetilde{S}_{T-1} \ .$$

Expanding with $\mathbb{E}_{T-1}[\widetilde{S}_T]$ yields

$$\widetilde{V}_{T-1}^{TS,\mathrm{HC},S} = \frac{\mathbb{E}_{T-1}[\widetilde{TS}_T^{\mathrm{HC},S}]}{1+\eta_T^{\mathrm{HC},S}} \ ,$$

where

$$\eta_T^{\mathrm{HC},S} = (1+i_T^{\mathrm{FC}}) \cdot \frac{\mathbb{E}_{T-1}[\widetilde{S}_T]}{\widetilde{S}_{T-1}} - 1 = (1+i_T^{\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_T^S]) - 1$$

is deterministic at the valuation date. Thus, the tax shield of period T is converted at the expected spot exchange rate of period T. The converted tax shield in HC is not deterministic but contains exchange rate risk, which is considered in the discount rate $\eta^{\mathrm{HC},S}$.

For the expected value of the tax shield of period $T-2$ follows

$$\widetilde{V}_{T-2}^{TS,\mathrm{HC},S} = \frac{\mathbb{E}_{T-2}[\widetilde{TS}_{T-1}^{\mathrm{HC},S}]}{1+\eta_{T-1}^{\mathrm{HC},S}} + \frac{\mathbb{E}_{T-2}[\widetilde{V}_{T-1}^{TS,\mathrm{HC},S}]}{1+\rho_{T-1}^{u,\mathrm{HC},S}} \ ,$$

where $\eta_{T-1}^{\mathrm{HC},S} = (1+i_{T-1}^{\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_{T-1}^{S}]) - 1$. In addition to the value of the tax shield that results from the debt level of period $T-2$, we have to add and discount the value of the tax shield of period $T-1$. From Section 4.3, we already know that $\rho^{u,\mathrm{HC},S}$ is the correct cost of capital to discount an uncertain quantity that is converted with spot exchange rates. From this, we can conclude the expected value of the tax shield at some period t for an infinite planning horizon as

$$\mathbb{E}[\widetilde{V}_{t}^{TS,\mathrm{HC},S}] = \frac{\mathbb{E}[\widetilde{TS}_{t+1}^{\mathrm{HC},S}]}{1+\eta_{t+1}^{\mathrm{HC},S}} + \frac{\mathbb{E}[\widetilde{V}_{t+1}^{TS,\mathrm{HC},S}]}{1+\rho_{t+1}^{u,\mathrm{HC},S}} \ , \tag{4.51}$$

where

$$\eta_{t+1}^{\mathrm{HC},S} = (1+i_{t+1}^{\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_{t+1}^{S}]) - 1 \ .$$

The tax shield of period $t+1$ is not certain from the view of period t but contains exchange rate risk. Thus, we cannot discount it at the risk-free interest rate but use the adjusted discount rate $\eta^{\mathrm{HC},S}$, where the risk-free interest rate in FC is multiplied by the expected change in spot exchange rates. Since the risk-free interest rate is deterministic, a covariance is not considered.

For the modified HC-valuation approaches, the modified expected tax shield amounts to

$$\begin{aligned}
\mathbb{E}[\widetilde{TS}_{t}^{\mathrm{HC},S^*}] &= \mathbb{E}[\widetilde{TS}_{t}^{\mathrm{FC}}] \cdot \mathbb{E}[\widetilde{S}_t] \\
\mathbb{E}[\widetilde{TS}_{t}^{\mathrm{HC},F^*}] &= \mathbb{E}[\widetilde{TS}_{t}^{\mathrm{FC}}] \cdot F_t \ .
\end{aligned}$$

The modified expected value of the tax shield follows from

$$\mathbb{E}[\widetilde{V}_{t}^{TS,\mathrm{HC},S^*}] = \frac{\mathbb{E}[\widetilde{TS}_{t+1}^{\mathrm{HC},S^*}]}{1+\eta_{t+1}^{\mathrm{HC},S}} + \frac{\mathbb{E}[\widetilde{V}_{t+1}^{TS,\mathrm{HC},S^*}]}{1+\rho_{t+1}^{u,\mathrm{HC},S^*}} \tag{4.52}$$

$$\mathbb{E}[\widetilde{V}_{t}^{TS,\mathrm{HC},F^*}] = \frac{\mathbb{E}[\widetilde{TS}_{t+1}^{\mathrm{HC},F}]}{1+i_{t+1}^{\mathrm{HC}}} + \frac{\mathbb{E}[\widetilde{V}_{t+1}^{TS,\mathrm{HC},F^*}]}{1+\rho_{t+1}^{u,\mathrm{HC},F^*}} \ . \tag{4.53}$$

Since future forward exchange rates are deterministic, the modified tax shield of period $t+1$, see Eq. (4.53), does not contain exchange rate risk and is certain from the view of period t. Thus, it is discounted at the risk-free interest rate in HC. In the second term of Eqs. (4.52) and

(4.53), the value of the tax shield of period $t+1$ contains financial risk. Depending on the chosen approach, it is discounted at the corresponding unlevered cost of equity. As shown at the end of Section 4.3.7, note that future market values of the tax shield that are computed using different HC-valuation approaches do not coincide. At the valuation date, all approaches yield the same result.

B.6 Derivation of the levered cost of equity and the WACC for active debt management

B.6.1 Derivation of a link between the levered cost of equity in HC and the levered cost of equity in FC

To derive a link between the levered costs of equity in HC, $\rho^{\ell,\mathrm{HC},S}$, and the levered cost of equity in FC, $\rho^{\ell,\mathrm{FC}}$, we compare valuation equation (4.38) to Eq. (4.26) for $j=S$. From the view of period t, we obtain

$$\widetilde{E}_t^{\ell,\mathrm{FC}} \cdot \widetilde{S}_t \stackrel{!}{=} \frac{\mathbb{E}_t[\widetilde{FtE}_{t+1}^{\mathrm{HC},S}] + \mathbb{E}_t[\widetilde{E}_{t+1}^{\ell,\mathrm{HC},S}]}{1+\rho_{t+1}^{\ell,\mathrm{HC},S}} \ .$$

From rearranging follows

$$\begin{aligned}
1+\rho_{t+1}^{\ell,\mathrm{HC},S} &= \frac{\mathbb{E}_t[\widetilde{FtE}_{t+1}^{\mathrm{HC},S}] + \mathbb{E}_t[\widetilde{E}_{t+1}^{\ell,\mathrm{HC},S}]}{\widetilde{E}_t^{\ell,\mathrm{FC}} \cdot \widetilde{S}_t} \\
&= \frac{\left(\mathbb{E}_t[\widetilde{FtE}_{t+1}^{\mathrm{FC}}] + \mathbb{E}_t[\widetilde{E}_{t+1}^{\ell,\mathrm{FC}}]\right) \cdot \mathbb{E}_t[\widetilde{S}_{t+1}]}{\left(\mathbb{E}_t[\widetilde{FtE}_{t+1}^{\mathrm{FC}}] + \mathbb{E}_t[\widetilde{E}_{t+1}^{\ell,\mathrm{FC}}]\right) \cdot \widetilde{S}_t} \cdot (1+\rho_{t+1}^{\ell,\mathrm{FC}}) + \frac{\mathrm{Cov}_t[\widetilde{FtE}_{t+1}^{\mathrm{FC}} + \widetilde{E}_{t+1}^{\ell,\mathrm{FC}}, \widetilde{S}_{t+1}]}{\widetilde{E}_t^{\ell,\mathrm{FC}} \cdot \widetilde{S}_t} \\
&= (1+\rho_{t+1}^{\ell,\mathrm{FC}}) \cdot (1+\mathbb{E}_t[\tilde{\delta}_{t+1}^S]) + \mathrm{Cov}_t[\tilde{r}_t^{\ell,\mathrm{FC}}, \tilde{\delta}_{t+1}^S] \ .
\end{aligned}$$

In Section 4.2, we describe that the expectation of the change in spot exchange rates does not depend on the information available at time t. Note that in the case of active debt management, the distribution of the return of the levered firm at some time t also does not depend on the occurred states before time t since financial risk is constant in every state of this period. It follows that the covariance does not depend on the available information at time t, such that the levered cost of equity $\rho_{t+1}^{\ell,\mathrm{HC},S}$ is deterministic.

The relations for the modified HC-valuation approaches follow similarly. Overall, we obtain

$$\rho_t^{\ell,\mathrm{HC},S} = (1+\rho_t^{\ell,\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) + \mathrm{Cov}[\tilde{r}_t^{\ell,\mathrm{FC}}, \tilde{\delta}_t^S] - 1 \tag{4.54}$$

$$\rho_t^{\ell,\mathrm{HC},S^*} = (1+\rho_t^{\ell,\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) - 1 \tag{4.55}$$

$$\rho_t^{\ell,\mathrm{HC},F^*} = (1+\rho_t^{\ell,\mathrm{FC}}) \cdot (1+\delta_t^F) - 1 \; . \tag{4.56}$$

B.6.2 Derivation of Eq. (4.28)

To derive a link between the WACC in HC $\rho^{\ell,\mathrm{HC},S}$ and the WACC in FC $\rho^{\ell,\mathrm{FC}}$, we apply the same pattern as in Appendix B.6.1. We compare Eq. (4.27), for $j = S$, to the FC-valuation approach, see Eq. (4.25). From the view of period t, we obtain

$$\widetilde{V}_t^{\ell,\mathrm{FC}} \cdot \widetilde{S}_t \overset{!}{=} \frac{\mathbb{E}_t[\widetilde{FCF}_{t+1}^{\mathrm{HC},S}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{\ell,\mathrm{HC},S}]}{1+\rho_{t+1}^{\tau,\mathrm{HC},S}} \; .$$

From rearranging follows

$$\begin{aligned}
1+\rho_{t+1}^{\tau,\mathrm{HC},S} &= \frac{\mathbb{E}_t[\widetilde{FCF}_{t+1}^{\mathrm{HC},S}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{\ell,\mathrm{HC},S}]}{\widetilde{V}_t^{\ell,\mathrm{FC}} \cdot \widetilde{S}_t} \\
&= \frac{\left(\mathbb{E}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{\ell,\mathrm{FC}}]\right) \cdot \mathbb{E}_t[\widetilde{S}_{t+1}]}{\left(\mathbb{E}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{\ell,\mathrm{FC}}]\right) \cdot \widetilde{S}_t} \cdot (1+\rho_{t+1}^{\tau,\mathrm{FC}}) + \frac{\mathrm{Cov}_t[\widetilde{FCF}_{t+1}^{\mathrm{FC}} + \widetilde{V}_{t+1}^{\ell,\mathrm{FC}}, \widetilde{S}_{t+1}]}{\widetilde{V}_t^{\ell,\mathrm{FC}} \cdot \widetilde{S}_t} \\
&= (1+\rho_{t+1}^{\tau,\mathrm{FC}}) \cdot (1+\mathbb{E}_t[\tilde{\delta}_{t+1}^S]) + (1-\theta_t) \cdot \mathrm{Cov}_t[\tilde{r}_t^{\ell,\mathrm{FC}}, \tilde{\delta}_{t+1}^S] \; .
\end{aligned}$$

Note that in the last step we used, that the risk-free interest rate is deterministic, such that its covariance amounts to zero.

B.6.3 Derivation of Eq. (4.31)

Inserting the formula for the WACC in FC, see Eq. (4.23) into Eq. (4.28) yields

$$\begin{aligned}
\rho_t^{\tau,\mathrm{HC},S} &= (1+\rho_t^{\ell,\mathrm{FC}} \cdot (1-\theta_{t-1}) + i_t^{\mathrm{FC}} \cdot (1-\tau^{\mathrm{FC}}) \cdot \theta_{t-1}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) \\
&\quad + (1-\theta_{t-1}) \cdot \mathrm{Cov}[\tilde{r}_t^{\ell,\mathrm{FC}}, \tilde{\delta}_t^S] - 1 \\
&= \left((1+\rho_t^{\ell,\mathrm{FC}}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) + \mathrm{Cov}[\tilde{r}_t^{\ell,\mathrm{FC}}, \tilde{\delta}_t^S]\right) \cdot (1-\theta_{t-1}) \\
&\quad + \theta_{t-1} \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) + (i_t^{\mathrm{FC}} \cdot (1-\tau^{\mathrm{FC}}) \cdot \theta_{t-1}) \cdot (1+\mathbb{E}[\tilde{\delta}_t^S]) - 1 \; .
\end{aligned}$$

By inserting Eq. (4.54), we obtain

$$
\begin{aligned}
\rho^{\tau,\mathrm{HC},S} &= (1+\rho^{\ell,\mathrm{HC},S})\cdot(1-\theta_{t-1}) + (1+i_t^{\mathrm{FC}}\cdot(1-\tau^{\mathrm{FC}})\cdot\theta_{t-1})\cdot(1+\mathbb{E}[\tilde{\delta}_t^S]) - 1\\
&= \rho^{\ell,\mathrm{HC},S}\cdot(1-\theta_{t-1}) - \theta_{t-1} + (1+i_t^{\mathrm{FC}}\cdot(1-\tau^{\mathrm{FC}}))\cdot(1+\mathbb{E}[\tilde{\delta}_t^S])\cdot\theta_{t-1}\\
&= \rho^{\ell,\mathrm{HC},S}\cdot(1-\theta_{t-1}) + \Big((1+i_t^{\mathrm{FC}}\cdot(1-\tau^{\mathrm{FC}}))\cdot(1+\mathbb{E}[\tilde{\delta}_t^S]) - 1\Big)\cdot\theta_{t-1}\ .
\end{aligned}
$$

Defining $\eta^{\tau,\mathrm{HC},S}$ as in Eq. (4.32) yields Eq. (4.31) for $j=S$. The modified WACCs $\rho^{\tau,\mathrm{HC},S^*}$ and $\rho^{\tau,\mathrm{HC},F^*}$ follow analogously.

B.6.4 Derivation of Eq. (4.34)

in the following, we show a detailed computation of Eq. (4.34) for $j=S$. Note that for the return of the levered firm in FC, the same adjustment formula holds as for the levered cost of equity, that is,

$$
\tilde{r}_{t+1}^{\ell,\mathrm{FC}} = \tilde{r}_{t+1}^{u,\mathrm{FC}} + (\tilde{r}_{t+1}^{u,\mathrm{FC}} - i_{t+1}^{\mathrm{FC}})\cdot\left(1-\frac{\tau^{\mathrm{FC}}\cdot i_{t+1}^{\mathrm{FC}}}{1+i_{t+1}^{\mathrm{FC}}}\right)\cdot L_t^{\mathrm{FC}}\ .
$$

For the covariance of the return of the levered firm and the change in spot exchange rates follows

$$
\mathrm{Cov}[\tilde{r}_{t+1}^{\ell,\mathrm{FC}},\tilde{\delta}_{t+1}^S] = \mathrm{Cov}[\tilde{r}_{t+1}^{u,\mathrm{FC}},\tilde{\delta}_{t+1}^S] + \mathrm{Cov}[\tilde{r}_{t+1}^{u,\mathrm{FC}},\tilde{\delta}_{t+1}^S]\cdot\left(1-\frac{\tau^{\mathrm{FC}}\cdot i_{t+1}^{\mathrm{FC}}}{1+i_{t+1}^{\mathrm{FC}}}\right)\cdot L_t^{\mathrm{FC}}\ . \tag{4.57}
$$

By inserting the adjustment formula for $\rho^{\ell,\mathrm{FC}}$, see Eq. (4.24), into Eq. (4.54), we obtain

$$
\begin{aligned}
\rho_{t+1}^{\ell,\mathrm{HC},S} = &\left(1+\rho_{t+1}^{u,\mathrm{FC}} + (\rho_{t+1}^{u,\mathrm{FC}} - i_{t+1}^{\mathrm{FC}})\cdot\left(1-\frac{\tau^{\mathrm{FC}}\cdot i_{t+1}^{\mathrm{FC}}}{1+i_{t+1}^{\mathrm{FC}}}\right)\cdot L_t^{\mathrm{FC}}\right)\cdot(1+\mathbb{E}[\tilde{\delta}_{t+1}^S])\\
&+ \mathrm{Cov}[\tilde{r}_t^{\ell,\mathrm{FC}},\tilde{\delta}_{t+1}^S] - 1\ .
\end{aligned}
$$

Inserting Eq. (4.57) and applying the formula for the link between $\rho^{u,\mathrm{HC},S}$ and $\rho^{u,\mathrm{FC}}$, see Eq. (4.10), yields Eq. (4.34) for $j=S$, that is,

$$
\rho_{t+1}^{\ell,\mathrm{HC},S} = \rho_{t+1}^{u,\mathrm{HC},S} + \left(\rho_{t+1}^{u,\mathrm{HC},S} - \eta_{t+1}^{\mathrm{HC},S}\right)\cdot\left(1-\frac{\tau^{\mathrm{FC}}\cdot i_{t+1}^{\mathrm{FC}}}{1+i_{t+1}^{\mathrm{FC}}}\right)\cdot L_t^{\mathrm{FC}}\ .
$$

Eq. (4.34) for $j=S^*$, and Eq. (4.35) can be obtained similarly.

5 Conclusion

5.1 Summary and implications

Corporate valuation with discounted cash flow (DCF) methods requires assumptions about a firm's financing strategy. Since interest on debt is deductible from taxable income, the financing strategy has an immediate influence on the market value of the firm. In corporate valuation practice, usually active or passive debt management is applied. However, considering the variety of theoretical and empirical findings on the capital structure behavior of firms, it becomes obvious that these pure financing strategies cannot accurately display the real financing behavior of firms. This led to the development of mixed financing strategies such as discontinuous financing. This is a useful advancement but the application and consequences for the market value had been unclear. Furthermore, in the highly relevant valuation of cross-border investments, many challenges occur that had not been sufficiently addressed in the literature. Among other things, it was ambiguous how to consistently integrate different currencies with the associated risks and debt financing into the valuation equation. Therefore, in a first step, the aim of this thesis was to analyze mixed financing strategies and make them more accessible for corporate valuation practice. In a second step, problems of cross-border DCF valuation were addressed and corresponding solutions were presented.

The first study, *Valuation with mixed financing strategies* investigates mixed financing strategies in a two-phase model. By analyzing hybrid financing, we show how passive and active debt management can be implemented into such a two-phase model. We refine this model by combining passive debt management in the explicit forecast phase and discontinuous financing in the steady-state phase. Thereby, we also present a more intuitive valuation approach for discontinuous financing, making it more accessible for corporate valuation practice. We transfer the approach of L- and D-hybrid financing resulting in L- and D-discontinuous financing. In the case of an L-financing strategy, the leverage ratio of the steady state is deterministically defined at the valuation date. This can require a substantial refinancing at the end of the explicit forecast phase. This is not necessary for a D-financing strategy, where the debt level at the beginning of the steady-state phase is deterministic. However, as a consequence, the leverage ratio of the steady-state phase is stochastic at the valuation date. Depending on whether or not such a refinancing activity seems plausible for a firm, the financing strategy should be chosen accordingly.

To investigate the differences in market values that result from different financing assumptions, we conduct a theoretical comparison, which we corroborate by simulations. We compute the deviation of market values when a firm's financing behavior corresponds to one of the presented mixed financing strategies but a pure financing strategy is assumed in the valuation. Especially when a firm has a high leverage ratio, the deviations can amount to more than 10%. The discrepancies are smaller if active debt management instead of passive debt management is used. Overall, the study contributes to the ongoing theoretical research on mixed financing strategies but also supports corporate valuation practice in choosing the best financing strategy to approach a firm's real financing behavior.

The second study, *Terminal value calculation with discontinuous financing and debt categories*, continues the research of the first study but focuses on the steady-state phase. Characteristics of discontinuous financing in the steady-state phase are analyzed. In particular, we show that the levered cost of equity is period-specific, and the assumption of discontinuous financing results in an inconstant financing risk. We suggest a solution by introducing debt categories which is based on similar assumptions as discontinuous financing. Furthermore, we illustrate both financing strategies with an example and compare them to passive and active debt management. Thereby, we show that when comparing standard discontinuous financing and debt categories, deviations are very small. The deviations become larger when one of those mixed financing strategies is compared to active debt management. Significant deviations emerge when standard discontinuous financing or debt categories is compared to passive debt management. Overall, the study contributes to the research on discontinuous financing by clarifying the application in a steady-state phase and solving occurring problems by introducing debt categories.

Altogether, the first and the second study provide guidance for corporate valuation practice in choosing a suitable financing strategy. A two-phase model, that is usually used for the planning of cash flows, should also be used for the planning of debt levels. Whereas passive debt management is a valid choice for the explicit forecast phase, it should not be assumed in the steady-state phase. There, it is more plausible that a firm chooses its debt levels based on its development. A careful analysis should clarify whether active debt management or a mixed financing strategy is an appropriate assumption for the steady-state phase. If discontinuous financing is assumed, it can be interpreted as an approximation for debt categories. For the transition between those phases, one has to choose between L- and D-financing. By illustrating market value deviations between different financing strategies, the importance of these choices are highlighted.

The third study, *Cross-border discounted cash flow valuation* addresses fundamental problems of international valuation. Compared to the specific problems of debt financing, that are considered in the other studies, the entire DCF calculation is examined in order to derive a consistent cross-border valuation framework. In particular, conditions for the application of the GCAPM are presented, from which deterministic costs of capital are derived. After formulating a consistent model for the valuation of an unlevered firm with valuation approaches in FC and HC, the results are generalized to the valuation of a levered firm under the assumption of passive and active debt management. Thereby, we establish a framework in which all valuation approaches yield the same result at the valuation date. Furthermore, advantages and disadvantages of the different approaches are discussed.

We outline that for the valuation of multinational companies, the HC-valuation approach with spot exchange rates should be applied since the corresponding cost of capital is based on the domestic capital market. For the other approaches, all foreign capital markets would have to be analyzed to derive costs of capital. However, for the HC-valuation approach with spot exchange rates, correlations between exchange rate risk and business risk have to be estimated. A neglection of covariances in the conversion of cash flows, that are discounted at a cost of capital that is based on the domestic capital market, can yield deviating valuation results. Overall, this study provides a sound basis for the application of DCF methods in a cross-border valuation.

Since even the implementation of pure passive and active debt management has not been sufficiently analyzed before, mixed financing strategies have not been applied in an international context. However, the findings on mixed financing strategies of the first and the second study remain valid in a cross-border setting: For the planning of debt levels, a two-phase model should be applied, for which the financing strategies should be chosen carefully. To implement valuation equations with mixed financing strategies, the derived adjustment formulas for the levered cost of equity from the second study can be used and, based on our results in the third study, transferred to an international setting.

5.2 Limitations and outlook

This thesis and the studies therein are subject to limitations, which mainly originate from modeling assumptions. It is often not possible or also not fruitful to incorporate too many different aspects into one model. Instead, the presented models concentrate on certain aspects to highlight the direct implications.

First, in all studies, debt is assumed to be risk-free. This restrictive assumption is generally not valid for corporate valuation practice but helps to illustrate the consequences of different assumptions on debt financing. This limitation can be easily relaxed by applying the cost of debt instead of the risk-free interest rate. Such a procedure is common in corporate valuation practice but still constitutes a simplification since it does not take the costs of financial distress correctly into account. In the literature, there are a vast variety of findings on the integration of insolvency risk and the costs of financial distress (see, for example, Kruschwitz et al. 2005; Almeida and Philippon 2008; Friedrich 2015, 2016; Krause and Lahmann 2016; Lahmann et al. 2018). We give an overview of approaches to integrate the risk of default for discontinuous financing and debt categories in the second study. However, there does not exist a widely accepted model that is applied in corporate valuation practice.

Second, in the literature, there exist other mixed financing strategies that are not explicitly analyzed in this thesis. A short overview is given in the introduction. Further mixed financing strategies can be obtained, for example, by combining active and passive debt management differently, or by applying a three-phase model (Koller et al. 2020, p. 289) that has an additional phase to smoothen the transition from the explicit forecast phase to the steady-state phase. Following up on this, the definition of a steady-state phase can be examined in more detail. The assumption that the expectation of all input parameters grows at a constant growth rate is widely accepted but this is an ideal situation that is unlikely to be achieved. In particular, a firm often has multiple business units with a different steady state development (Dierkes and Schäfer 2021). Furthermore, the expected inflation has to be taken into account correctly (Bradley and Jarrell 2008). Holland (2018) examines an improved method for the estimation of terminal value for mature companies. Schwetzler (2018) analyzes assumptions of the growth rate in a steady state in connection with a given technology. The definition of a steady state of a multinational company is even more challenging since assumptions regarding exchange rates and subsidiaries in different countries have to be made.

Third, we have analyzed a simple setting of a cross-border valuation and have excluded problems like legal requirements, international taxation and country risks. Especially the consideration of country risk premiums displays an ongoing discussion (see, for example, Krapl and O'Brien 2016). For corporate valuation practice, it is easy to include a risk premium in the cost of capital (see, for example, Damodaran 2003). An overview of existing approaches to account for political risk with a critical assessment and a presentation of a new approach can be found in Bekaert et al. (2016). However, from a theoretical point of view the integration of a country risk premium in the cost of capital is difficult to justify (for criticism on Damodaran's country risk premium, see Kruschwitz et al. 2011). Consequently, from a theoretical point of view, the country risk premium should rather be included in the planning of cash flows.

Fourth, it is possible to account for a more detailed planning of debt levels in multiple currencies. As described above, the presented results on mixed financing strategies can be transferred to a cross-border setting but future research could also analyze characteristics of the capital structure of multinational companies. Then, our results may serve as a basis for the integration of the resulting financing strategies.

This dissertation has expanded the research on mixed financing strategies and addressed challenges in cross-border valuations, but some further questions remain unanswered. From a theoretical perspective, future research should examine analytically the consideration of insolvency risk and the costs of financial distress. In addition, an analysis of practical and realistic assumptions regarding the steady-state phase of national and international companies may be beneficial. Moreover, a model to estimate covariances in a cross-border valuation is needed. Furthermore, a theoretically sound model for an integration of country risks that is relevant for corporate valuation practice should be addressed. From an empirical perspective, future research should focus on the capital structure of multinational companies and the significance of country risk premiums. Moreover, simulations should be used further to illustrate deviations of market values that are computed under inconsistent assumptions in a cross-border valuation.

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Statement of contribution
to each study of the cumulative dissertation

To the three studies of the cumulative dissertation, I personally contributed as follows:

1. *Valuation with mixed financing strategies* co-authored by Stefan Dierkes:
   I contributed to the development of the concept of this study. I was responsible for the literature research, many analytical calculations, and for most of the writing.

2. *Terminal value calculation with discontinuous financing and debt categories* co-authored by Stefan Dierkes:
   I contributed to the development of the concept of this study. I was solely responsible for the literature research, analytical calculations, the implementation of examples, and for most of the writing.

3. *Cross-border discounted cash flow valuation* co-authored by Stefan Dierkes:
   I contributed to the development of the concept of this study. I was solely responsible for the literature research, most analytical calculations, and for the writing.

Göttingen, 17.05.2023

Place, Date

Imke de Maeyer

Ph.D. program in Economics

Declaration for admission to the doctoral examination

I confirm

1. that the dissertation "Corporate valuation with mixed financing strategies and cross-border relations" that I submitted was produced independently without assistance from external parties, and not contrary to high scientific standards and integrity,

2. that I have adhered to the examination regulations, including upholding a high degree of scientific integrity, which includes the strict and proper use of citations so that the inclusion of other ideas in the dissertation are clearly distinguished,

3. that in the process of completing this doctoral thesis, no intermediaries were compensated to assist me neither with the admissions or preparation processes, and in this process,

   - no remuneration or equivalent compensation were provided

   - no services were engaged that may contradict the purpose of producing a doctoral thesis

4. that I have not submitted this dissertation or parts of this dissertation elsewhere.

I am aware that false claims (and the discovery of those false claims now, and in the future) with regards to the declaration for admission to the doctoral examination can lead to the invalidation or revoking of the doctoral degree.

Göttingen, 17.05.2023

Place, Date

Imke de Maeyer