

**MARKET INTEGRATION
ANALYSIS AND TIME SERIES ECONOMETRICS-
CONCEPTUAL INSIGHTS FROM MARKOV-SWITCHING MODELS**

Doctoral Dissertation

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DEDICATIONS

To the

*Power (Holy Spirit) and Glory (Jesus Christ)
of the most high God*

&

*my
Grandma:
Maame Afua Abudwo*

ABSTRACT

The concept of market integration (MI) and inter-markets price dynamics in international trade, commodity markets and industrial organisation domains have directly been linked to market efficiency, competitiveness and various policy strategies. Consequently, measurement and testing issues in MI analysis have received considerable attention over the years. The broadness of the concept in particular has however resulted in introduction and development of diverging measurement techniques. Two major lines of MI assessment methods have emerged within agricultural markets studies; namely, price transmission econometrics- formulated within time series framework and the Parity Bound Model (PBM). The later is an arbitrage-based measure of inter-markets outcomes evaluated along spatial equilibrium conditions. Major advancements have been achieved in these methodological lines in their respective settings over the last decade. Thus, notwithstanding the fact that insights from the above two lines of market integration analysis raise important market policy, measurement and theoretical questions under specific inter-market conditions, they have not been combined effectively so far. Formulating a robust technique that comprehensively confronts market integration analysis (MIA) without seriously ignoring fundamental theoretical concepts and their implications has remained a challenge. While the time series characteristics of markets inter-relationships carry important policy and methodological implications, they impose analytical complexities when other crucial elements of market integration concept such as transactions cost, arbitrage and spatial equilibrium conditions are to be directly reflected.

In view of the above challenge, the study operationalised a working definition for MI as both *process and outcome* of inter-market relations manifested in an existence of *one price (in relation to cost of trade)*, price transmission and or physical flow of goods between the markets. In effect, various weaknesses and strengths of existing tools were theoretically explained in section three of the study. Notably, how the concept of tradability and time dynamics in arbitrage responses can lead to misleading conclusions under the PBM approach has been systematically explained and demonstrated by the study. In section three, the regime switching implications imposed by spatial equilibrium and arbitrage conditions were linked to the concept of multiple equilibria in time space.

To accommodate both inter-markets processes and outcomes, we have proposed Markov switching models as an alternative regime switching tool to both the PBM and current time series price transmission econometrics tools. Specifically, the proposed Markov switching model (MS-VEM) combines the basic threshold autoregressive structure from the PTE and arbitrage-based equilibrium conditions implied by the PBM. Based on the theoretical foundation built in section three, all arbitrage conditions are decomposed into their respective time path characteristics within the concept of rent irreversibility. Following the modelling technique of *arranged autoregression (usually applied in threshold models)*, we have shown that the complications imposed by transactions cost can be eliminated by sample splitting techniques. We have consequently, demonstrated in the thesis through a synthesised exercise that the flexibility of Markovian formulations allows them to handle both adjustments dynamics that underpin the PTE and the equilibrium conditions that drive the PBM.

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LIST OF ABBREVIATIONS

| | |
|----------------|---------------------------------------------------------------------|
| B-TAR | : Band threshold autoregressive process |
| ECM | : Error correction model |
| EMSM | : Endogenously selecting Markov-switching model |
| Eq-TAR | : Equilibrium threshold autoregressive process |
| ESTJ | : Enke-Sameulson-Takayama-Judge |
| HMM | : Hidden Markov model |
| LOP | : Law of one price |
| LR | : Likelihood ratio |
| MCMS | : Multi-chain Markov switching model |
| MCMS-EM | : Multi-chain Markov switching equilibrium model |
| MCMS-VEM | : Multi-chain Markov switching vector equilibrium model |
| MI | : Market integration |
| MIA | : Market integration analysis |
| MS-EM | : Markov-switching equilibrium model |
| MSAH(m)-AR(p) | : Markov-switching autoregressive heteroskedastic model |
| MSAH(m)-VAR(p) | : Markov-switching vector autoregressive heteroskedastic model |
| MSIH(m)-VAR(p) | : Markov-switching intercept heteroskedastic vector autoregressive |
| MSIH(m)-AR(p) | : Markov-switching Intercept heteroskedastic model |
| MSMH(m) -AR(p) | : Markov-switching mean heteroskedastic model |
| MSMH(m)-VAR(p) | : Markov-switching mean heteroskedastic vector autoregressive model |
| MS-VECM | : Markov-switching vector error correction model |
| MS-VEM | : Markov-switching vector equilibrium model |
| PBM | : Parity bound model |
| PTE | : Price transmission econometrics |
| Rd-TAR | : Returning drift threshold autoregressive process |
| SSSM | : State-space models with regime switching |
| TVECM | : Threshold vector error correction model |
| SETAR | : Self-exiting threshold autoregressive process |
| STAR | : Smooth transition model |
| TAR | : Threshold autoregressive model |
| TC | : Transactions costs |
| VECM | : Vector error correction model |

SECTION ONE

1.0 INTRODUCTION

1.1 Background and Motivation of Study

Economic efficiency and welfare issues have underpinned many market reforms and arguments for free market economic policies in many countries. In market economies, price mechanisms ensure that competitive forces within demand and supply relationships lead to *Pareto* optimal allocation of scarce resources. While perfect competition conditions are rarely met in practice, the efficiency of the price mechanism as a means of resource allocation under a broad range of realistic conditions is widely acknowledged (Brümmer et al. 2005).

On the basis of this, economists have and continue to study the functionality of markets to appropriately design, recommend and assess market policies through price transmission and market integration analysis. That is, the degree of markets inter-relationships determines the strength and effectiveness of price mechanism in resource allocation. Without integration of markets for instance, price signals will not be transmitted from supply deficit regions to surplus markets; prices will be more volatile; agricultural and food producers for instance will not specialise according to long-run comparative advantage, and gains from trade will not be realised (Baulch 1997). Hence, the importance of understanding price transmission and market integration mechanisms in market economies, especially the emerging and developing economies as a whole, cannot be overemphasised. This is due to crucial positions market liberalisation; parastatal reforms, trade and price policies among others occupy on their economic development agenda.

Market economists have developed a variety of empirical methods for studying price transmission and market integration to this effect (see Fackler and Goodwin 2001; Meyer and Cramon-Taubadel 2004 and Abdulai 2007 for recent review). These econometric techniques have grown rapidly from simple bivariate correlation analysis of price series to increasingly diverse and sophisticated econometric techniques. Notable improvements have been made,

especially in the time series domain, with the introduction of cointegration and error correction models (ECM) during the late 1980s and early 1990s. These innovations in particular distinguished non-spurious from spurious relationships between (commonly non-stationary) prices, and by providing deeper insights into the equilibrating dynamics, generally attributed to arbitrage, that underlie the former. The price transmission methods, as noted above, are fundamentally grounded on the neo-classical price theory in which arbitrage forces maintain market equilibrium. Consequently many market integration studies have used or use price series, since by the foregoing theoretical implications, any fairly significant long-run relationship between two markets must be reflected by their price series dynamics.

However, market integration in its engulfing conceptual settings has been proved by studies in the last decade to be more complex than usually assumed. These researches have as a result demonstrated that there are pitfalls associated with the general cointegration methods to the analysis of market integration (e.g. Baulch 1997; McNew & Fackler 1997; and Barrett 2001). These shortfalls are results of the conceptually diverse conditions that define market integration, for which price dynamics, though of major component, contribute only a part.

Major criticisms to price transmission methods have been propelled by equilibrium conditions that trade flow and transfer cost data incorporate into market integration concept. Cointegration and error correction modelling techniques which ignore these data; and also assume linear relationships between market prices tend to violate consistent market integration condition of discontinuities in trade, implied by spatial arbitrage conditions. Similarly, the nature of price formation in multi-market systems and trade flow reversals can lead to price series that are not cointegrated even though the markets in question are integrated. Consequently, as noted by Baulch (1997), markets that are well functioning are often diagnosed as exhibiting incomplete and /or lagged price adjustments.

These insights have spurred applied economists to further refine the empirical methods that they use to analyse price transmission and market integration. Two major strands have emerged; the extension of VECM to threshold and other switching models (Goodwin and Piggot 2001, Meyer 2004, Sephton 2003, Serra et al. 2005 and Brümmer et al. 2005); and the parity bound models (PBM) and extensions (Baulch 1997, Barrett and Li 2002, Park et al. 2002 and Negassa et al.

2004) which use mixture distribution models that directly incorporate transfer and trade flow data (binary).

Threshold cointegration models allow for nonlinearity and discontinuity in the equilibrating dynamics that link prices, but maintain the hypothesis that there is a unique equilibrium relationship between the prices under consideration. This tends to be strong assumption when consistent market integration conditions implied by Enke-Samuelson-Takayama-Judge (ESTJ) spatial equilibrium theory where trade flow behaviours, transfer cost and arbitrage conditions are taken into account. For instance when trade flows reverse – one each for the export and import regions – it may be reasonable to expect more than one equilibrium point or when trade restrictions and other policy barriers hold, the nature and level of transactions cost distort or restrain the inter-market relations to switch between different arbitrage conditions.

The PBM approach, though explicitly accounts for all market integration conditions, does not explicitly reflect any possible time series nature of the system. It instead, treats each observation in the series individually based on independently estimated market regimes. Failure to account for the time series nature of the data (e.g. when trade flow in one period affect price in subsequent periods, an element of feedback response should be expected) may lead to false conclusions. Again, when trade flow data are not available or in form required, the PBM will provide biased conclusions regarding integrated and segmented periods of the inter-markets process. That is, without explicitly accounting for tradability by either physical trade flow or price transmission, all periods of failed arbitrage will be categorised as segmentation, even though imperfect integration might be the case as spatial equilibrium theory posits.

The PBM techniques and the associated literature (Baulch, 1997 and Barrett & Li, 2002) in particular have however, helped to understanding the relationship between market integration, price transmission and efficiency much better as they reflect the nature of markets inter-relationships or price transmission process within arbitrage, spatial equilibrium and tradability concepts of market economics theory. Nevertheless, there is the need for further refinements in a manner that will account for the potential time series features and by exploring the advances brought about by the advent of cointegration and other time series innovations of threshold

models in explicitly defining the nature of possible dependencies that guide the complete data generating process of the equilibrating system.

While major methodological progress has been achieved over the years in the measurement and testing of price transmission and market integration, a robust technique that comprehensively confront market integration analysis (MIA) without seriously ignoring fundamental theoretical concepts and their implications still remains a challenge. Thus, insights from the above two major lines of market integration analysis raise important market policy, measurement and theoretical questions. However these have not been combined effectively so far. While the time series characteristics of markets inter-relationships carry important policy and methodological implications, they impose analytical complexities when other crucial elements of market integration concept such as transactions cost, arbitrage and spatial equilibrium conditions are to be directly reflected.

Meyer (2004), along the lines of threshold cointegration approaches models market integration to account for transfer cost. Brümmer et al. (2005), apply Markov-switching model to reflect nonlinearity in Ukrainian wheat market in vertical price transmission analysis. These, point to the potential role hidden Markov models (HMMs) and their extensions can play in market integration and price transmission analysis, since they are capable of handling complex systems regarding both time series implications of the sequence and the inference on the intrinsically unobserved behaviours (Hidden) of the system, with much flexibility and relatively more statistical elegance. That is, the use of hidden (unobservable) states makes the hidden Markov models generic enough to handle a variety of complex real-world time series, while the relatively simple prior dependence structure (the “Markov” bit) still allows for the use of efficient computational procedures (Cappé et al. 2005).

1.2 Objectives of the Study

From the foregoing perspective, the study is tasked to model and measure price transmission and market integration by exploring recent innovations of hidden Markov models (HMMs). Specifically, we seek to,

- (i) deepen current market integration concept to demonstrate in detail how ignoring
 - time series nature of the inter-market process can affect and constrain the current MI assessment approaches and PBM techniques in particular.
 - equilibrium conditions (inter-markets outcomes) of current time-series methods on MI conclusions.
- (ii) model market integration along the path of the PBM by
 - incorporating the dynamics in describing the equilibrating structure
 - reflecting arbitrage outcomes in spatial equilibrium conditions
- (iii) implement and compare the proposed models developed in (ii) with existing price transmission econometric models and the PBM by using synthesised market data.

1.3 Organisation of the Study

The study is structured into six major sections. Section one presents background of market integration measurements reflecting the problem statement and study objectives. We survey and review theoretical and conceptual issues of market integration in section two. This section demonstrates the complexity of the concept of market integration from market equilibrium and arbitrage concepts. Major methodological approaches to market integration analysis (MIA) are also presented and reviewed. Here the importance of addressing both the shortfalls of price transmission econometrics (PTE) and parity bound models (PBM) are highlighted. Section three defines our theoretical proposition that underpins our conceptual framework for the proposed methodology. Competing statistical tools for approaching the problem are highlighted in section four. In this section variants of hidden Markov models are proposed and their consistency demonstrated from the theoretical foundation established within spatial equilibrium and tradability theory upon which existing PTE and PBM are based. We analyse synthesised market data with the proposed Markov-switching vector equilibrium model (MS-(V)EM) and compare MI conclusions with existing ones and PBM in particular in section five. Section six concludes the study and highlights some policy and methodological implications for market integration analysis.

SECTION TWO

2.0 THEORETICAL CONCEPTS AND MARKET INTEGRATION METHODS

This section contains two major sub-sections. We review the concept of market integration from classical market economic theory and methods used in recent empirical studies. The various inter-related economic concepts of tradability, market efficiency, competitive equilibrium and the law of one price as they define markets inter-relationships and have been implied in market integration and price transmission studies are highlighted. As will be shown soon, the concept of market integration is indeed broader and can imply many complications than usually assumed by professionals and policy makers alike. Consequently, though the various tools have seen rapid refinements, they tend to be limited with respect to a given conceptual notion of market integration focus.

2.1 Market Integration Concept

Based on the broadness of the concept of market integration many experts and policy makers have viewed it from a particular notion or criterion of interest. Specifically, the concept can be inferred by an indicator of a *process of markets inter-relationships*, evidenced by *tradability* and the resultant *co-movements* of market prices in particular, on one side. On the other hand, it can be evaluated by an *outcome of the inter-market process*, gauged by *arbitrage conditions*. In the strong sense market integration can be defined by the *outcome criterion* where existence of perfect competitive equilibrium between markets ensures that arbitrageurs clear the market of any arbitrage opportunities. In its weak sense, it may be defined by the *process* of inter-market relationship assessed by co-movement of two or more markets indicators over a given time period.

These two major frames of MI definition reflect those available in the literature. For instance, Gonzalez and Helfand (2002), evaluate market integration on *evidence of common trade and*

information behaviours among the markets in question; Chen and Knez (1995) see it as *existence of law of one price (LOP) or no-arbitrage opportunities between markets*; while others focus on *extent to which demand and supply shocks arising from one market location are transmitted to other locations* (Fackler 1996; McNew 1996; McNew and Fackler 1997; Fackler and Goodwin 2001). Barrett and Li (2002) define it as *tradability or contestability between markets (but with focus on physical trade as tradability in their application)*.

Indeed, all of the above definitions of market integration require some degree of “flow of goods and/or information across space, time, and form” (Barrett 1996). Market integration has therefore been viewed and measured from either evidence or existence of *one price* or *price co-movement*. Given the time series nature of market data and richness of price data in particular, many empirical researchers have favoured the markets co-movement assumption (*process*), especially in vertical market integration analysis (see Granger and Elliot 1967; Goletti and Babu, 1994; Alexander and Wyeth, 1994; Dercon, 1995; Brorsen *et al*, 1985; Wohlgenant, 1985; Kinnucan and Forker, 1987; von Cramon-Taubadel, 1998). Many of the international trade studies often measure market integration by the law of one price (LOP) criterion or purchasing power parity (PPP) at an aggregate level (Serra et al. 2005).

With respect to these two lines of market integration definitions, co-movement or arbitrage-based criteria (which we denote by *process* or *outcome* criterion respectively), the following basic inter-linked economic theories are assessed to highlight how they direct market integration (MI) measurement and evaluation.

2.1.1 Tradability and contestability

At the heart of the measures of markets inter-connectedness lies the concept of tradability. In general a good is tradable when it can be sold across market borders or in other regions other than where it is produced. To this respect, transportability of the good at any point in time, propelled by arbitrage forces or transfer costs constraints, determines the level of tradability. In terms of market integration, a product is "tradable" between two markets if the good is actually traded or if market intermediaries are indifferent about exporting and not exporting the good from one location or country to the other if arbitrageurs face zero marginal returns

(contestability). Hence, a mere physical observation of trade between market locations provides *prima facie* evidence that spatial markets are interconnected and, therefore, integrated. Tradability signals the transfer of excess demand from one market to another, as captured in actual or potential physical flows. By this criterion prices need not be equilibrated across markets, implying a consistency with *Pareto inefficient* distributions (Barrett 2005), though prices co-movements may transpire. Such situations might be due to presence of imperfect competition or introduction of trade barriers- tariff, transport constraints among others, or very huge unobservable transactions cost.

A perfectly tradable good for a given two market points is subject to the law of one price. Because in such situations, it should be easy to move goods to where they are needed without any transportation impediments. This means an existence of Walrasian efficient markets, where arbitrage opportunities are cleared by markets intermediaries, either by information or physical flow of goods. The absolute version of this law of one price states that prices will equalize across freely trading areas and that identical goods sell for the same common-currency price in different locations (countries), while the relative version allows for transaction costs.

Tradability as MI conceptualization in effect can imply both co-movement (process) and outcome manifested by the LOP. However, as noted above, measuring MI by tradability that is captured through trade flow or prices co-movements may imply *Pareto inefficient* distributions. Consequently, the primary approach that has dominated the spatial market integration studies focuses instead on the notion of competitive equilibrium and *Pareto efficiency* manifest in zero marginal profits to arbitrage. That is, while tradability, measured by observation of trade is sufficient to imply market integration, it blurs many important economic and policy issues. Hence, MI studies usually supplement or incorporate other conceptual insights with tradability measure, especially in efficiency and arbitrage settings.

2.1.2 Market efficiency and arbitrage conditions

The concept of market integration in international trade, commodity markets and industrial organisation domains, has directly been linked to market efficiency, competitiveness and their policy implications. In these fields therefore, market integration measures usually seek to

determine the pattern, magnitude and degree of price formation structures and mechanisms via equilibrium specifications. These approaches throw more light on distribution of welfare effects of market and trade policy scenarios and strategies. For spatially distinct markets, market efficiency requires the minimization of inter-market transfer costs and quasi rents from binding quotas in addition to the attainment of competitive spatial equilibrium (Barrett, 2001). If transaction costs of trade are excessively high (e.g., due to trade barriers, poor transport infrastructure, etc.), markets can be in competitive spatial equilibrium and yet not be socially efficient. Also, as indicated above, tradability may hold at *Pareto inefficient* distributions of welfare as a result of imperfect competition or trade restrictions (quota) that limit sufficient trade flows to clear arbitrage opportunities.

Impliedly, MI studies have followed approaches that can at least infer a general picture of market efficiency, demonstrated by a violation of the LOP, perfect competitive market equilibrium or by the extent and nature of tradability as manifested by price adjustments processes. Two major lines of MI evaluations have followed; one group of recent studies combines competitive spatial market equilibrium and *Pareto efficiency* manifest in zero marginal profits to arbitrage, while the other utilises the process criterion in the form of prices co-movements founded on implicit assumption of perfect competition equilibrium. Thus, underlying many market integration analyses is the ESTJ (Enke, 1951; Samuelson, 1952; Takayama and Judge, 1971) spatial equilibrium theory, where market efficiency and competitive equilibrium and their respective MI outcomes are directly distinguished. Inherently, these measures imply both firm-level profit maximization and long-run competitive equilibrium at market level. Generally spatial market integration occurs when the competitive equilibrium condition holds, irrespective of whether trade occurs but does not imply welfare maximization unless the costs of commerce and the quasi-rents associated with binding trade quotas are minimized (Barrett, 2005).

2.1.3 Competitive spatial market equilibrium

The classical specification of the LOP can be thought of as an existence of long-run competitive market equilibrium. Thus, if markets are efficient, in the sense of competitive equilibrium where expected marginal profit to arbitrage is zero, we should expect prices to equilibrate across space

after all transfer costs¹ are accounted for. *Under such circumstances, the markets are said to be integrated.* Following ESTJ spatial equilibrium theory, three consistent conditions ensue, based on trade flow restrictions and arbitrage conditions. Spatial competitive equilibrium implies that:

$$E\{P_{At}\} \leq P_{Bt} + \tau_{ABt} \quad 2.01$$

Thus, if we take P_{Bt} and τ_{ABt} as given, then P_{At} is expected to be at least equal to P_{Bt} since in this setting, market A is importing from B . E is the expectations operator, P_{At} is the price in market A in time t , and τ_{ABt} is the transfer cost from B to A in time t . By spatial competitive equilibrium condition in (2.01), two market conditions follow;

$$E\{P_{At}\} = P_{Bt} + \tau_{ABt} \quad 2.02$$

$$E\{P_{At}\} < P_{Bt} + \tau_{ABt} \quad 2.03$$

In (2.02) where equality holds, the product is tradable between markets and the welfare gains from competitive equilibrium emerge whether or not trade flows actually occur. Baulch (1997) refers to this condition in spatial market integration as the competitive equilibrium condition under tradability or perfect integration by Barrett and Li (2002).

From (2.03) the negative expected profit to arbitrage means no attractive opportunities for marketing intermediaries to trade and exploit. This is consistent with spatial competitive equilibrium with non-trading activities (segmented competitive equilibrium), since in such cases there might be so high transfer costs that arbitrage is unprofitable in expectation (Samuelson 1952) for rational arbitrageurs to conduct trade. In this case however, the LOP in its strict form does not hold. Thus, if trade occurs and is unrestricted, the marginal trader earns zero profits and (2.02) prevails. Under this situation, prices in the two markets co-move perfectly. However, when some sort of trade restrictions exists, a third equilibrium condition holds:

¹ We define this to include all cost of conducting trade between the two markets; including transactions cost.

$$E\{P_{At}\} > P_{Bt} + \tau_{ABt} \quad 2.04$$

In (2.04) there exist positive expected returns to inter-market trade, signaling foregone arbitrage opportunities (Fackler and Goodwin, 2001). Here markets are characterized by imperfectly competitive equilibrium in which positive marginal profits to arbitrage are unexploited due, for example, to oligopsonistic or oligopolistic behavior or to binding quantitative restrictions on trade (e.g., quotas). The theory, in effect implies,

- i) multiple competitive equilibria in time space (switching equilibria)
- ii) (perfect) co-movement of prices under equation (2.02) and (2.03)
- iii) that competitive equilibrium may hold without market efficiency (2.03)

In sum, spatial market equilibrium implies that markets are inter-connected or integrated along a long-run relation defined by transactions cost and the nature of trade restrictions. Since transmission of market information or goods between markets are crucial for maintaining spatial market equilibrium over time, it follows that (perfect) integrated markets must exhibit price co-movements over time, if tradability holds. However, since under the conditions above, transactions cost plays very important role, prices may not co-move if rent to trade falls below the cost of trade.

Based on above conceptual notions, time series price transmission methods have been utilised in MI analysis, with recent innovations that incorporate the long-run relations and the potential role of the transactions cost component. In the same direction, there is a consensus about market integration defined by the arbitrage condition (LOP) in all fields of applied economics. Switching techniques have been utilised to capture market integration along the multiple equilibrium framework of the ESTJ theory. In the next section we review these two major lines of market integration measurement tools and critically highlight their respective strengths and shortfalls given conceptual insights of MI definition based on above inter-related market theories that under-pin these methods.

2.2.0 Review of Major Market Integration Tools

This review excludes pure spatial econometric approaches to market integration analysis; specifically, those that consider spatial interaction with respect to distance, market sizes and location (e.g. Gravity models). Our focus is therefore placed on market integration approaches that are based on long-run or equilibrium relationships as the foregoing concepts dictate. That is, time series price transmission econometrics and arbitrage-based regime switching tools (e.g. parity bound models) as two major strands of market integration analysis.

In market analysis in general, economists usually prefer to utilise all possible information to infer demand and supply mechanisms- from prices and quantities produced and traded, as well as cost data, and transactions cost in particular. However, all such information may not be available at, and or in desired form under a given circumstance at a given point in time. With assumptions, guided by theoretical economic concepts, many researchers have resorted to either price-based methods (price transmission econometrics-PTE) with the implicit notion that prices dynamics reflect market equilibria of demand and supply processes; or regime switching methods (parity bound models -PBM) that utilise more than price data in equilibrium representation. We review the major specific tools below.

2.2.1 Price Transmission Econometrics

As discussed earlier, since a *process* conceptualization of market integration is informative and the fact that all market data are rarely available, price-based methods have dominated the MI literature over the years (see Abdulai, 2007). The application of price transmission tools are founded on the assumption of co-movement of two market prices in at least the long-run; and the richness and availability of price series vis-à-vis others.

This strand of measures, *co-movement of prices*, has revolved through many innovations; from the classical price correlation/bivariate regression through cointegration analysis and its recent extended versions. For instance Meyer (2004) applies Hansen and Seo's (2002) threshold vector form of the error correction model to infer transaction cost component in market integration analysis while van Campenhout (2007) relaxes the constant transactions cost component implied

by threshold models (see also Buyst et al. 2006). Others make use of innovation accounting and a variant of principal component analysis to directly measure market integration in same cointegration framework. See von Cramon-Taubadel (1998), Ashe et al. (1999), Abdulai (2000, 2002 and 2007) and Balcombe and Morisson (2002); for some general developments in price transmission and the cointegration framework. In these perspectives, the strength of arbitrage defines the price relationships along a continuum that ranges between two extreme cases of, the strong form of the law of one price and completely disintegrated markets. The nature of the markets under study or the distortions that characterise the markets determine how the two price series may behave; it may be that prices adjust less than completely, or slowly rather than instantaneously and according to various dynamic structures or being related in a non-linear manner (Rapsomanikis et al. 2006).

The history of price dynamics in market analysis in general has long-lived, perhaps from the concept of market equilibrium in the wider spectrum of market efficiency analysis. In commodity markets, Farrell's (1952) empirical investigation on irreversible demand functions; and Lele (1967), Granger and Elliot (1967) and Tweeten and Quance (1969) price-based assessment of markets can be considered as some of the earliest efforts to employing price transmission econometrics in market integration analysis.

2.2.1.1 Correlation and bivariate methods (Pre-cointegration)

Traditionally, PT econometrics utilised simple correlation or bivariate regression framework. Thus on the intuition that prices of integrated markets move together, price series with high(low) correlation coefficients meant market integration (segmentation). Given price series from two markets A and B , as defined above, the degree of linear association between the markets can be measured by the sign and magnitude of the correlation coefficient, r . For the two price series P_{At} and P_{Bt} , and their means \bar{P}_A and \bar{P}_B respectively, the correlation coefficient is;

$$r = \frac{\sum_{t=1}^T (P_{At} - \bar{P}_A)(P_{Bt} - \bar{P}_B)}{\sqrt{\sum_{t=1}^T (P_{At} - \bar{P}_A)^2 \sum_{t=1}^T (P_{Bt} - \bar{P}_B)^2}} \quad 2.05$$

The correlation coefficient ranges between -1 and +1. By this approach significantly positive r indicates well integrated markets.

Various forms of the general regression specification in time series framework have been applied with specific interest (causality, symmetry, cointegration, dynamic adjustments etc) about markets inter-relationships in both short- and long-run settings. The basic structure as was applied in the earlier studies of bivariate regression took the form;

$$P_{At} = \alpha + \beta_1 P_{Bt} + \mu_t \quad 2.06$$

The μ_t is the error term which is assumed to be independent and identically distributed with mean zero. Parameters α and β in (2.06) defined the markets relationships –whether integrated or not. The above specification also implied a direction specific influences between prices P_{At} and P_{Bt} . With price asymmetric concerns in the commodity markets for instance, Tweeten & Quance (1969) use a dummy variable technique to estimate irreversible supply functions with respect to decreasing and rising prices as represented in (2.07).

$$P_{At} = \alpha + \beta_1^+ D_t^+ P_{Bt} + \beta_1^- D_t^- P_{Bt} + u_t \quad 2.07$$

This was extended by Wolfram (1971) and Houck (1977); and Ward (1982) with first differences of the increasing and decreasing phases of the exogenous prices and with some lag-structures as in (2.08) below (see Meyer and Cramon 2004 for recent review).

$$\Delta P_{At} = \alpha + \sum_{j=1}^K (\beta_j^+ D^+ \Delta P_{Bt-j+1}) + \sum_{j=1}^L (\beta_j^- D^- \Delta P_{Bt-j+1}) + \gamma_t \quad 2.08$$

In regression framework Granger (1969) proposed causality tests, which improves greatly on the simple bivariate correlation tests. In this way price co-movements can be tested with respect to the direction of influence.

$$P_{At} = \sum_{j=1}^K \alpha_j P_{At-j} + \sum_{j=1}^K \beta_j P_{Bt-j} + \mu_{1t} \quad 2.09$$

$$P_{Bt} = \sum_{j=1}^K \varphi_j P_{At-j} + \sum_{j=1}^K \vartheta_j P_{Bt-j} + \mu_{2t} \quad 2.10$$

From equations (2.09) and (2.10), Granger causality can be tested by testing for the statistical significant of the coefficient parameters, β_j and φ_j . For instance, P_{At} Granger causes P_{Bt} if $\sum_{j=1}^K \varphi_j$ (j is lag length) in equation (2.10) is significantly different from zero, while $\sum_{j=1}^K \beta_j$ of (2.09) is not. The P_{Bt} Granger causes P_{At} if the opposite scenario holds. These are termed uni-directional causality. If both β_j and φ_j test significantly different from zero, then a form of feedback relation exists between the two prices and there exists bilateral causality between the prices. Test for independence follows, thus if both β_j and φ_j are not significantly different from zero. Some authors improved this method to overcome common auto-correlation by detrending (see Piece and Haugh 1977). While these models have some advantage over correlation coefficients as they allow for lagged or leading effects in price inter-relationships, results can still be spurious since they did not take into account seasonality and other implications of non-stationarity.

In parallel, many authors also raised criticism about the classical correlation and bivariate regression models as represented in specifications above (see Blyn 1973, Ravillion 1986, Delgado 1986, Heytens 1986, Sexton et al. 1991, Goodwin and Grennes 1994 and 1998, Benson et al. 1994, and Silvapulle and Jayasuriya 1994). Blyn (1973) raised concern about short- and long-run behaviours of the market and proposed that long-run relations assessed by making use of the residual of (2.06) after taking care of any possible time and seasonal trends; Granger and Newbold (1974) with similar concern demonstrated how non-stationarity results in spurious regression. See also Harris and Barbara (1979), Timmer (1987) and Timmer and Alderman (1979) who advocated for a variant of multivariate form of MI analysis with spatial considerations. Delgado and Christopher (1986) suggested extensions to price variance decomposition, whilst Ravillion (1986) proposed a dynamic structure in both short- and long-term perspectives.

In the MI literature, Ravillion's model became the standard tool as it provided more comprehensive assessment of markets inter-relationships and resolved many of the shortfalls of the previous approaches. Especially, it allowed for short and long-run dynamics, autocorrelation and spurious correlation. He assumed a radial market system with a single central market and several local markets linked to the central (urban) market by traders. Again, he assumed that while there may be some trade among rural markets, it is the trade with the central market that dominates local price formation. Thus, price shocks originate from the central market. If we define price of the central market by P_{At} and others by P_{Bt} where in this case P_{Bt} is the price of the B^{th} local market at time t with $(B = 1, 2, \dots, N)$, then his model can be represented as:

$$P_{At} = \sum_{j=1}^K a_{Aj} P_{At-j} + \sum_{B=1}^N \sum_{j=0}^K b_{Aj}^B P_{Bt-j} + c_A X_{At} + \mu_{At} \quad 2.11$$

$$P_{Bt} = \sum_{j=1}^K a_{Bj} P_{Bt-j} + \sum_{j=0}^K b_{Bj} P_{At-j} + c_B X_{Bt} + \mu_{Bt} \quad 2.12$$

An exogenous variable X_{Bt} was also allowed to capture external influences, example inflation. Ravillion suggested the following testable hypotheses of the parameters to imply:

1. *Market segmentation*- central market prices do not influence prices in the B^{th} local market if $b_{Bj} = 0$ ($j=0 \dots K$)
2. *“Strong Form” Short-run Market Integration*- Prices shocks in central market are fully and instantly transmitted to the local market. Here, past prices in the central market have no lagged effect on future local prices. This is tested by the joint hypothesis that $b_{B0} = 1$ and $a_{Bj} = b_{Bj} = 0$ for all $j=(1, \dots, K)$
3. *“Weak Form” Short-run Market Integration*- Prices shocks in central market are fully and instantly transmitted to the local market. In this case, past prices in the central market have no lagged cumulative effect on future local prices. It is tested by the joint hypothesis that $b_{B0} = 1$ and $\sum (a_{Bj} + b_{Bj}) = 0$ for all $j=(1, \dots, K)$
4. *Long run market integration*- long run equilibrium is the one which the market prices are constant over time $P_{Bt} = P_B$; $P_{At} = P_A$ and in effect $u_{Bt} = 0$ for all t . This requires

$$\text{that; } \sum_{j=1}^K a_{Bj} + \sum_{j=0}^K b_{Bj} = 1$$

In general, Ravallion's model was an innovative achievement compared to the Bivariate Correlation/regression and Granger Causality as it made provisions for other variables that affect prices in general, and more importantly some suggestions to address the effects of non-stationarity. Moreover, this model was more comprehensive in MIA; it differentiated between market segmentation, short-run market integration and long-run market integration. The major shortfalls of this model stem from its underlying assumptions. The assumption of radial market system in which central market prices are exogenous is deemed abstract.

Again ignoring the impact of trade amongst local markets seems to be very strict assumption. Like its predecessors, the linear relationships assumed for prices and the fact it directly excludes inter-market transfer costs from the model makes it susceptible to incorrect rejection and conclusion of the market integration hypothesis (see Fackler, 1996 and McNew, 1996).

2.2.1.2 Cointegration and error correction models

While the various extensions reflected some improvements in MI analysis, namely, asymmetry, dynamic adjustment structure, multi-market considerations; they did not address the problem of spurious regression associated with non-stationary series as raised by Granger and Newbold (1974). As demonstrated by von Cramon and Loy (1996), in the asymmetric price transmission literature, when the two price series are integrated I(1) with a cointegration vector then any of the representations above, is inconsistent (see Granger, 1981). Thus the existence of cointegration process places a restraint on the price dynamics in the long-run perspective with the so-called error correction specification (Engle and Granger 1987).

Suppose (2.13) represents the general symmetric representation for two related non-stationary prices (P_{At} and P_{Bt}) of same order, cointegration would

$$\Delta P_{At} = \beta_0 + \sum_{j=1}^K (\beta_j \Delta P_{Bt-j+1}) + \gamma_t \quad 2.13$$

imply that there is a long-run equilibrium relationship between the non stationary price series and in effect the long-run adjustment process also explains any changes in the prices, ΔP_{At} . Hence error correction representation of (2.13) ensues in (2.14), with its vector version stated in (2.15) below

$$\Delta P_{At} = \alpha_0 + \sum_{j=1}^K (\alpha_j \Delta P_{Bt-j+1}) + \phi ECT_{t-1} + e_t \quad 2.14$$

$$\Delta P_t = \alpha_0 + \sum_{j=1}^K (\alpha_j \Delta P_{t-j+1}) + \beta P_{t-1} + e_t \quad 2.15$$

In equations (2.15), the β components represent the cointegration vector, which identifies the linear combinations of the non stationary variables in the price vector P_t . The vectors are defined as:

$$\begin{aligned} \Delta P_t &= [P_t - P_{t-1}], \\ \beta &= -[I_k - \beta_1 - \beta_2 - \dots - \beta_K] \quad \text{and} \\ \alpha_j &= [\alpha_{j+1} + \alpha_{j+2} + \dots + \alpha_K]; \end{aligned}$$

where $k = 1, 2, \dots, K-1$. Thus, in consistent with ECM in equation (2.14), a stationary ΔP_t means that

$$[\beta P_{t-1} + \sum_{j=1}^{K-1} (\alpha_j \Delta P_{t-j+1})]$$

is strictly stationary. Cointegration basically implies that if P_t is not stationary, for instance when it contains unit roots, then a stationary ΔP_t implies that $[\beta P_{t-1}]$ must be stationary, as can be easily solved from (2.15). In this case, the matrix β is singular and can be written as $\beta \equiv \phi \varphi$, where ϕ is an $(m \times c)$ matrix, φ is a $(c \times m)$ matrix of c cointegration vectors, with $c = \text{rank}(\beta)$. From VECM (2.15), if the vector $\mathbf{z}_t \equiv [\varphi P_{t-1}]$ is stationary (this is analogous to the

ECT-error correction term- in (2.14), Engel and Granger (1987), two-stage procedure), reflecting long-term relationships among prices, then $[\beta P_{t-1}] \equiv \phi \mathbf{z}_t$ (see Hamilton 1994 and Johansen, 1988, 1991; Maddala and Kim 1998 and Enders 2005 for comprehensive review of cointegration systems).

The above framework has been utilised in MI measurement with a given component of PT in focus. Generally when researchers find the presence of cointegration between two market prices, they conclude market integration (see references above e.g. Ashe et al. 1999 and some counter arguments from Barrett and Li 2002, McNew and Goodwin 1997 among others). In commodity markets and from policy perspectives the ECM representations have brought considerable insight into long-run market relationships/price dynamics with great policy interest. The ϕ vector contains the parameters of the error correcting coefficients which measures the rate of correction or adjustment to restoring the long-run equilibrium relationship. Thus, in addition to cointegration the VECM representation throws more light on the adjustment process in both short- and long-run responsiveness to price shocks which reflects arbitrage and market efficiency in general terms.

Again, the asymmetric version initially proposed by Granger and Lee (1989) and its consistent specification test by Enders & Granger (1998) and Enders & Siklos (2001), directly reflects a sort of market inefficiency and special form of inter-markets nonlinearity (see Cramon 1998 for asymmetric issues). Unlike the Ravillion's model, cointegration establishes long run equilibrium between series without requiring the series to be stationary and does not require any assumptions, or any restrictions on the market structure like the radial market structure.

2.2.1.3 Threshold autoregression models

From spatial market perspective and transaction costs constraints, it has become clear that many of the price adjustment processes follow nonlinear- threshold patterns (Goodwin & Piggott 2001; Azzam 1999 and Baulch 1997). Thus, a situation whereby the magnitude and speed of adjustments depend on say the size of the shock, than a mere positive or negative shocks as has been the style and focus of the asymmetric price transmission literature. These models have

assumed linear error correction in a form of constant adjustment parameters where a constant proportion of any deviation from the long-run equilibrium is corrected. Specifications (2.14) and (2.15) easily extend to include this notion as in Meyer and von Cramon (2004);

$$\Delta P_t = \begin{cases} \alpha_1 + \beta_{1,0}P_{t-1} + \sum_{j=1}^K (\phi_{1,j}\Delta P_{t-j+1}) + e_t & \text{if } z_t < \tau_1 \\ \alpha_2 + \beta_{2,0}P_{t-1} + \sum_{j=1}^K (\phi_{2,j}\Delta P_{t-j+1}) + e_t & \text{if } \tau_1 \leq z_t \leq \tau_2 \\ \alpha_3 + \beta_{3,0}P_{t-1} + \sum_{j=1}^K (\phi_{3,j}\Delta P_{t-j+1}) + e_t & \text{if } z_t > \tau_2. \end{cases} \quad 2.16$$

Recently, Serra et al. (2005) have directly extended the classical TAR model to analyse MI by applying nonparametric techniques. We elaborate on their formulation since the models we present later in this study follow a similar construction (i.e. direct equilibrium representation). As will become obvious in section five, when dynamic structures pertain with relatively deeper threshold band, imposed by transactions cost, then band threshold effects ensue if the markets are characterised by competitive equilibrium. Equilibrium threshold effects, Eq-TAR, however obtains if adjustments or observations in or outside the threshold band demonstrate a sort of reversion to an equilibrium point within the band. Unlike, the usual TC-based TAR effects, a form of adjustment activities can also occur within the band.

Balcombe et al. (2007), attempt to generalise the traditional TAR formulation such that presence of threshold effects can be directly linked to either b-TAR or Eq-TAR. Since we are more interested in non-linear structures that are caused by both TC-based threshold constraints and switching equilibrium outcomes based on prices differences, we believe both Eq/b-TAR formulations share common place in market equilibrium analysis, if the notion of trader indifference within parity bound is to be reflected (see section five for detailed proposition). The band-threshold autoregressive (b-TAR) models of price differentials, which are often used in the analyses of the law of one price and other arbitrage-based models in market integration analysis (see Obstfeld and Taylor 1997) can be deduced from standard autoregressive (AR) model of price differentials as follows:

$$R_t = \beta R_{t-1} + u_t \quad 2.17$$

where R_t represents the price differentials ($P_{At} - P_{Bt}$) or rent; u_t is a white noise error term; and β is a parameter that indicates the extent to which price differentials adjust in the period that follows a price shock. In this framework, a value of one or closer means that a shock has a permanent effect on price differentials. On the other hand, if a shock tends to quickly die out over time, then it will be equal or close to zero. For threshold effect, the following relation holds between changes in price differentials and previous values:

$$\Delta R_t = \rho R_{t-1} + u_t \quad 2.18$$

where $\rho = \beta - 1$

A TAR model occurs when the size of the lagged price differentials leads to different behaviors in the adjustment process in a regime fashion. In this case, ρ vary according to whether the shock, u_t , is bigger or smaller than certain threshold values. As in the co-integration version of (2.16) above a “neutral bands” within which prices might not be linked to one another due to transactions costs can be estimated. A three-regime TAR can be represented as:

$$\Delta R_t = \begin{cases} \rho_1 R_{t-1} + u_{t1} & \text{if } -\infty < R_{t-1} \leq \tau_1 \\ \rho_0 R_{t-1} + u_{t2} & \text{if } \tau_1 < R_{t-1} \leq \tau_2 \\ \rho_2 R_{t-1} + u_{t3} & \text{if } \tau_2 < R_{t-1} < +\infty \end{cases} \quad 2.19$$

where τ_1 and τ_2 are the threshold parameters. Further elaborations are made in section five where we impose our theoretical proposition on the above structure within the context of dynamic market equilibrium and integration (tradability) conditions. The above formulation (2.16 and 2.19) can be implemented following threshold tests by Balke & Fomby (1997), Tsay (1989), Goodwin & Holt (1999) and Goodwin & Piggott (2001). Meyer (2004) applies a variant of Hansen and Seo’s (2002) two regime threshold cointegration in VECM, to infer transactions cost component. In these models the threshold band are usually assumed constant over time.

With some regularity assumptions, van Campenhout (2007), restricts the adjustment parameter within the band and include time trend in the threshold parameter in a symmetric TAR model.

As Meyer and von Cramon (2004) note, TVECM (TAR) can improve the specification in cases where transaction costs are present or the data generation process follows such a nonlinear pattern. But as in all regime-switching models the number of thresholds to be included and their tests in meaningful economic interpretation are still under investigation. In recent studies, some authors have extended the PTE tools to accommodate regime switching conditions (see papers by Kostov and Lingard 2004, Brümmer et al. 2005). We leave the details of these extensions to the next section, as they form the basis of our proposed regime switching approaches.

In summary, as has been highlighted from above, the complexity and innovations of PTE can be summarised into the following three components as identified in Rapsomanikis et al. (2006) based on Balcombe and Morisson (2002):

- *co-movement and completeness of price adjustments*- PTE measures the extent and how changes in prices in one market are transmitted to the other
- *dynamics and speed of price adjustments*- PTE explains the process by, and rate at which, changes in prices in one market are filtered to the other market or levels
- *asymmetric response*- PTE reveals how upward and downward movements in the price in one market are symmetrically or asymmetrically transmitted to the other.

The price transmission econometrics models through their rapid refinements with the advent of cointegration, ECM and threshold extensions still do not provide a comprehensive framework for MI analysis, as the theoretical concepts highlighted above imply. Nevertheless, it is probably the most useful tool in policy orientation as it provides a flexible structure which various market characteristics that define market integration and efficiency, namely, instantaneous and gradual price transmissions, transactions cost effects on adjustment processes, nonlinearity in the equilibrating process, can be specified and tested.

From classical equilibrium theories prices are said to be the focal strings of market economies and reflect the market behaviour as dictated by supply and demand forces. However, given the wide range of phenomena from which prices dynamics result from, many questions remain as to

how one can statistically explore price data to make meaningful economic judgment. For instance as it is known from cointegration, the concept implies that prices in short run may drift apart, but co-move in the long run, as it is expected and in consistence with the concept of market integration theory. It may be misleading in assessing MI if for example; prices in spatially separated markets have a common stochastic trend reflecting say, inflation. In this case the cointegration parameter will be equal to one reflecting a proportionality of unity or a common global shock, by implying that price transmission is complete.

Price transmission testing framework does not identify the factors that affect market integration and price transmission, whether the dynamics are shaped by say transaction costs, policy intervention that insulates the domestic markets, trade quota or by the degree of market power exerted by agents in the supply chain. In effect, MI via price data or data without some market specific or an attempt to complement the results with some qualitative information on the major factors that may determine the extent of transmission may result in situation where one measures something different from what is intended.

Some authors have argued and proposed particularly from spatial market analysis that since PTE usually mixes market efficiency and competitive equilibrium analysis in inferring MI, models for MIA should be able to draw distinction between spatial market integration and spatial market efficiency that make use of more than price data. The PBM is proposed, as it is claimed to be robust and overcomes these weaknesses of the conventional methods of testing for market integration. The next section presents and reviews the PBM along Baulch (1997), Park et al. (2002), Barrett and Li (2002) and Negassa et al. (2004) as a switching regime technique for MIA based on arbitrage conditions rather than prices dynamics.

2.2.2 The Parity Bound Model (PBM)

The parity bound approach to MI analysis stems on efforts to discriminate between consistent arbitrage conditions and competitive market equilibrium that defines market efficiency. The development of the parity bounds model (PBM) represents an attempt to utilizing all available market data- prices, transfer costs and trade flows binary and volumes- possibly simultaneously, to describe markets along their long-run conceptual settings. Specifically, these models seek to

draw distinction between spatial market efficiency and spatial market integration (Barrett and Li 2002). They can be seen as a variant of the arbitrage-based models usually applied in financial market literature (see Chen and Knetz 1995).

We present the basic model of Baulch (1997) which extends on those of Sexton et al. (1991) and Spiller and Wood (1988); and highlight on some recent refinements. Let two markets prices, P_A and P_B , represent markets A and B respectively. We also assume that they are located in different localities but deal with (common) tradable commodity. The markets can be described in terms of arbitrage possibilities that may exist between them at any point in time. From the spatial equilibrium theory as elaborated earlier, three arbitrage conditions may prevail at any given time; if trade occurs between the markets the prices will be equal or differentiated by transactions cost. In this case if transactions cost is denoted by τ , then the following three conditions may exist, see equations (2.02)-(2.04) above; $P_{At} = \tau_{ABt} + P_{Bt}$, $P_{At} < \tau_{ABt} + P_{Bt}$ or $P_{At} > \tau_{ABt} + P_{Bt}$.

The transactions cost component is usually modeled as a random variable with time varying mean transfer costs, γ , and random component ν at time t . Classical PBM approach to MI analysis posits that in efficient and integrated markets, arbitrageurs ensure that no any arbitrage opportunity exist (i.e. rent to trade is zero- $R_t = 0$). When the markets are characterised by non-zero rent to trade ($R_t \neq 0$), then the markets are not integrated, although competitive spatial equilibrium and hence efficiency may prevail as discussed under spatial equilibrium theory above. From tradability concept, however, observing trade between the markets is sufficient to imply market integration and therefore identifies the later two cases (2.03 and 2.04) with trade flow observations as weak form (imperfect) integration. Baulch (1997) identifies the above market conditions in a probabilistic model where the price differential at any point in time is compared with exogenously estimated transactions cost. He specifies the PBM under the assumption of normal plus half-normal distribution along the stochastic production frontier applications. In this respect, the rent, R_t , series is assumed to be generated by one of the following process:

$$R_t = \begin{cases} v_t + \nu_t & R_t > 0 & \text{Regime 2} \\ v_t & \text{if } R_t = 0 & \text{Regime 1} \\ v_t - \nu_t & R_t < 0 & \text{Regime 3} \end{cases} \quad 2.20$$

where ν_t is a one-sided positive half-normal error which is independent of v_t . This error structure denotes periods in which rent levels differ significantly from the expected normal zero profit levels. The v_t error component describes perfect integration conditions where rent levels do not significantly differ from zero and as such are represented by a normally distributed error with mean α (Baulch 1997 uses zero mean) and variance σ_u^2 . The v_t is usually assumed symmetric around the transactions cost component. The ν_t 's effect under regime 3, implies how imbalance the two market forces- demand and supply forces- are in the presence of relatively higher transactions cost; while on regime 2, it measures the extent to which rent exceeds transactions cost when the spatial arbitrage conditions are violated, the so-called failed arbitrage (Park et al. 2002). Given the above (2.20) error structure, in probability setting the PBM can be thought of as a threshold model in the PTE case (and a Band-TAR in particular) whereby there exists a band of rent points within which price differentials are not mean reverting (Meyer 2004, Balke and Fomby 1997). While the Band-TAR models use grid search procedures to basically focus on the nature of price dynamic adjustments, the PBM seeks to categorise and estimate the probability of the rent series into autarky, successful arbitrage and arbitrage failures assuming a stochastic transactions cost (Park et al. 2002)². In the later no effort is directly put on establishing any long-run relationship (cointegration) *a priori*.

If f_t defines the probability function for equation (2.20) under the assumption of normal plus half-normal distribution, the three regimes can be specified as (2.21) with likelihood function (2.22); and (2.23 & 2.24) if trade information is further utilised to distinguish between perfect and imperfect integration as in Barrett and Li (2002):

² We build on these two insights and define a switching dynamic equilibrium model (MS-VEM) in section four.

$$\begin{aligned}
f_t^1 &= \frac{1}{\sigma_v} \phi \left[\frac{R_t - \alpha}{\sigma_v} \right] \\
f_t^2 &= \left[\frac{2}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \phi \left[\frac{R_t - \alpha}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \times \left[1 - \Phi \left[\frac{-(R_t - \alpha) \sigma_u / \sigma_v}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \right] \\
f_t^3 &= \left[\frac{2}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \phi \left[\frac{R_t - \alpha}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \times \left[1 - \Phi \left[\frac{(R_t - \alpha) \sigma_u / \sigma_v}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \right]
\end{aligned} \tag{2.21}$$

$$L = \prod_{t=1}^T \left[\lambda_1 f_t^1 + \lambda_2 f_t^2 + (1 - \lambda_1 - \lambda_2) f_t^3 \right] \tag{2.22}$$

$$\begin{aligned}
f_{ABt}^{1Td} &= f_{ABt}^{1nT} = \frac{1}{\sigma_v} \phi \left[\frac{R_{ABt} - \alpha}{\sigma_v} \right] \\
f_{ABt}^{2Td} &= f_{ABt}^{2nT} = \left[\frac{2}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \phi \left[\frac{R_{ABt} - \alpha}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \times \left[1 - \Phi \left[\frac{-(R_{ABt} - \alpha) \sigma_u / \sigma_v}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \right] \\
f_{ABt}^{3Td} &= f_{ABt}^{3nT} = \left[\frac{2}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \phi \left[\frac{R_{ABt} - \alpha}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \times \left[1 - \Phi \left[\frac{(R_{ABt} - \alpha) \sigma_u / \sigma_v}{(\sigma_u^2 + \sigma_v^2)^{1/2}} \right] \right]
\end{aligned} \tag{2.23}$$

In (2.23) where trade flow data is included in the estimation, six market conditions prevail. We use notations ABt to imply direction specific rent and transactions cost (*even though, at time t arbitrage will dictate only one directional rent in a long run representation for economically tradable goods, unless classical multiple equilibrium conditions hold*). In principle, the three *equilibrium conditions* as identified in (2.20)- when the marginal profit to arbitrage is equal to, greater or less than zero- with-trade (Td) or without-trade (nT), define six possible states (regimes) which further reduce to four *market conditions* of interest as indicated in Barrett and Li (2002). The λ_k are probabilities, describing the six regimes and the error parameters are defined by α , σ_u^2 and σ_v^2 . From equations (2.21) - (2.23), ϕ is the standard normal density function and Φ is the standard cumulative distribution function.

Market equilibrium theory is consistent with cases where the profit to arbitrage, rent (R_{ABt}), is zero with or without trade, and where $R_{ABt} < 0$ without trade. Impliedly, whether trade occurs or not with $R_{ABt} > 0$ is inconsistent with *market equilibrium*. However, from the tradability and contestability concepts above, *market integration* holds whenever trade occurs and or the equilibrium condition is binding ($R_{ABt} = 0$). In effect, four of the six possible regimes define market integration. Although, from classical market theory when $R_{ABt} > 0$, whether there is trade or not the markets are classified as segmented, (i.e. when the law of one price is violated). The tradability concept in this respect seems plausible since in practice one cannot observe all possible elements of transaction cost- such as subjective risk premia, discount rates or quasi-option value. Trade flow information can therefore offer indirect evidence on the effects of unobservable or omitted transactions costs (see Barrett and Li 2002), as well as lag price adjustments.

The PBM in this context estimates the joint probability of the rent (R_{ABt}) and trade (Td_{AB}) in maximum likelihood estimation as specified under the distribution function (2.23) and (2.24) below with trade flow information. A_{jit} is an indicator variable that takes value one if trade is observed and zero otherwise.

$$L = \prod_{t=1}^T \left(A_{jit} \cdot [\lambda_1 f_{jit}^{1Td} + \lambda_3 f_{jit}^{2Td} + \lambda_5 f_{jit}^{3Td}] + (1 - A_{jit}) \cdot [\lambda_2 f_{jit}^{1nT} + \lambda_4 f_{jit}^{2nT} + \lambda_6 f_{jit}^{3nT}] \right) \quad 2.24$$

The regime probabilities can be estimated by maximising the extended likelihood (2.24), with logarithm of the likelihood function, subject to the constraints, $\lambda_k \geq 0 \quad \forall k$ and $\sum_k \lambda_k = 1$. Baulch's case assumes that $\alpha = 0$ and a constant A_{jit} , which implies that trade either always occurs or never at all; assumptions Barrett and Li (2002) consider strong.

As noted by Park et al. (2002), the standard PBM as specified in (2.21), does not allow tracking of adjustment paths and the effects of say policy changes on the probabilities of different trade regimes. Trying to avoid identification problem in their specification, they defined policy regime-periods within which the parameters are assumed to be constant. Negassa et al. (2004)

generalises that of Park et al. (2002) to reflect both instantaneous and gradual change in regime probabilities due to policy changes in studying the grain market of Ethiopia. Their idea can be seen as identifying structural changes in PTE systems. Extending the standard PBM in this direction they make provision for transition phase and re-formulate (2.22) as (2.25) below.

$$L = \prod_{t=1}^T \left[\lambda_1 f_{jit}^1 + \delta_1 D_t f_{jit}^1 + \lambda_2 f_{jit}^2 + \delta_2 D_t f_{jit}^2 + (1 - \lambda_1 - \lambda_2 - \delta_1 D_t - \delta_2 D_t) f_{jit}^3 \right] \quad 2.25$$

The δ_k in (2.25) measures the structural change in the probability parameter of being in regime k due to the policy changes and D_t is a transition-phase dummy variable, which characterizes the alternative time path of structural change in regime probabilities (see Ohtani and Katayama 1986, Moschini and Meilke 1989). The dummy in D_t is usually specified in a three-way style. Thus, if the beginning date of market policy impact is S_1 , and the beginning of the full policy impact is S_2 then D_t takes the value zero (0) for all dates before S_1 and one (1) for all dates after S_2 . The period between S_1 and S_2 is the transition-phase period. While in some applications both S_1 and S_2 are considered unknown others usually directly associate beginning of policy implementation dates with it (e.g. Park et al. 2002 and Negassa et al. 2004).

When one expects an abrupt change to the system then transition phase does not exist, and $S_1 + 1$ is equal to S_2 . When S_2 is greater than $S_1 + 1$, then it means the system takes some time for policy adjustment, which depends on how flexible traders and other market agents are in making investment or disinvestment decisions to marketing policy or technological changes. It also depends on the level of uncertainties to and the extent of awareness of traders about the policy changes and effects on their margins and operation.

Major challenges that one usually encounters in implementing PBM is how to get all data types in a form that is usually required. For instance the very data inputs that PBM claims to throw more light on the system dynamics than PTE are usually not available or in the form needed for comprehensive time series analysis. However, as an alternative, and as is usually the case where time series transactions cost data is not available, some authors estimate the transfer costs using the PBM, based on decomposition of the observed spatial price differentials (e.g., Park et al. 2002), even though, this may implicitly assume a time invariant transfer cost component. Others

in effect, do the estimation of the transfer cost data either by using the marketing cost computed from surveys and adjusting for inflation (e.g., Baulch 1997) or inflating the time series transport cost data by a certain percentage to account for the unobserved components of transfer costs (see Negassa et al. 2004 and references indicated).

In summary, the PBM improves on MI assessment as it marries the theoretical concepts of markets inter-relationships and market data. However, the PBM and its extensions also have some weaknesses. The results are often sensitive to the distributional assumptions made, especially the half-normal distributions seem questionable if one could perceive the system as regime switching process governed by multiple equilibria mechanism as the ESTJ theory posits. Again, the constraints to accurately estimating the transfer and transaction costs might also bias the results (see Goodwin & Piggott 2001).

Moreover, the various assumptions usually imposed on the system to avoid identification problem may affect the estimated parameters (see van Campenhout 2007). Another important issue with PBM is that, it does not directly accommodate price or rent dynamics in the inter-market relationships. To avoid the danger of specification bias, lower frequency data are usually resorted to in PBM applications (see e.g. Park et al. 2002, Barrett and Li 2002). However since in MIA the dynamics (the rate at which shocks are corrected) fundamentally drive any short- or long-run markets processes, resorting to data aggregation as it is usually the case, can seriously bias the analysis than normally assumed³, as trade-rent may possess some lag relations and the fact that tradability may hold without physical trade flow.

2.3.0 Summary and Concluding Remarks

In sum, the above theoretical review shows that market integration is a broad concept and hence its definition can be vague. For instance tradability represented by physical trade flow is sufficient to imply MI but without price transmission such approach can be very biased since tradability can also imply information flow between markets without physically observing trade.

³ Since in practice, many market data have strong lag relations, reducing the frequency implies, throwing away about a third of already relatively short commodity market series. Given the relatively many regimes implied by the structure of the PBM as a regime switching model, results can easily be influenced by transient shocks, if they are not fairly distributed over time. Moreover, since segmentation implies a form of random walk process, lower frequency data may miss some crucial periods.

MI can again hold without price transmission when threshold effect persists. Outcome-based notions tend to be less informative in policy wise but do distinguish between the various interconnected economic concepts that define markets behaviours over time. The broadness of the concept usually results in situation where each of the divergent measurement approaches, tends to work well under one case but sometimes inconsistent in the other.

Since the past decade, the growing advances in market integration and price transmission measurements have generated popular view that the traditional models for commodity markets integration analysis within a linear setting are inconsistent under many real world situations in explaining observed movements in market phenomena. The diversion to recent non-linear and regime switching versions indicate a direction to the domains of models with implicit assumption of multiple equilibria, since MI and ME are intrinsically interlinked. As implied by ESTJ spatial equilibrium theory, and the fact that many commodity markets and trade regions are characterised by changing policy schemes, technological innovations on transactions cost and their associated uncertainties on market decisions, make it quite appealing to suspect that many commodity markets would relate differently in particular periods of large transactions cost, more liberalized market schemes, policy uncertainties and in strategic planning phases in time. In applied economics in general, presence of such features make it difficult to explain aggregate long-run behaviour using traditional linear models. Hence the important contribution made by the PBM and other regime switching tools in particular as they possess much flexibility to accommodate various theoretical views that underpin markets inter-relationships.

From the review, it is worth noting that while price formation structures dwell heavily on demand and supply interactions, it is only under (perfect) competitive market equilibrium that one can both assume long-run measure of no-arbitrage and efficiency. In effect, though market prices are outcomes of a process (interactions among many market variables) through demand and supply mechanisms and as such contain richness of market information, the many implied information from market prices are unobserved and dependent on the underlying market equilibrium condition. Price transmission analysis that do not accommodate all the possible underlying equilibrium conditions tend to address a particular form of MI and may be biased where other conditions hold.

Nevertheless, PTE readily reveal both theoretical and policy implications of market dynamics over time. In fact, the degree and speed of price adjustment processes to re-establishing equilibrium can, to some extent, help understand how markets function efficiently as well as how theoretical scenarios are gauged through empirical findings. In many cases where competitive equilibrium are expected, price transmission analysis -time series econometrics or partial equilibrium models- have played a major role in policy prescription and addressing distributional issues of welfare effects from market policy scenarios. But since in competitive equilibria analysis, comparative static long or short-run equilibrium implies a dynamic process, market efficiency is concerned with whether optimal amount of trade is occurring to ensure price differentials that result from demand and supply shocks are exhausted. Within cointegration and error correction frameworks insights about both short and long run inter-markets relations are provided under perfect competitive assumptions.

The distinction between MI and competitive ME is important for meaningful MI analysis, especially with limited information on tradability. Thus, though the two concepts are intrinsically intertwined they can imply different welfare outcomes and in effect policy concerns. That is;

“In order for markets to fulfil the promise they offer for risk management, efficient distribution of production according to comparative advantage, clear transmission of policy signals, and maintenance of micro-level incentives to innovate, there should be neither segmented competitive equilibria nor effective trade quotas Given limited data, in particular a paucity of data on transactions costs and trade volumes, and the intrinsic limitations of existing empirical methods, economists still have only a fragile empirical foundation for reaching clear, strong judgements about spatial market integration as a guide for corporate or government policy”.(Barrett, 2005)

Depending on the nature of trade policy environment, distortions that characterise the markets and transactions costs involved in conducting trade, price series may behave in various ways of relationships. Thus, while MI can be evaluated via any of the above two major methodological frame under specific assumptions of the market, each has a potential weakness when a complete conceptual foundation of MI theory is to be inferred. Each measurement tool depends on the specificity of the market under consideration and as such fails to distinctively address the

relationship between the law of one price, competitive spatial market equilibrium and implied efficiency, nature of arbitrage dynamics; and market integration within each model frame.

In fact, it has been established that many macroeconomic variables are characterised by different nonlinear forms that require thoughtful or more robust econometric tools. In market analysis for example, while transactions cost may deter arbitrage to a certain threshold of price/rent variations between two market points, the behaviour of the long-run relationship between the two markets may indeed be far from constant or linearity due to economic uncertainties and policy changes among others. In practice while threshold (ECM) and PBM models are usually used in that order to address market behaviour in commodity markets assessments as noted above, in any respect, each specification may be unable to capture the system behaviour where the very driving assumptions of the other are of prime important and are also to be represented. While TAR models are capable of characterising systems by their dynamic processes in magnitude and speed of price (rent) adjustments and hence to infer process of integration, the PBM as a static probabilistic model accounts directly for the nonlinear-discontinuities in long-run relationship to define arbitrage conditions (outcomes) which explicitly ignores any adjustment dynamics of the process and their implied time series effects.

Recent extensions of PTE into regime switching (MS-VECM) provide room for inferring arbitrage conditions (market outcomes). These models do not however, account directly for transactions cost effect on the adjustment processes and usually do not impose equilibrium conditions. Impliedly, it is conceivable to construct a more robust model for MI assessment by merging these two major methodological blocks, where transactions cost can be inferred from the price differentials in a regime switching fashion to accommodate arbitrage conditions when such data is not available.

In the next section we define a conceptual framework to highlight how markets inter-relationships over time fit in a wider non-linear dynamical system and demonstrate how the complexity of MI strongly suggests such modelling framework.

SECTION THREE

3.0 CONCEPTUAL FRAMEWORK AND THEORETICAL PROPOSITION

The conceptual framework, based on the preceding chapters on market equilibrium theory and arbitrage dynamics, is presented in this section to bring to bear the MI issues we want to address. We demonstrate that MI concept falls within a more complex dynamical system, and as such recent advances in regime switching models can be utilised to sufficiently describe MI concept from empirical analysis, especially in time space. It is concluded that MI concept is consistent with multiple equilibrium theory as it is implied by the Enke-Samuelson-Takayama-Judge spatial equilibrium theory and market policy changes.

3.1 Theoretical Proposition

Economic change and market policy dynamics have fundamentally altered the structure and performance of many commodity markets and their price formation processes. In market equilibrium theory, as demonstrated by Baulch (1997) and Barrett and Li (2002) based on Enke-Samuelson-Takayama-Judge (ESTJ) spatial equilibrium theory above (Enke 1951; Samuelson 1952; Takayama and Judge 1971), three main market equilibrium conditions can be identified in general terms. Our analytical focus and models are built along same fundamental logic but differ by directly incorporating the unobserved structures defined by tradability and arbitrage concepts in the basic model structure.

As noted in the previous section, trade and arbitrage forces lead to price transmission and in effect rent adjustments conditional on prevailing transactions cost⁴ (TC). Thus, from PTE stand point (threshold modelling) as represented by figure 1 below, transactions cost constrains price transmission and in effect exhaustion of arbitrage opportunities to a given threshold, (defined by the transactions cost component- in red line from figure 1). The figure presents relationship

⁴ Transactions cost here implies the cost of trading between the two markets (includes transfer cost)

between price differentials and the underlying transactions cost levels in time series settings. Standard band-threshold autoregression formulation of equation (2.19) defined and explained in previous chapter, below (3.01);

$$\Delta R_t = \begin{cases} \rho_1(R_{t-1} - \tau_1) + u_1 & \text{if } \infty > R_{t-1} \geq \tau_1 \\ \rho_0 R_{t-1} + u_2 & \text{if } \tau_1 > R_{t-1} > \tau_2 \\ \rho_1(R_{t-1} + \tau_2) + u_3 & \text{if } \tau_2 \geq R_{t-1} > -\infty \end{cases} \quad 3.01$$

was used to generate the series on symmetric transactions cost component of $\tau_1 = \tau_2 = 2.2$ in absolute terms. Unlike (2.19) however, symmetric adjustments also ensue for the rent correction process (ρ_1 holds for all cases outside the threshold band). All parameters remain as explained under (2.19).⁵ Since we are interested in equilibrium formulation, no trend component or direct costs of trade- transportation cost components were included. To incorporate the idea of spatial competitive equilibrium, physical trade associate periods in which arbitrage opportunities are attended to. By this representation all distortions to profit or rent levels above the corresponding transactions cost component (in red) are corrected by arbitrage pressures.

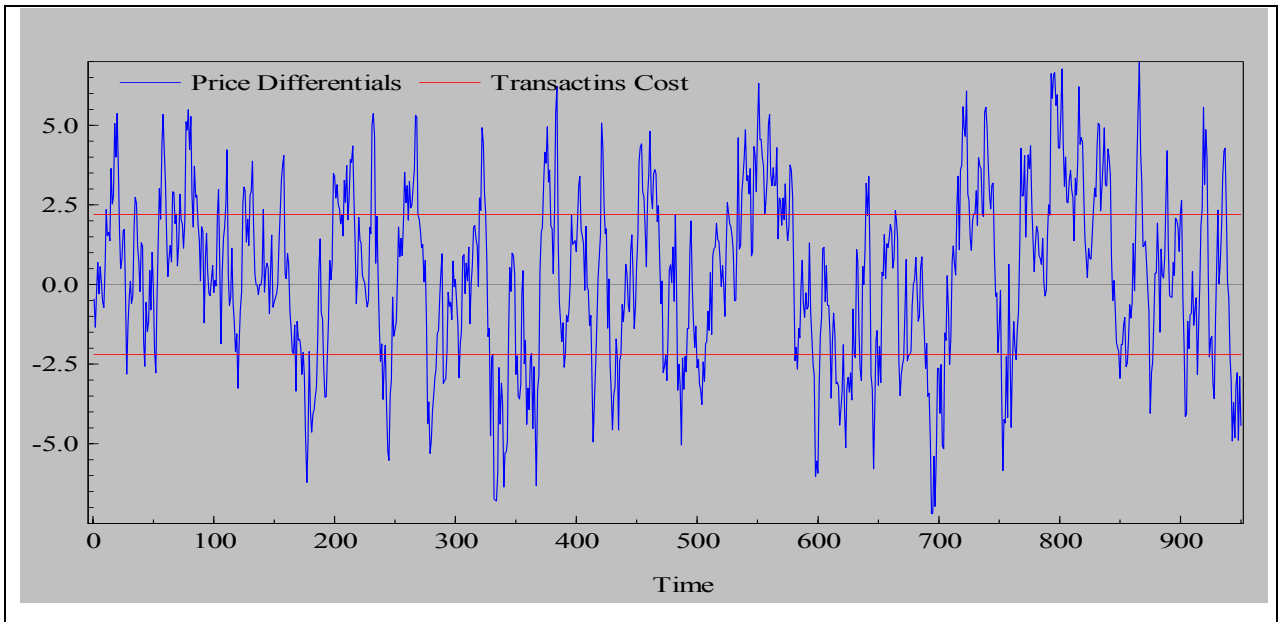


Figure 1: Band TAR illustration within integrated market structure

⁵ Parameters ρ_1 ; ρ_0 ; $u_1 = u_2 = u_3$ and $\tau_1 = \tau_2$ were set at -0.35; 0; 1.38 and 2.2 respectively (see section five for details on DGP)

Given the random walk nature ($\rho_0=0$; $\beta_0=1$), and the fact that once price differential is located within the threshold band no adjustments are expected, the TC-based threshold effects in dynamic equilibrium process define b-TAR structure. In this case the markets are said to be integrated, and MI assessment reduces to assessing how fast deviations from the TC are cleared (defined by ρ_1).

With reference to the preceding theoretical insights in section two, figure 1 implies a particular scenario of inter-market relations where in the long-run all profit to trade is zero (perfect integration if ρ_1 does not differ significantly from one in absolute terms). The ESTJ theory however, postulates that at least three long-run market equilibrium conditions are possible. That is whether in the long-run rent to arbitrage is greater, equal or less than the inter-market transactions cost. Classical threshold methods in effect study a particular form of MI, imperfect or perfect integration, as long as different levels of the rent adjustment processes are not accommodated in the model to account for any unexploited margins or potential losses due to switching inter-market equilibria structure.

In figure 2 below, the basic MI concept as already highlighted is illustrated. Here price differentials are characterised by periods of market integration and segmented equilibria (due to TC-based threshold effect). Given the TC levels, again, defined by the red lines, in some periods (e.g. from time points 71-120, 341-390, 511-550, and 831-885) strong persistence holds as arbitrage forces fail to clear all inter-market rent levels (in these episodes, $\rho_1=|0.5|$ and in addition rent corrects to a 2.8 level instead of 2.2 that is implied by band-threshold effects, to reflect a margin of unexploited rent under the maintained assumption that TC is 2.2) while rent fully clears in all other periods once the threshold level is exceeded. This portrays inter-market situations where barriers exist against full trading even if costs of trade fall far below prices differentials.

Traditional regime switching approaches to MI along Baulch (1997) or PBM do not accommodate dynamic adjustments that are modelled in the threshold models of the PTE domain. On the other hand existing threshold models do not reflect the switching nature of the arbitrage conditions, especially in periods of segmented equilibrium or imperfect integration

($R_t \neq 0$). From the ESTJ theory, without any information on trade flow volumes/quota and price transmission analysis (adjustments), price differentials less or greater than inter-markets transactions cost that are usually defined as ‘integration’ or ‘segmentation’ respectively, mixes up MI and spatial ME concepts (see Barrett and Li 2002).

For instance, under market conditions presented in figure 2, price transmission occurs in all the cases, once TC level is exceeded, which implies that tradability holds. Yet the ‘trade’ is not sufficient to exhaust all the inter-market arbitrage in the four episodes indicted above due, for example, to trade quota or imperfect competitive practices.

It follows that, if the inter-market price differentials are conditioned on the TC, two distinct market adjustment processes hold;

- (i) *where any arbitrage opportunity or losses to trade that result from demand and supply shocks are fully cleared or adjusted by arbitrage forces through demand and supply mechanisms and*
- (ii) *in situations where such distortions are not fully cleared by market intermediation efforts.*

Consequently, real market segmentation will imply cases where, given the TC levels, any indication of tradability- price transmission or physical trade flow- does not hold in the presence of arbitrage opportunities or potential losses. In Barrett and Li (2002), to motivate a switching regime model, effort is made to distinguish between the above possible market scenarios into market integration/segmentation and spatial market equilibrium/disequilibrium conditions by tradability criterion as described in section two. That is, in addition to price differentials and TC, trade flow binary is used to infer tradability in the analysis to distinguish between perfect and imperfect market integration conditions on one side and segmented equilibrium or disequilibria on the other.

In figure 2, as an illustration, trade flow binary is superimposed on the rent series to describe the distinction between MI and ME without taking into accounts the adjustment processes. The grey

dots indicate periods in which trade is observed. By this approach Barrett and Li (2002) distinguish the market inter-relationships into four distinct market conditions:

- 1) *Perfect integration: Price differential equals TC with or without trade*
- 2) *Segmented equilibrium: Price differential is less than TC with no trade*
- 3) *Imperfect integration: Price differential is either greater or less than TC with trade*
- 4) *Segmented disequilibrium: Price differential is greater than TC with no trade*

To illustrate the position that ignoring the measure of price adjustment can overstate the crucial segmented states, we re-adjust the trade flow volumes with respect to the series described above, such that some periods under imperfect market integration conditions do not correspond to physical trade flows (see chapter five for demonstration). Here we follow the position that price transmission occurs throughout the inter-market process once tradability holds, at least in a form of b-TAR process. On this position, the series presented in figure 2, have no periods of real market segmentation (given TC-based threshold effects, i.e. segmented disequilibrium). We assumed that tradability is associated with or without physical trade, but price transmission holds once the threshold point is exceeded.

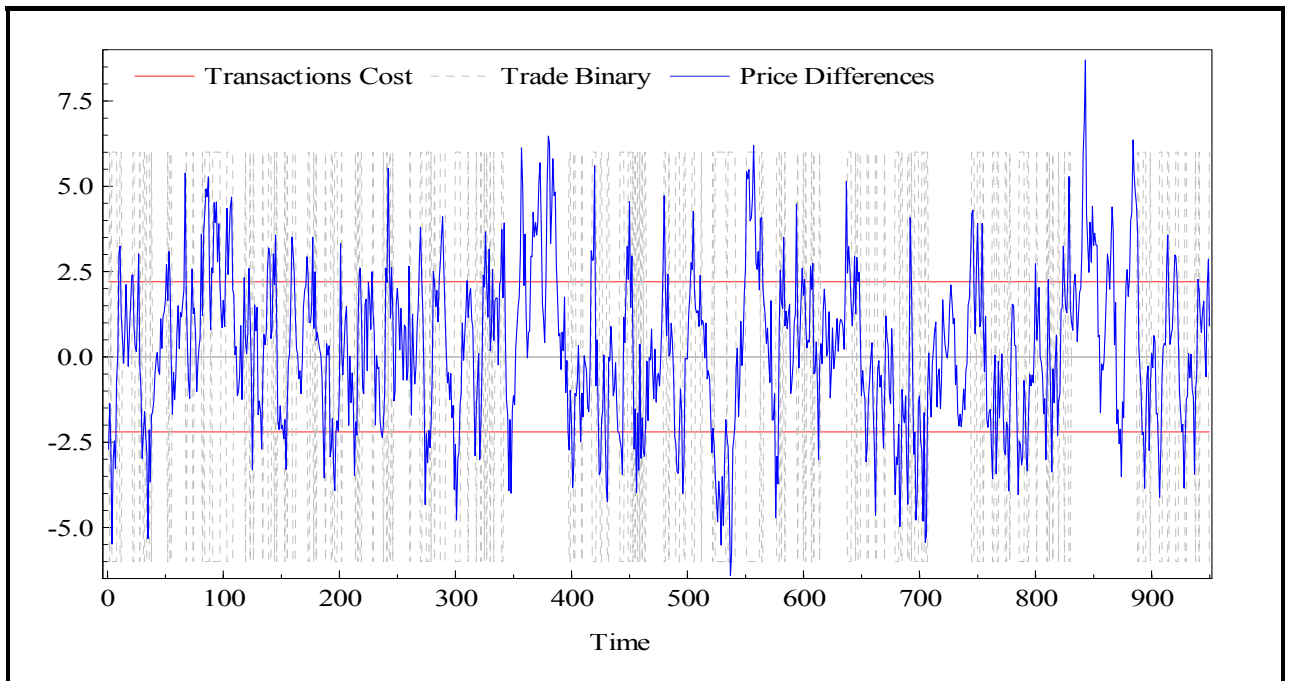


Figure 2: Band-TAR illustration within switching inter-market conditions

In effect, periods around (341-390 and 831-885) do not correspond to periods of physical trade. With this categorisation, the PBM is used to identify the four inter-market conditions stated above in a mixture distribution model. The tradability concept seems plausible since in practice one cannot observe all possible elements of transactions cost, such as subjective risk premia, discount rates or quasi-option value. Trade flow volumes or binary therefore offers indirect evidence on the effects of unobservable or omitted transactions costs and as a result, brings further insights into MI, by distinguishing between perfect and imperfect integration from spatial market equilibrium framework. However, since observing trade is not a necessary condition to ensure tradability and the fact that rent and trade depend on each other in the equilibrating process, two time series implications are worth noting. First, since trade often occurs even when rent differ from TC, and the fact that in many inter-market trading, contracting and transportation lags push traders to respond to inter-market prices and rent variations before actual transactions are made, observed trade flow may not correspond to a given rent level, as such transient shocks may incorrectly be picked as imperfect or segmented conditions.

On the second notion, it is consistent from foregoing argument that price transmission as directed by tradability without physical flow of goods can also occur even when rent differs from TC at expectation. By inferring tradability only from observed trade without adjustment processes of the system can lead to erroneous conclusions of the distinctions portrayed in figure 2 above. For instance, in figure 2, price transmission holds for time points (341-390 and 831-885, once TC level is exceeded) but under this illustration, these episodes wrongly fall outside market integration regimes into segmented equilibrium / disequilibrium states. In sum, by inferring tradability only on physical trade flow implicitly carry the assumption that physical trade flow is necessarily associative with tradability once perfect competition/integration does not hold. This is very strict if the multiple equilibria implications of the ESTJ equilibrium are to be fully accommodated in MIA especially in time space; and as well, if information flow implications on tradability defined by Walrasian transfer and Pareto inefficiency are to be taken into account. Dynamic adjustments in the market equilibrium process do not only contain rich information for explaining the extent and degree of MI but can further help distinguish MI and ME conditions.

Given the foregoing theoretical basis, to model MI process, we first characterise the system variables along PTE⁶ via equilibrium adjustments specification and then factor in the non-linear and non-constant elements in a probabilistic setting as in PBM and other regime switching techniques. This is to reflect the regime dynamics of the underlying equilibrium structure as well as the adjustment processes directed by arbitrage forces. Given the insightful role of tradability in addition to rent levels in market analysis, we suggest direct modelling of trade and rent variables to reflect possible uncertainties and dynamics about the implied equilibrium relations that may characterise the markets. In this setting, the impact of tradability as implied by Walras' law must be accounted for and reflected by the speed and depth of rent adjustments following shocks to the system.

Suppose that the price differentials or rent, (R_t^7) and trade flow series (Td_t) are directly observable but the underlying data generation process of these series is not directly observed. If this unobserved process that governs the system is defined by the equilibrium structure (conditions), then one can infer the process by a regime switching system given the observed data. The nature and form of such M -state market process are defined based on recent notions about transactions cost behaviour and arbitrage conditions in commodity market equilibrium theory. Under this dynamical assumption, particularly about the equilibrium processes, the R_t series are not unconditionally treated as independent observations but are modelled as stochastic (permanent) component that represents transactions cost; and transitory components (arbitrage levels) which imply equilibrium correction process (denoted as ρ_1 in (3.01)). That is, if the transactions cost component cannot be directly observed, appropriate decomposition technique is utilised to infer the transactions cost element of the R_t directly from the model, similar logic of Park et al. (2002) and other Band-TAR models.

Departure from the scenario in figure 1 to 2 underpins the regime switching approach that is based on the ESTJ equilibrium conceptualisation. When trade volumes and transactions cost series are readily available, the system can be extended to various consistent forms of

⁶ We follow threshold autoregressive specification of (Obstfeld and Taylor, 1997; Goodwin and Piggott, 2001, Serra et al. 2005).

⁷ We use R_t to define both rent and "price differentials", since in the study direct costs of trade (transportation and other direct transfer costs) are accounted for - not included.

multivariate representations, where inter-dependencies among the variables can directly be taken into account. Consequently, we conceptualise that:

- (i) *When trade information is available, it must be used alongside price transmission or band-TAR analysis in order not to miss important inter-market conditions. Two options follow; either the trade information is used as dummy on the dynamic model or it is modelled in a pair wise fashion within the vector-equilibrium-representation model. In the later framework, feedback relations between changes in rent levels and trade volumes to restoring market equilibrium can be directly accommodated. Since emphasis of MIA is to distinguish between the three basic market equilibrium conditions as outlined in section two with respect to spatial arbitrage conditions, the system is specified in a regime switching framework. Impliedly, the distribution of the (R_t) series in regime switching set up is therefore governed by the state dynamics (equilibria conditions), assumed to be stochastic and not directly observable as opposed to the b-TAR model.*
- (ii) *Again, if one intends to measure MI by the six-state market equilibrium conditions noted by Barrett and Li (2002) then the two variables are fundamentally governed by distinct state processes, though some interdependencies are strongly expected. This holds in that such specification/categorisation draws directly on both restricted structures of efficiency-based (three arbitrage conditions) and tradability-based (trade or no trade) concepts of MI. To this effect, two modelling implications hold; inter-state dependencies and non-binding simultaneous switching processes (no co-breaking⁸) between trade flow volumes and arbitrage outcomes. As a result, we conceptualise and propose multi-chain switching-equilibrium model based on these apparent interdependence and lags structures that might characterise trade-rent relationship. In this sense, the 2-state trade flow variable is not restricted to co-break with the 3-state equilibrium conditions, implying two distinct underlying chains but perceptibly with some interdependencies.*

⁸ Co-breaking as introduced in Hendry (1996) implies simultaneous regime switching of multiple time series.

Obviously, while the time series characteristics of markets inter-relationships carry important theoretical, policy and methodological implications in MIA, they impose analytical complexities when other crucial elements of market integration concept such as tradability, arbitrage and spatial equilibrium conditions are to be directly reflected. The complexity of MI analysis can therefore be viewed from basic assumptions that drive PTE such as threshold effects, asymmetry, degree and extent of price/rent adjustments; and that of PBM, namely, non-constant trade patterns, non-constant transaction cost and their resultant spatial arbitrage and market equilibrium conditions as implied by tradability. To reflect all these complications of MI in a model suggests a combination of the two major strands of MI approaches. That is, the adjustment processes bedeviled by the time series characteristics of the system are gauged through (TVAR)TECM on one hand; and the stochastic switching processes (a form of multiple equilibria) based on spatial arbitrage conditions along ESTJ equilibrium model are incorporated via regime switching on the other. Our proposed model for analysing MI is therefore a variant of

- i) the switching state-space model or state space model with regime-switching (SSSM), when transactions cost data is not readily available but to be decomposed from the rent series *as extension of band-TAR (see Ghahramani and Hinton 1998 and Kim and Nelson 1999)*; or
- ii) the multi-chain/multivariate regime switching AR(EM) model in cases where price, trade and transactions cost series are available along (Krolzig 1997 and Otranto 2005).

These proposed models for MIA dwell on unified modelling variants of hidden Markov modelling (HMMs) based on data availability and ones interest about the system dynamics of the markets. Within these frameworks, both equilibrium correction dynamics and switching equilibrium structures along spatial arbitrage conditions can be simultaneously specified.

In multi-chain regime switching models in particular, each variable is allowed to follow its own state level dynamics, though these regime dynamics may exhibit some inter-dependencies. Since tradability implies both physical trade flow and cases where traders are indifferent to trade, pairing both the ESTJ three equilibrium states and the two tradability implied states with

equilibrium adjustment process provides a unified comprehensive frame for MIA. This is illustrated in figure 3 below.

3.2 Complete model structure with tradability implications

Figure 3 shows complete equilibrium conditions as implied by both trade and rent dynamics. The C_{it} are defined for each observed variable's state process (trade volumes and rent levels) and are denoted as C_{et} and C_{dt} for rent equilibrium and trade states respectively. As explained above, this defines a multi-chain regime switching equilibrium model, where the assumption of co-breaking is relaxed. From the figure below, each state evolves independently of the other; however under some market specific conditions some relations can be specified as noted above.

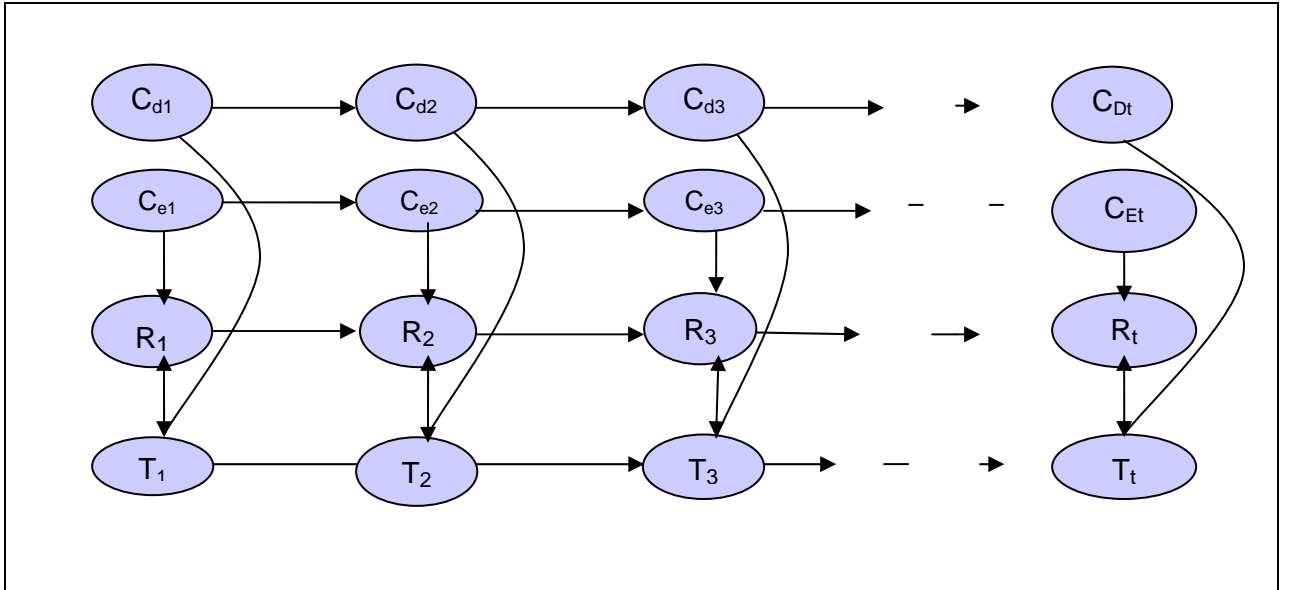


Figure 3: Complete Market Integration Conceptualisation

For instance, it is conceivable that structural changes in transactions cost levels and the rent equilibrium-state dynamics may together drive the dynamics of the trade-regimes. That is, when markets are characterised by segmented equilibrium, any substantial structural changes in transactions cost caused by say policy/technological changes would trigger trading activities between the markets, while those that raise transactions cost under competitive equilibrium

conditions would halt/constrain trade. In simple case an arrow joins C_{et} and C_{dt} as one possible scenario of state level inter-dependencies.

3.3.0 Summary and Concluding Remarks

In this section, a conceptual foundation has been introduced within theoretical implications of ESTJ spatial equilibrium context and time series characteristics of the equilibrating process. Specifically, we have established from ESTJ equilibrium theory that, since;

- 1) market integration can be assessed by arbitrage conditions (outcomes), i.e. no arbitrage, arbitrage failure or autarky ruling, a particular form (*time-space*) of multiple equilibria is consistent assumption for capturing nonlinearity in MI processes. In this respect if static equilibrium process is assumed, the PBM in its basic form can be represented by a three-state hidden Markov model.
- 2) in typical PTE the basic representation of market integration is described by the adjustment parameters (especially of the ECT), which naturally implies arbitrage process, with relatively high frequency data, rent and or trade dynamics can directly be reflected in the equilibria representations without assuming *a priori* market integration.
- 3) tradability can hold without physical trade flow, inferring tradability with only physical trade flows particularly in instances of arbitrage failures can result in misleading conclusions of MI and ME; even though in addition to rent and prices data physical trade flows patterns provide insightful details of the markets relationships over time- as it helps to discriminate between MI and ME
- 4) point three above holds, a flexible modelling implication ensues; that is, where TC is accounted for, regime switching techniques that can accommodate regime shifts in the adjustments parameters can also discriminate between segmented regimes and imperfect integration without directly using trade flow binary or volumes (see section five for synthesised data application).

It is concluded that MI falls within a complex dynamical system which can be gauged by multiple equilibria process. In order to infer from both the adjustment processes and the underlying data generation mechanism defined by arbitrage conditions, we have proposed a Markovian forms of regime switching techniques, namely; SSSM and MS/MC-VAR based on data availability. We have demonstrated that once, the MI concept is well dissected, time series data of prices and fair knowledge about TC can be utilised to infer the very insights of tradability that physical trade flow carries.

In general, the HMMs framework can directly accommodate the view of non-constant and non-linear long-run price-trade integrating factors that link spatial markets. Since the suggested models are generalizations of the classical hidden Markov model (HMMs) we highlight its basic statistical structure and meaning to motivate those in our case as extensions in the next section.

SECTION FOUR

4.0 PROPOSED METHODOLOGY

Along the basic rational of the ESTJ equilibrium theory and the conceptualizations made above, we introduce variants of regime switching techniques in hidden Markov modelling framework as a comprehensive approach to MI analysis. We concentrate on models that employ MS-VEM in situations where all the three basic market data are available. In this framework we directly accommodate the non-linear, long-run price-trade integrating factors that link spatial markets. We do not implement the SSSM but argue that the TAR model can be generalised into state-space modelling framework to accommodate switching arbitrage conditions.

4.1.0 Overview of Proposed Methodology

Hidden Markov variants with dynamic adjustment techniques have seen dramatic applications in many economic fields since Hamilton's groundbreaking work in 1989 (see also Krolzig 1997, Kim and Nelson 1999 and Cappé et al. 2005). In fact, the HMM concept has been one of the most successful statistical tools for complex patterns and systems analysis across almost all fields of scientific domain where interests are focused on sequence and systems identification, classification and dynamics over time. Since the models we propose in this study are generalizations of the classical hidden Markov model (HMMs) we highlight the basic statistical structure and meaning to motivate those in our case as extensions.

4.1.1 Hidden Markov models (HMMs)

This sub-section presents a relatively detailed concept of the HMM as it forms the basis of our methodological approach to market integration analysis, which has not been directly or widely applied in agricultural commodity markets analysis.

Classically, a hidden Markov model is doubly stochastic process with an underlying stochastic process that is not directly observable but can be observed only through another stochastic

process that produces the sequence of observations (Cappé et al. 2005). Generally, depending on what one intends to model and the purpose for which the HMM is to be used, the process can be defined in terms of the joint probability distribution of the variables or through a functional representation, the so-called general state-space model. For econometrics interest we prefer the later proposition as it can easily be generalised to incorporate critical economic structures. Although in the traditional time series setting, ‘state-space models’ are usually used to describe models of the linear Gaussian autoregressions. We use the term as in its general form for describing any HMM represented in functional relationship between hidden and observed variables along Cappé et al. (2005).

We begin the overview of HMM by using the variables in our MI concept. If we assume that the rent series (price differentials), $\mathbb{R} = \{R_1, R_2, \dots, R_t\}$ are independent series but generated by a non-linear process defined by M -state arbitrage conditions, then the system can be thought of as generated from a multiple equilibria system with switching rent levels that are represented by $R_t = 0$, $R_t < 0$ or $R_t > 0$ after taking transactions cost into account. From this scenario, regime one may imply equilibrium of normal economic profit $R_t = 0$ (rent to arbitrage is zero), while in regimes two and three the cost of trade unduly increases to imply autarky ruling and arbitrage failure regimes (i.e. price differentials fall far below and above the implied transactions cost) respectively. This position can be seen as direct representation of the PBM in a hidden Markov sense. To reflect the market dynamics in MI analysis, we define the \mathbb{R} in functional representation.

The issue is that it is more desirable in MIA to model explicitly the state processes that reflect ESTJ spatial equilibrium to infer inter-market relationships. In this case though, the series might be independently distributed but perhaps only as conditional on the underlying latent equilibrating structures. This underlying latent process is parameterised as Markovian, which imposes dependence structure on the system. If we can describe and identify a market system by such critical variables –in this case the three arbitrage outcomes- that explain the inter-markets dynamics, then such variables are referred to as its *state* variables. In the course of time the system’s features (arbitrage conditions) may vary in response to changing state variables and thus would exhibit dynamic behavior. Such changes in state variables are called *state-*

transitions. If we denote the state variable of the system by C_t (equilibrium condition), where t indicates discrete time of length T , then we can define a sequence $C_0, C_1, C_2, C_3 \dots C_t, C_{t+1}$ which is termed the process trajectory. At each discrete time slot t , the system takes a move to one of the states according to a set of state transition probabilities. We denote the state at time t as c_t . Thus, if the process is assumed to evolve randomly, then the probability of observing $C_{t+1} = c_t$ is given by :

$$P(C_{t+1} = c_t | C_1, \dots, C_t) \quad 4.01$$

However in Markov chains the probability of observing C_{t+1} does not depend on the history of the sequence as in (4.01), but only on the previous state C_t . That is, (4.01) reduces to:

$$P(C_{t+1} = c_t | C_1, \dots, C_t) = P(C_{t+1} = c_t | C_t) \quad 4.02$$

This property of Markov processes draws on the statistical concept of conditional independence. The conditional independence of X and Y given Z carries the interpretation that if knowledge of Z is available, then knowledge of Y does not change one's knowledge of X and vice versa. In Markov process it is the knowledge of a third random variable $Z(C_t$ in our case) that determines whether $X (C_{t+1})$ and $Y(C_1, \dots, C_{t-1})$ might or might not be independent of each other (*see Appendix A for short overview*). Hence, though deriving conditional independence assumption with proposition (4.02) seems strict in many applied fields (and especially in economic settings), the Markovian models have proved very useful in many complex system analysis where such assumption imposes a great deal of computational and analytical convenience/flexibility. Even in obvious situations where the conditional independence assumption cannot be strictly adhered to as in many economic time series, the state structure can be reformulated to carry the Markovian property, see specification (4.03). Simply, the Markovian assumption implies that given say n previous random variables, the current variable is conditionally independent of all other earlier variables other than the n previous ones. Therefore, an n^{th} -order Markov chain may always be converted into an equivalent first-order chain by;

$$C'_t = \{C_t, C_{t-1}, C_{t-2}, \dots, C_{t-n}\} \quad 4.03$$

where C_t is an n^{th} -order Markov chain, C'_t is a first-order Markov chain since;

$$\begin{aligned} & P(C'_t | C'_{t-1}, C'_{t-2}, \dots, C'_1) \\ &= P(C_{t-n:t} | C_{1:t}) \\ &= P(C_{t-n:t} | C_{t-n-1:t}) \\ &= P(C'_t | C'_{t-1}) \end{aligned} \quad 4.04$$

The transformation (4.04) implies that with relatively large state space, a first-order Markov chain may represent any n^{th} -order Markov chain, see Cappe et al. (2005) and Bilmes (2002 and 2006). The above characterisation of the Markovian property carries important analytical flexibility in time-series econometric applications where dynamic processes usually involve complex lag-structures. If the state variable C_t , can take values ($i=1, 2, \dots, M$), then the statistical evolution of a Markov chain is determined by the state transition probabilities

$$a_{ij}(t) = P(C_t = j | C_{t-1} = i) \quad 4.05$$

While the transition probabilities can in general be a function of both the states at successive time steps and of the current time t , we will assume that it is time invariant at this stage of our analysis. Such a time-independent chain is called time-homogeneous (homogeneous), meaning;

$$a_{ij}(t) = a_{ij}, \quad 4.06$$

for all t .

The transition probabilities in a homogeneous Markov chain are determined by a transition matrix A , where $a_{ij} = (A)_{ij}$. The rows of A form potentially different probability mass functions over the states and hence, A is also known as a stochastic transition matrix. That is, a matrix whose element lies between zero and one and the rows sum up to one. Given the Markov chain, the probability of observing a given sequence say $C = \{c_1, c_1, c_3, c_1, c_1, c_2, c_2, c_2, c_3, c_3\}$ is

$$\begin{aligned}
P(C | A, \pi) &= P(c_1, c_1, c_3, c_1, c_1, c_2, c_2, c_2, c_3, c_3 | A, \pi) \\
&= P(c_1)P(c_1 | c_1)P(c_3 | c_1)P(c_1 | c_3)P(c_1 | c_1)P(c_2 | c_1) \\
&\quad P(c_2 | c_2)P(c_2 | c_2)P(c_3 | c_2)P(c_3 | c_3) \\
&= \pi * a_{11} * a_{13} * a_{31} * a_{11} * a_{12} * a_{22} * a_{22} * a_{23} * a_{33}
\end{aligned} \tag{4.07}$$

where π is the initial state transition parameter. It follows that, the probability of a state sequence C_1, \dots, \dots, C_T can be calculated as the product of the transition probabilities:

$$\begin{aligned}
P(C | A, \pi) &= P(C_1)P(C_2 | C_1)P(C_3 | C_1, C_2) \dots P(C_T | C_1, \dots, C_{T-1}) \\
P(C | A, \pi) &= P(C_1)P(C_2 | C_1)P(C_3 | C_2) \dots P(C_T | C_{T-1}) \\
P(C | A, \pi) &= \prod_{t=1}^{T-1} \pi C_t C_{t+1}
\end{aligned} \tag{4.08}$$

If the state process or variables are observed, they form the output of the Markov chain. In many applied fields, these critical variables are not directly observed, but generate and emit some other system variables, R_t which are observable. Thus, these state variables and sequence that generate the observed R_t series are hidden. Hence the sequence of R_t depends very much on that of the underlying hidden variables, C_t in our case. The observation transition probabilities b_j are therefore defined as,

$$P(R_t | C_t = j) = b_j(R_t) \tag{4.09}$$

As noted above, hidden Markov model is doubly stochastic process with the state variable not directly observable but produces the sequence of the observations variable. From joint and conditional probability theory the two doubly stochastic processes can be expressed and defined as follows;

$$P(R_{1:T}, C_{1:T}) = P(R_T, C_T | R_{1:T-1}, C_{1:T-1})P(R_{1:T-1}, C_{1:T-1}) \tag{4.10}$$

$$= P(R_T | C_T, R_{1:T-1}, C_{1:T-1})P(C_T | R_{1:T-1}, C_{1:T-1})P(R_{1:T-1}, C_{1:T-1}) \tag{4.11}$$

$$= P(R_T | C_T)P(C_T | C_{1:T-1})P(R_{1:T-1}, C_{1:T-1}) \tag{4.12}$$

$$\begin{aligned}
&= \dots\dots\dots \\
&= \overbrace{P(c_1)}^{\pi_i} \prod_{t=2}^T \underbrace{P(C_t | C_{t-1})}_{a_{ij} = A} \prod_{t=1}^T \overbrace{P(R_t | C_t)}^{b_j(R_t)} \tag{4.13}
\end{aligned}$$

Formally specification (4.13) defines HMM in general form and can be seen to compose of the so-called five-tuple $[C, \mathbb{R}, \Pi, A, B]$ where;

- 1) $C = \{1, 2, \dots, M\} = \{c_1, c_2, \dots, c_T\}$, comprises of the M hidden state variable sequence
- 2) $\mathbb{R} = \{R_1, R_2, \dots, R_T\}$ comprises of the T -length observed variable sequence
- 3) Π is the initial state distribution $\pi_i = P(c_1 = i)$; $i \in M$.
- 4) A is the state transition probability $A = \{a_{ij}\}; i, j \in M$.
- 5) B is the observation variable probability distribution $B = b_j(R_t) = P(R_t | C_t = j)$

From equation (4.13), when a given system can be modelled in HMM then both the observation sequence and the underlying state sequence probabilities can be calculated from the conditional dependencies among the variables given the model parameters. Hence in HMM, three interest or questions are of prime concern:

- 1) Given a model $\Theta = (\Pi, A, B)$, and an observation sequence $\mathbb{R} = \{R_1, R_2, \dots, R_T\}$ how efficiently can the observation sequence generated by the model be computed? $P(\mathbb{R} | \Theta)$?
- 2) With model Θ and observation sequence \mathbb{R} , what is the underlying state sequence that best explains the observations? And
- 3) Given the observation sequence and a space of possible models, how do we adjust the parameters to settle on a model Θ that maximises $P(\mathbb{R} | \Theta)$?

The above three algorithmic frame of estimation, fundamentally defines the statistical estimation tool for HMMs (see Cappé et al. 2005; Bilmes 2002, 2006 and appendix B for detailed steps). However, specific computational complexities arise as to what distributional and or dynamic structural assumptions one imposes on the system. For instance, in state-space models of many

applied economics analysis the forward/backward probabilities are evaluated via Hamilton (1989) and or Kim (1994) filters.

Our application of HMM as an alternative to PBM presents richer statistical inference of MI (based on the ESTJ market equilibrium conditions), since the rent series cannot strictly be assumed as independent series but at least being conditional on the prevailing underlying market equilibrium condition which cannot be observed, the basic tenet of HMMs. In fact, as Bilmes (2006) demonstrates, HMMs represent dependency information between temporally disparate observation variables that is indirectly encoded in the hidden variables.

If under the true probability distribution, two random variables possess extremely large mutual information, an HMM approximation might fail because of the required number of states that might be required to sufficiently reflect such dependencies in the HMM. The problem with HMMs as in this case is how they are used; the conditional independence properties are inaccurate when there are too few hidden states, or when the observation distributions are inadequate. Many authors have argued that specifying HMM with enough hidden states and a sufficiently rich class of observation distributions, can accurately model any real-world probability distribution (see Bilmes 2002). However, in economic settings where various institutional noise and macro-level aggregations blur true outcomes, theoretical foundations play crucial role in model formulations.

An important dependence consideration that has been extended on HMM in some specific applications and in many econometric time series, concerns the additional information that might exist on an observation R_t in an adjacent frame (say R_{t-1}) that is not supplied by the hidden variable C_t . In this case the conditional independence property ($R_t \perp\!\!\!\perp R_{t-1} | C_t$) is invalid. This has resulted in hybrids of HMMs, notably, correlation or conditionally Gaussian HMMs in Engineering/Speech processing (see Bilmes 2006 for review) and Markov switching / state space representations in econometrics (see Hamilton 1989, 1994; Krolzig 1997 and Kim and Nelson 1999). Under such conditions an additional dependence is added between adjacent observation vectors. Applications of Markovian approaches in econometrics have been generally based on the Markov regime switching of Hamilton (1989), which in general moves along the switching linear Gaussian autoregressions (see also Krolzig 1997; Kim and Nelson 1999;

Hamilton and Susmel 1994; Krolzig 2002; Brümmer et al. 2005 and Otranto 2005 all as extensions).

Thus, as typical of most economic series, the non stationarity and dependence structures tend to be strong such that the conditional distribution of say price series P_{t+1} , given all past variables does not depend only on C_{t+1} (the underlying hidden chain) but also on P_t (and possibly more lagged P -variables). That is, conditional on the state sequence C_t , the P_t forms a non-homogeneous Markov chain, and obviously the conditional distribution of P_t does not only depend on C_t and P_{t-1} but also on other lagged C_s and P_s (see Cappé et al. 2005). As explained under our conceptualizations, the observation variable R_t does not have as a parent only of the hidden variable C_t but also the variables R_{t-l} for $l = 1, 2, \dots, K$ for some K lag-length. While it is widely acknowledged that increasing the number of states under classical HMM as introduced above can adequately model implications of lag and other dependence structures (e.g. applications of Krolzig 1997 and Rossi and Gallo's 2005 HMM in econometrics settings), these lag structures and their adjustment processes carry in themselves important policy and theoretical interpretations in economics.

Motivation for exploring HMMs in MIA is based on the fact that MI analysis basically reduces to identifying the sequence of the state of the markets behaviours as defined by the equilibrium and arbitrage conditions. If all economic time series data of the markets were available or observable, one could easily conclude such patterns from transactions costs, trade quotas/volumes and price series as equilibrium theories postulate. Thus, if we had complete knowledge about transactions cost and price series, then profit levels could easily be constructed to classify the market into successful and failed arbitrage as well as disintegration conditions based on trade flow data. Obviously, identifying such patterns over time under real dynamic and uncertain economic circumstances as is the case for commodity markets inter-relationships requires tools that can efficiently infer latent structures from available observed variables.

From the above fundamental insights into HMMs that our proposed methodology falls, we will in the next subsection characterise MI conceptualisations detailed in section three in these modelling framework. Particularly, we will not deal with their specific statistical and

computational issues but will refer to appropriate sources since they dwell heavily on those considered above. Hence, only the key issues of MI concept, especially implications of multiple equilibria, that motivate application of HMMs are summarised intuitively within MS-VEM and SSSM specifications to accommodate crucial time series characteristics of the MI process.

4.1.2 Multivariate Markov-switching Market Equilibrium Model

As noted by Barrett (2005), at the heart of many spatial market integration analysis lies the ESTJ theory of spatial equilibrium. This theory in general terms implies multiple equilibria system defined by prevailing arbitrage conditions and corresponding trade flow structure. In similar to PBM modelling settings, we treat the implied multiple equilibria in time-space as regime shifts that govern the market inter-relationships. Again, these regime shifts are viewed as stochastic rather than deterministic process, which imposes limitations on the application of traditional approaches such as using appropriate dummy variables or split sample analysis techniques in capturing the break points on time lines defined by the equilibria points. While more advanced probabilistic models are recommended to estimate such processes, for more meaningful interpretation and improved estimation, other variables that are directly related to the regime shifts are preferred to be included in economic modelling. As noted by Krolzig et al. (2002), this however involves computational difficulties and specification complications. As an alternative they extended the univariate MS of Hamilton (1989) into a multivariate framework to model UK labour market.

Brümmer et al. (2005) follows their approach to analyse vertical market integration of the Ukrainian wheat market based on price series. While long run market inter-relationships can be assessed via price cointegration, the role of other market variables, e.g. trade flow volumes & transactions cost effects within the above equilibrium structure, are very crucial to comprehensively explain market integration without imposing any strict structural assumptions. Thus though, rent series (from prices) can be used in a regime-switching model to infer multiple equilibria and in effect MI as we demonstrate with Band-TAR/MS-EM models in section five,

some crucial assumptions or decomposition must hold.⁹ We also employ a variant of MS-VAR (equilibrium model), as direct spatial vector equilibrium model with Markovian state propagation mechanism.

As noted above, Markov switching models are special form of classical HMMs where the observed series do not only depend on the state variable but also on some lag variables (especially AR are incorporated). Unlike the MS-VECM of Krolzig et al. (2002) and Brümmer et al. (2005), we do not assume a priori any form of market integration and therefore do not resort to cointegration analysis to define the equilibrium representation structure. Based on ESTJ equilibrium theory and TAR models, the rent series (defined at relatively high data frequency points by direct construction from prices) is specified to account for any dynamic adjustments that govern each of the equilibrium points. As noted earlier we follow two lines of specification under this Multivariate MS; by assuming co-breaking of state processes for system variables in one case and relaxing the assumption of co-breaking in the other (see Otranto 2005).

4.1.2.1 Markov-switching vector equilibrium model (MS-VEM)

We generalize the ESTJ spatial equilibrium model into a MS-VEM (vector equilibrium model) which improves on the PBM in two directions- by directly allowing equilibrium adjustment process and reflecting any possible dependence structure of the system in a vector representation. Indeed, PBM as a mixture distribution model is characterised by serially independently distributed regimes where the transition probabilities are independent of the history of the regime. Thus, if one restricts MS process to say the intercept with Gaussian errors and independent switching regimes for a 3-level spatial equilibria model, observationally, such system is equivalent to the PBM, since given the estimated transactions cost, the PBM can be seen as a structure with non-normal error innovations around a time-invariant intercept (see section five for further elaboration and example). Hence our MS-VEM, with regime shifts in some of the parameters can be viewed as a compressed but unified alternative model for MI analysis that combines both fundamental features of PTE and PBM in (4.14 and 4.15/16).

⁹ At least transactions cost must fairly be estimated.

$$R_t = u_t^* \quad \Rightarrow u_t^* = u_t + \delta_{c_t} \quad 4.14$$

In equation (4.14) the PBM version is stated, where R_t of the equation defines $T \times N$ observed series ($N=2$ in our case- rent and trade variables (volumes); and T is the length of the series). In this case rent equals price differentials less TC. The u_t^* comprises of the state-dependent equilibrium rent and trade levels, δ_{c_t} ($R = 0$, under perfect integration case for rent) and error term, u_t which follows normal i.i.d (the variance can also be allowed to vary across regimes). Since the transactions cost variable is directly deducted from the price differentials, we ignore it from the regime switching formulation that represents the inter-market conditions. Equation (4.14), is directly formulated along the PBM where when rent variable ($R = 0$) the MS-VEM is same as the PBM since they reduce to u_t , with or without significant trade levels. Under conditions where c_t does imply that ($R \neq 0$), then δ_{c_t} switches between significantly positive or negative mean to imply periods of inter-market segmentation or imperfections. While under the PBM δ_{c_t} is inherent random value given u_t , under MS-EM it is specified as constant within each state but propagates on a hidden stochastic process over time.

In equation (4.15) the system is defined within time dynamics to account for rent adjustments that are of special interest in the PTE framework with the possibility of switching system parameters.

$$R_t = \phi_{l(ct)} R_{t-1} + \delta_{ct} + u_t \quad 4.15$$

In the application section, where general formulation along Eq-threshold models is adopted, δ_{c_t} takes a constant value across regimes such that segmentation or imperfection condition is measured by the rate of rent correction ($\phi_{(c_t)}$) given $R = 0$ (see section five). With this, we do not switch δ_t but $\phi_{(c_t)}$ as the measure of inter-market anomaly; reduced to assessing how deviations from ($R = 0$), persist over time as in the PTE. Impliedly, the normal equilibrium level (perfect competitive equilibrium, $R = 0$) for instance, has zero rent (mean) and high

correction rate with or without substantial trade levels. The state variable c_t indicates which one of the M possible market equilibria (regimes) that governs the system at time t . As elaborated above, the state variables are assumed to be Markovian, which implies that the conditional probability density of the observed R_t series vector is defined by

$$P(R_t | R_{t-1}, c_t) = \begin{cases} f(R_t | R_{t-1}, \Phi_1) & \text{if } c_t = 1 \\ \vdots & \\ f(R_t | R_{t-1}, \Phi_M) & \text{if } c_t = M \end{cases} \quad 4.16$$

In general the R_{t-1} vector represents the observations $\{R_{t-j}\}_{j=1}^{\infty}$ in the vector autoregression system and the parameter vector Φ assumed to be dependent on the ruling state at time t . Like the classical HMMs, the state propagation mechanism follows an ergodic Markov chain with a finite number of states $c_t = 1, \dots, M$ represented by the transition probabilities;

$$A_{ij} = \Pr(c_{t+1} = j | c_t = i), \quad \sum_{j=1}^M A_{ij} = 1 \text{ for all } i, j \in \{1, \dots, M\}. \quad 4.17$$

Some structural insight on the model is needed at this point to allow economically consistent interpretation of the trade variable. As can be recalled from the ESTJ theory, three possible arbitrage regimes, with or without trade ensue. In equation (4.14) therefore, by implicitly imposing the assumption of co-breaking on trade flow and rent state processes on the model structure requires that the number of states on the common equilibrium mechanism (state process) of the two variables are increased to reflect MI by the combined outcomes of both tradability and efficiency-based concepts. In all, the combined system produces six distinct equilibrium points (the four market conditions of Barrett and Li, 2002) or three-state symmetric system as assumed in Meyer (2004), when strong time dynamics hold in the time series settings.

It is important to note from above six regime structure due Barrett and Li (2002) that imposing two-state structure on tradability with trade flow series implies that price transmissions associate physical trade in periods of imperfect integration. When this is not the case as conceptualised in section three, then trade flow does not co-break with rent dynamics (see figure 2 in chapter

three). Under this scenario the vector system, like the PBM counterpart will wrongly categorise imperfect integration as segmented, if the number of states are not increased accordingly.

Even though flexibility of the Markovian frame can allow further states, the theoretical support may not far fetch since other institutional noise may become explicit. It follows that if strong time dynamics pertain, adjustment parameters will differ significantly at least between the imperfect integration and segmented equilibrium/disequilibrium states. Alternatively, multi-chain (two-chain for trade and three for arbitrage conditions) process that does not assume co-breaking is more appropriate. For statistical and computational issues of Markov-switching models refer to Hamilton (1989, 1994) and Krolzig (1997 and 1998).

4.1.2.2 Multi-chain Markov-switching vector equilibrium model (MCMS-VEM)

Under this specification we relax the assumption of co-breaking of the state variables in the system so that overlapping regime processes can directly be inferred from the model as in PBM of Barrett and Li (2002) in the following;

$$R_t = \phi_{(ct)} R_{t-1} + \delta_{ct} + u_t \quad 4.18$$

As in equation (4.14 & 4.15), (R_t) of equation (4.18) defines $T \times N$ observed series ($N = 2$ in our case- rent and trade variables; and T is the length of the series) and all other parameters remain as already defined. The only difference here is that, unlike the MS-VEM, the state variable c_t at time t is a vector of length N , which implies N multiple chains (C_t) . The MS-VEM, which inherently assumes co-breaking can only capture MI representation as demonstrated in Barrett and Li (2002) by increased number of states, specifically six if no symmetric structure is imposed on the system (see the illustration, above). With the MCMS-VEM however, each variable's state mechanism is explicitly defined to accommodate any consistent theoretical view point (see Gallo and Otranto 2006).

In this case the unrestricted six-state system of MI within PBM specification is represented by three- and two-state regime switching processes on the rent and trade variables respectively. While the MS-VEM can be seen as a restricted form of MCMS-VEM with observationally similar form, it is not nested in the later as the functional structure of the latent variables C_t differs significantly (see Otranto 2005). If we define $C_t \equiv \{c_{1m}, c_{2m}, \dots, c_{NM}\}$ then c_{1m} represents the state associated with variable $R_{t(nm)}$, where M is number of states in a chain. The first subscript of c_{1m} represents the variable number in the time series vector (impliedly, with rent and trade, $R_{t(1)}$ and $R_{t(2)}$ obtains). Given the transition probability matrix $A = \Pr[C_t | C_{t-1}]$, if we impose symmetry on the system to simplify the MI space to be two-state for each variable, for the sake of clarification (say rent is either equal to zero or otherwise; and tradability defined by *with* or *without* trade flow), then $N = M = 2$. In this case the state vector C_t can assume four different values $C_t \equiv \{c_{11}, c_{12}, c_{21}, c_{22}\}$ and the matrix A is a 4 x 4 matrix which correspond directly to say perfect integration (c_{11} combining either c_{21} or c_{22}), which reads, rent is zero at expectation with trade equals zero or not ; imperfect integration (c_{12} and c_{22}) and segmented market conditions (c_{12} and c_{21}).

The rationale behind the Multi-Chain Markov Switching model is the flexibility to specify multivariate process such that the switching mechanism across regimes makes it possible to express the state for one variable dependent on the lagged states of all the variables in the system. Gallo and Otranto (2006) utilise the MCMS to test for Asian stock markets interdependencies, contagion and independence. From our conceptual settings, the functional dependence structure of the regime dynamics and more importantly the state overlaps of rent-trade state level relations makes the MCMS likely candidate to correct potential misleading conclusions of calibrating MI via the six-regime market conditions implied by the PBM.

Since the properties of the MCMS are founded on same theoretical views as those of standard Markov switching models, Otranto (2005) suggests filtering and smoothing procedures described by Hamilton (1989) and Kim (1994). Because of computational complications associated with this type of modelling, as noted in Krolzig (1997), some restrictions are required

on the general model (4.18) in order to make it tractable, and also to retain consistent interpretation of the results according to the specific application at hand.

4.2.0 Summary and Concluding Remarks

In this section, we have demonstrated that given the flexibility of hidden Markov models and the fact that market equilibrating processes fall within a complex time series system, HMMs (Markov switching in particular) methods can directly be adopted in market integration analysis. It has been argued from the basic structures of HMMs representations and existing regime switching models of MI measurement that Markovian framework is consistent for MIA based on the dynamics and nonlinearity of markets inter-relations as implied by spatial market equilibrium and tradability concepts, and their resultant arbitrage conditions.

Specifically, from spatial equilibrium and tradability theories, since market integration can be assessed by;

- (a) arbitrage conditions (outcomes), i.e. no arbitrage, arbitrage failure or autarky ruling,*
- or*
- (b) periods with or without trade.*

HMMs are flexible regime switching tools for MI assessment. It is also demonstrated that since in typical PTE the basic representation of market integration is described by the adjustment parameters (especially of the ECT), which naturally implies arbitrage process, with relatively high frequency data, rent and or trade dynamics can directly be reflected in the equilibria representations along TAR settings without assuming a priori market integration in switching framework. In effect, two variants of HMMs have been proposed for MIA, defined within two major lines, by taking into account both short- and long-run processes and roles of various market data:

- 1) Markov switching equilibrium model MS-(V)EC*
- 2) Markov-switching multi-chain model MSMC.*

Thus, MI dwells not just on whether the two price series are inter-related, but more importantly how they differ conditional on the transactions cost component. The models above combine these two tenets of MI notions in equilibrium framework. Put differently, given the transactions cost, prices dynamics defined by adjustments in rent series and switching equilibrium conditions are represented. Our choice for Markovian framework is based on its flexibility.

It is however obvious, as it is always in economic issues, that the models outlined here are more or less specific given ones knowledge and underlying theoretical assumptions about the markets in question. Although given the strong growing evidence of non-linear time series dynamics in market economic systems our proposed models can be seen as a benchmark for integrated and robust tools for MI analysis. The basic models, as defined above can be extended to take into accounts all sorts of conceptually consistent notions of market integration- asymmetry and more importantly imposing variational-restrictions on the TC component to account for a particular policy effects.

In the next section we implement Multivariate MS models by analysing an ideal market data along side classical MI (PBM and b-TAR) tools with a synthesized series.

SECTION FIVE

The aim of this section is to implement our proposed MS-VEM model in the previous section and to evaluate how they can identify market integration patterns from empirical perspectives. We approach this by using synthesised ideal market data (prices, trade flow volumes and transactions cost) where crucial inter-markets conditions are imposed under guided assumptions. These assumptions are based on market equilibrium and arbitrage theories that drive classical approaches as outlined in the previous sections. Our focus is to evaluate how MS-(V)EM can be used to infer the very insights PBM and TAR models generate in complex non-linear market equilibrium conditions.

5.0. MODEL IMPLEMENTATION AND APPLICATIONS

5.1 Characteristics of Data Types Used in the Experiment

To implement the model proposed in section four (MS-EM), ideal market data of price differentials and trade flow volumes are generated. Along the theoretical proposition developed in section three, the data sequences are allowed to propagate interdependently over time. The relationships are also allowed to accommodate threshold effects as may be implied by presence of transactions cost in inter-market trading. In addition, three different long-run profit-structures (rent components) are imposed on the system to reflect regime switching processes within arbitrage conditions as formulated by the PBM. To fully analyse MI within market equilibrium context, in one setting (data sets A & B), the regime dynamics are allowed to reflect only cases of market integration where tradability (implied by rent adjustments or physical trade) prevails throughout the process within band-threshold structure. In the other frame (data sets C & D), we relax the assumption of tradability by imposing autarky conditions in some periods; irrespective of size of the realizable inter-market rent in relation to transactions cost.

Specifically, the series are generated within band- TAR models as these form the basis of non-linear models applied in MI analysis. In line with PBM we impose static equilibria assumption in the *form of the LOP*, whereby no lags relations were allowed in case of data sets A & C. Adopting time-series procedures, an autoregression process is imposed on data sets B & D,

where shocks to the system take some time to correct. Again, in both perspectives we apply the concept of tradability as discussed in the previous sections, with the position that physical trade flow is not a necessary condition. Moreover, relatively large sample size (950) was used to ensure that at least each model characteristics are included. This is important because an ideal single market data set¹⁰ is constructed in the analysis.

5.2.0 Characterising Market Equilibrium Conditions in Time-space

To illustrate how the time series characteristics of the market system are to be captured, the structure of the alternative threshold model is adopted. In this respect and as explained under the theoretical proposition, we assume that R_t is not strictly direction specific but an arbitrage opportunity that pertains at time t . We assume that direct cost of trade is not included in R_t and as such price differentials represent margins, unless otherwise stated. The following equilibrium representations considered in section two is expanded in lines of the theoretical foundation upon which the application of MS-(V)EM is based. We thus, reframe inter-market equilibrium conditions in time dynamics based on the TAR form below, recalled from section two:

$$\Delta R_t = \rho R_{t-1} + u_t \quad 5.01$$

$$R_t = \beta R_{t-1} + u_t \quad 5.02$$

which follows that $\rho = \beta - 1$.

With threshold effects we get a regime switching process of the form;

$$\Delta R_t = \begin{cases} \rho_1 R_{t-1} + u_1 & \text{if } \infty > R_{t-1} \geq \tau_1 \\ \rho_0 R_{t-1} + u_2 & \text{if } \tau_1 > R_{t-1} > \tau_2 \\ \rho_1 R_{t-1} + u_3 & \text{if } \tau_2 \geq R_{t-1} > -\infty \end{cases} \quad 5.03$$

¹⁰ Bootstrap and Monte Carlo simulations were performed under PBM for assessing maximisation issues of the model.

Specification (5.03) defines a perfect market condition with threshold effects, within symmetric adjustment structure when ρ_1 is not significantly different from negative one (-1). The meaning of this representation in market and trade analysis is that once traders initiate trade there is the tendency for full arbitrage, reverting shocks instantly to normal profit levels. Thus, if initiating trade involves a relatively fixed transactions cost then within the threshold band as it is assumed in MI analysis under the PBM, traders do not react to price differentials, and do not initiate trade. If price differential exceeds the TC, two options for trading obtain. Arbitrage may lead to full clearance of the market, the global equilibrium $R = 0$. Or trade revert the price differentials to the level of the TC. When the latter holds, the so-called band-TAR effects pertain, while the former is what the literature refers to equilibrium threshold (Eq-TAR).

That is, in equilibrium representation, rent (R_t) is defined by absolute price differentials less cost of inter-market trade as already discussed including the TC component. When the markets are perfectly integrated, any increase or decrease in price in market B at time t , under static framework, is immediately responded to by price changes in market A before time $t-1$. If b-TAR obtains, equation (5.03) in effect becomes;

$$\Delta R_t = \begin{cases} \rho_1(R_{t-1} - \tau_1) + u_1 & \text{if } \infty > R_{t-1} \geq \tau_1 \\ \rho_0 R_{t-1} + u_2 & \text{if } \tau_1 > R_{t-1} > \tau_2 \\ \rho_1(R_{t-1} + \tau_2) + u_3 & \text{if } \tau_2 \geq R_{t-1} > -\infty \end{cases} \quad 5.04$$

Consequently, rent (R_t) to arbitrage equals zero ($R_t = 0$) if transactions cost does not impose price response constraints (equation 5.01). The threshold models as a result assess markets inter-relationships and functionality based on the size of ρ_1 in (5.03). As noted in the previous sections, the degree of market integration is then inferred. The main rationale underlying the general three-regime specification of the PBM is that, unlike the implications of equations (5.01 and 5.02), rent to inter-market trading can differ significantly from zero ($R_t = 0$); or price differentials from TC if the markets are not integrated or imperfect behaviours exist at any given time t . In the settings of equations (5.03/4) above, the threshold conditions under competitive market equilibrium are violated, if shocks beyond the normal TC based threshold (τ_s) do not revert to zero ($R_t = 0$) or TC . Complete market integration conditions in equilibrium context

alter the dynamic threshold space of (5.04) as in (5.05) below. To explain the inter-market conditions in time dynamics, we assume that conditions that violate the systems in (5.04) arise from either *extreme increases in cost of trade* such that at any given period t , trade cannot be profitable irrespective of the size of the price differences; or normal cost of trade (TC) prevails but traders do not attend to the profits due to say market and policy restrictions (e.g. price restrictions and bans) or lack of market information. System (5.05) below portrays all possible inter-market time dynamics as described above.

$$\Delta R_t = \begin{cases} \rho_1(R_{t-1} - \tau_1) + u_1 & \text{if } \infty > R_{t-1} \geq \tau_1 \quad \& Td = 1 & (i) \\ \rho_2(R_{t-1} - \tau_1) + u_1 & \text{if } \infty > R_{t-1} \geq \tau_1 \quad \& Td = 1 & (ii) \\ \rho_0(R_{t-1} - \tau_1) + u_1 & \text{if } \infty > R_{t-1} \geq \tau_1 \quad \& Td = 0 & (iii) \\ \rho_0 R_{t-1} + u_2 & \text{if } \tau_1 > R_{t-1} > \tau_2 \quad \& Td = 0 & (iv) \\ \rho_2 R_{t-1} + u_2 & \text{if } \tau_1 > R_{t-1} > \tau_2 \quad \& Td = 1 & (v) \\ \rho_1(R_{t-1} + \tau_2) + u_1 & \text{if } \tau_2 \geq R_{t-1} > -\infty \quad \& Td = 1 & (vi) \\ \rho_2(R_{t-1} + \tau_2) + u_1 & \text{if } \tau_2 \geq R_{t-1} > -\infty \quad \& Td = 1 & (vii) \\ \rho_0(R_{t-1} + \tau_2) + u_1 & \text{if } \tau_2 \geq R_{t-1} > -\infty \quad \& Td = 0 & (viii) \end{cases} \quad 5.05$$

ρ_0 , ρ_1 and ρ_2 indicate the strengths of rent correction. ρ_0 corresponds to periods of no rent adjustment while ρ_1 and ρ_2 imply a particular sort of rent correction and in effect market integration. Thus a relatively perfect MI system under (5.04) assumes that tradability (Td) holds throughout the period of the market evaluation and as already noted in section three ρ_1 indicates a full instantaneous rent correction when price differentials exceed TC. In effect ρ_2 is included in system (5.05) to imply imperfect integration market conditions where though, some correction holds substantial levels of rent are left unexploited. From the systems in (5.05), when tradability does not hold ($Td = 0$) in (iii) and (viii), the markets behave in similar ways as those in (iv). That is, since no element of inter-market rent correction exists in these periods, a sort of random walk process (ρ_0) also prevails outside the TAR-band. Case (v) seems plausible for classical TAR framework, in which threshold impacts are not necessarily due to TC constraints. If this occurs within the TAR band under ESTJ theorem, that is, if trade initiates while rent falls below the TC, then competitive equilibrium is violated. Under (ii) and (vii), trading activities occur but significant rent or losses to arbitrage remain due to insufficient trade

or to the fact that causes of the excessive rise in cost of trade have not been addressed, implying a weaker adjustment or imperfect integration (ρ_2). It is worth noting that the magnitude of the adjustment parameter also depends on the data frequency, which implies that MI conclusions depend on the richness of time-series data available. As noted earlier, Barrett and Li (2002), use binary variable on trade flows to distinguish between segmented and imperfect inter-market anomalies. We utilise both trade flow data and rent correction in the form of switching regression to discriminate between autarky/segmentation and imperfect market integration cases.

Thus, given market data over time, all the equilibrium conditions are decomposed into their time path dynamics. For instance, when imperfect integration is the case, significant changes in P_{At} will be partially matched by changes in P_{Bt} , resulting in higher rent levels and more rent persistence than would prevail under perfect market integration condition where full corrections ensue (ρ_1). Under market segmentation however, changes in either P_{Bt} or P_{At} do not trigger changes on the other corresponding price, which can also lead to higher price differentials/rent (conditional on potential cost of trade) and a form of random walk process. The PBM identifies any rent at time t , that significantly differ from the TC as a period of inter market anomaly; that is, segmentation or imperfect integration when trade flow is observed. We demonstrate that a three state Markov-switching structure can be adopted to capture same three differing rent structures. While a direct and more parsimonious application of a 3-state switching AR (autoregression) MS-VEM can identify the dynamic patterns of the system presented in (5.05) by the degree of rent irreversibility, to categorise the complete inter-market process directly with series that contain *TC-based* threshold component without any information on *TC* would require a pair-wise or hierarchical model to distinguish between conditions (iv) of (5.05) from (iii) and (viii).

Given the computational issues involve, and the basic objective of MI assessment, where all market data available is used and the evaluation along PBM that requires at least TC data, we adopt a two-stage modeling approach, where *TC* series is used to detect and remove the threshold effects as adopted in the PBM framework. As such an invariant mean/intercept Markov-propagation in the form of switching AR(p) structure is then imposed to distinguish

between the rent levels ($R_t = 0$, $R_t < 0$ and $R_t > 0$) based on the adjustments dynamics as implied by ρ_0 , ρ_1 and ρ_2 of system (5.05), assuming high frequency data.

5.3 Comparing Results from MS-(V)EM, PBM and TAR Models

5.3.1 Integrated Markets with Threshold Effects (simple non-linear series)

To motivate our analysis, a *perfect* integration system in which the data generation process follows a threshold autoregression is considered first (see equation 5.03/4). We assume that tradability holds bi-directionally. Impliedly, at any given time t , direction of trade depends on the size of price differences between the two markets points, relative to cost of trading. Here we generate a series from equation (5.04) above, by assuming $\rho_1 = -1$ at expectation, ($\beta_1 = 0$). This means that when trade occurs rent is fully and quickly corrected and as such price differentials revert to TC (τ) bound.

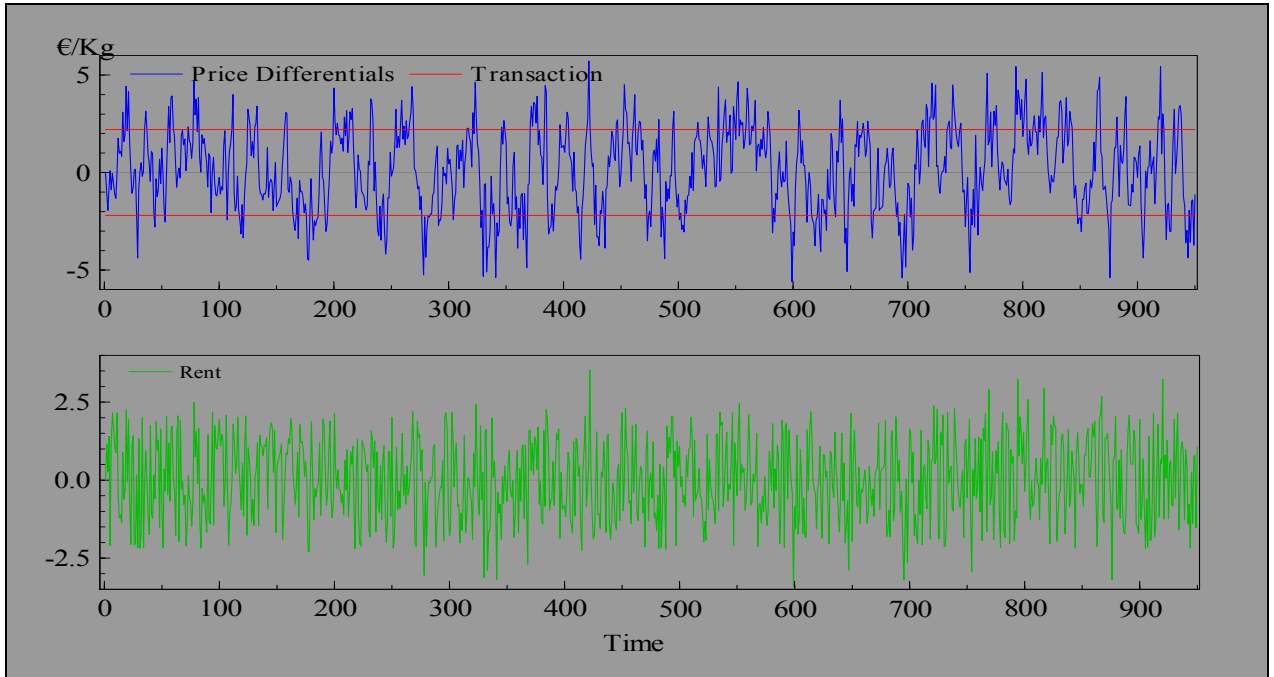


Figure 4: Simple Non-linear series (Full correction)

To focus on real inter-markets conditions beyond normal TC-created autarky conditions we specify $\tau_1 = \tau_2 = 2.2$, thus symmetric structure, with 1.38 innovation (u) variance and ($\rho_0 = 0$ at

expectation, ($\beta_0=1$)). While relatively large variance ensures that the TC can be exceeded, it tends to create relatively less persistence within and outside the band, especially if strong corrections, as implied by perfect integration conditions, exist.

The second series set relaxed the assumption of instantaneous one time full correction and incorporated some adjustment phase; where ($\rho_1=|0.35|$) is implied indicating that price differentials clear up but within some time period. In this case the equilibrium correction parameter was set at 0.35. This means that only 35% of equilibrium errors correct by the following time point. The figures (4) above and (5) below show the resultant series as price differentials and their respective rent levels depicted in blue and green lines respectively.

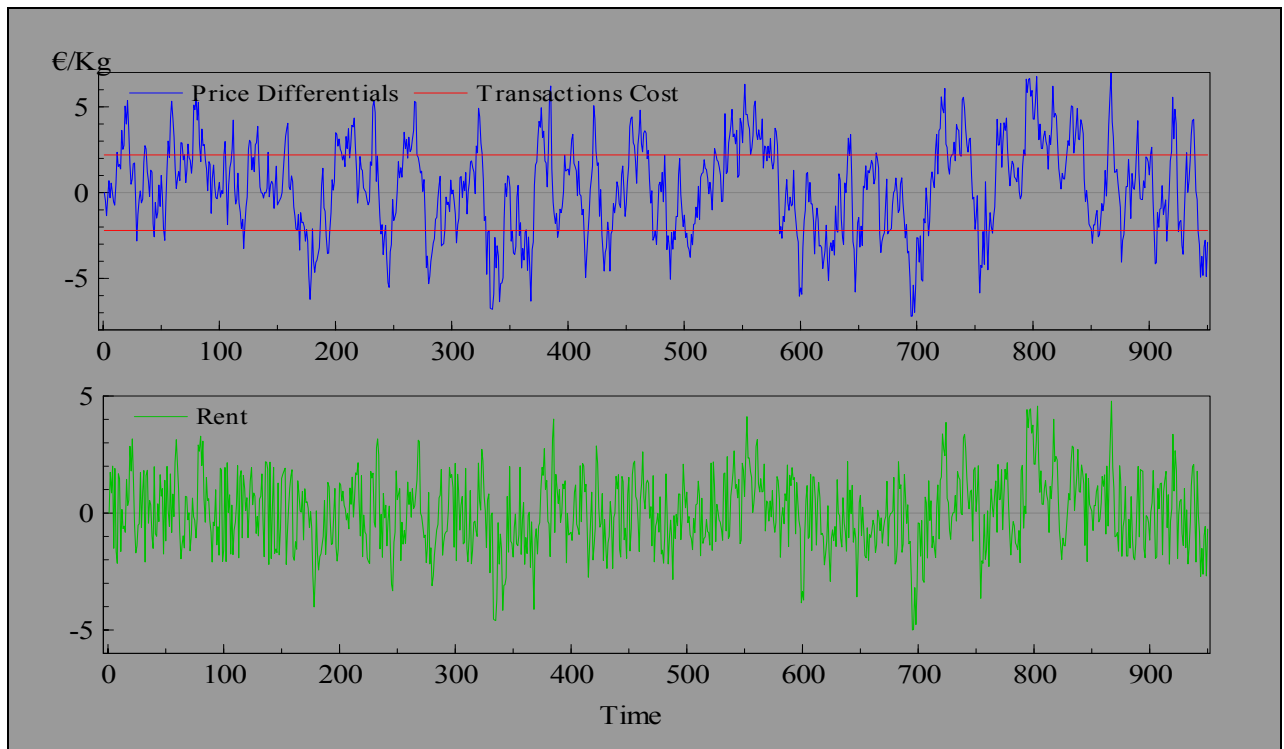


Figure 5: Simple Non-linear series (Rent correction over time)

We did not include trend in the process, as our primary concern is to construct rent series along TAR modeling framework that is based on MI concepts as posit by the ESTJ theory. We also did not present the results and issues of series that are characterised by asymmetric conditions, as these can directly be captured within the general self-exciting threshold autoregressive (SETAR) and Markov switching frameworks. The rent series represent prices differentials less

TC. Given these data sets, we impose the usual economic assumptions that drive MI assessment models on the DGP and employ the respective tools based on our knowledge of the true DGP¹¹. We then extend same theoretical assumptions in a unified framework and utilise the flexibility of Markov switching specification in same equilibrium modelling framework. For instance, if we assume that the true data generation process follows a perfect integration structure but *TC* constrains arbitrage responses, then MS-autoregression-heteroskedasticity (MSAH(2)-AR(p)) ensues as direct Markovian alternative to the (Eq-TAR) or (MSMAH(3)-AR(p)) as b-TAR version under a *perfect* competitive equilibrium process. Theoretically, an Eq-TAR or b-TAR process can be seen as a two-state regime switching process with shifts on the adjustment parameter if symmetric adjustments hold - that is, a form of random walk within the threshold band and autoregression outside the bounds, given the TC (Krolzig 1998 demonstrates the linkage between these two regime switching models).

5.3.1.1 Results from Band-TAR models

In this section we analyse the series presented above to conclude on the implied inter-market process by using the TAR as time series MI measurement tool. The TAR model as defined in equations (2.17) to (2.19) of section two and recalled above, (5.01) to (5.04) is applied to the above data sets. We assumed that the adjustment process of the market system is not linear as we expect that the rent series contain significant amount of transactions cost. If the adjustment process is characterized by threshold effects as a result of TC constraints, then rho (ρ_0) from equation (5.04) should not differ significantly from zero to reflect random walk nature of price differentials within the threshold band. However, when price differentials tend to exceed TC levels in absolute terms, rho (ρ_1) differs significantly from zero to correspond to periods in which beta β_i in equation (5.02) shows strong correction.

We utilise general SETAR set up with the Markov-switching package (MSVAR) of Krolzig (1998) on OX 3.2 platform. The SETAR framework can be used to capture TC asymmetries that are usually associated with direction specificity of trade. As noted already, this study follows the theoretical foundations of spatial market equilibrium structure, where the rent to arbitrage is

¹¹ Respective OX codes for the DGPs are available from the author

modeled directly given an independently estimated TC and trade flow volumes. Hence we do not construct our b-TAR model in cointegration framework (as in Meyer 2004, Hansen and Seo 2002 or Balcombe et al. 2007), but follow the structure of the PBM by directly studying the price differentials between the markets within time series framework (see Serra et al. 2005).

Since regime switching processes that are considered in MI studies assess market equilibrium conditions with respect to profit margins (that is, whether substantial rent goes unexploited, trade occurs while rent falls significantly below trading cost, or how fast equilibrium errors are corrected); test for market integration within spatial equilibrium structures can be reduced to testing for regime changes in the mean/expectation of rent levels in static equilibrating systems as implied by the PBM on one hand; or in degree of irreversibility of significant shocks to equilibrium level ($R_t = 0$) in dynamic equilibria settings. The later can be formulated in the form of switching AR parameters in equilibrium model (using price differentials (rent) and trade data)¹². Again, given that inter-markets anomalies tend to show persistence over time, the Markovian specification can detect the differing adjustment processes introduced in equation system (5.05). In short, our proposed alternative MI tool, MS-(V)EM, is simply a switching autoregression of Hansen (1992/1996) or MSAH(3)-AR(1). Given the theoretical complications of categorizing MI conditions under threshold conditions over time however, we also utilise TC and trade flow data to finally draw conclusions on inter-markets relations. Thus, unlike the threshold model however, we use TC data directly to concentrate out rent points that exceed the TC level.

Thus, since our threshold point is known a priori, the test for market integration is reduced to testing for rent persistence beyond TC-based threshold bounds. In effect following the proposition that threshold effects obscure the true picture of market integration and competitiveness, the series were split into two by isolating the TC-based threshold effects. This sample splitting approach is similar to the rational behind *arranged autoregression* technique usually applied in threshold analysis. An arranged autoregression orders the data according to the potential switching variable (see Petrucelli and Davies 1986, Tsay 1989, Balke and Fomby 1999). If threshold effects do not imply, then the MS-(V)EM formulation applies directly. Under

¹² This is preferred to switching mean, since under dynamic equilibrating structure, a form of r-walk process characterises periods of segmentation (local non-stationarity).

such circumstances, the already known results of the general hidden Markov variants in the form of switching mean, intercept, variance, the adjustment parameters or their combinations can directly be adopted as explained in section three of the thesis. We present the results from the TAR model for the series graphed above in table 1 below. Market integration outcomes implied by the Markovian alternative with implications on degree of integration is also portrayed in the table.

Table 1: TAR and MS-EM Estimates for Simple Non-linear Relations (A)

| | Linear Model | B-TAR- Price Differentials | | |
|-------------------------|---------------------|-----------------------------------|----------------------------|-----------------|
| Variable | | Regime 1 | Regime 2 | Regime 3 |
| Thresh. Point | | $R_1 \leq -1.73$ | $-1.78 \geq R_1 \geq 1.69$ | $R_1 \geq 1.69$ |
| Const | 0.0400 (0.048) | -1.9199 (0.331) | 0.0512 (0.061) | 2.0736 (0.290) |
| R(t-1) | -0.2645 (0.022) | -0.9014 (0.109) | -0.0515 (0.061) | -0.9634(0.099) |
| Reg Prob | 1.0000 | 0.1958 | 0.5505 | 0.2537 |
| Davies | | 91.663(0.000)*** | | |
| | | MS-EM | | |
| Full series-Rent | | | | |
| Const | 0.0137 (0.041) | 0.0149 (0.041) | 0.0149 (0.041) | ----- |
| R(t-1) | -0.9000 (0.031) | -0.8851 (0.235) | -0.9077 (0.116) | ----- |
| Reg Prob | 1.0000 | 0.3405 | 0.6595 | |
| LR (Davies) | | 0.0112 (Na) | | |
| Sample 2-Rent | | | | |
| Const | -0.0101 (0.073) | -0.0101 (0.073) | -0.0101 (0.073) | ----- |
| R(t-1) | -0.8378 (0.057) | -0.8344 (0.045) | -0.8398 (0.163) | ----- |
| Reg Prob | 1.0000 | 0.3684 | 0.6316 | |
| LR (Davies) | | -0.0016 (NA) | | |

Source: Own Analysis with MSVAR 3.1: ***, **, *, 1, 5 and 10% levels of significance

Results from the null (linear AR) model are presented in column two of the table, while estimates from the SETAR model are shown in columns three. Two issues are evident from table 1. The estimated threshold points of (-1.78 and 1.69) from the TAR model seem to relatively under estimate the true threshold points of 2.2 in absolute terms. This can be attributed to the adjustment parameter imposed on ρ_1 vis-à-vis the innovations variance used.

Notwithstanding, the estimated values for rho (ρ_1) (-0.9014 (0.109) and -0.9634 (0.099)) and (ρ_0) (-0.0515 (0.061)) with standard errors in parenthesis¹³) strongly point to rapid adjustment process that characterises the system when the threshold point is exceeded; and near random walk process within the band as (ρ_0) does not differ significantly from zero.

The estimated intercept points (-1.9199(0.331), 0.0512(0.061) and 2.0736 (0.290) for regimes one, two and three respectively also reflect the symmetric nature of the system and closeness of the intercept term to the true TC level of 2.2). The test for the presence of threshold effects against the null of linear representation strongly favours the former as indicated by the likelihood ratio statistic and highly significant p-value for Davies statistic¹⁴. A stylised symmetric specification from Hansen's (1992) bootstrap based test also supported the non-linear formulation. The proportions of observations assigned to each regime are over-stated for the traded regimes-one and three- (about 45 against 55 percentages), given that 33 percent of all the observations were observed with trade (it seems to cover some of the transient shocks).

In the second half of the table the results from the MS-EM that imposes a switching adjustment parameter model with invariant mean (with rent series) on the process are presented. A three-state mean/AR switching model was also estimated as suggested by Krolzig (1998) on the price differences (R_t). Though linearity was rejected, in the later formulation, the three adjustments parameters did not differ significantly from one another (-0.6277 (0.102), -0.7003(0.083) and -0.5876(0.065)) for states one, two and three respectively and with standard errors put in brackets. The inability of the Markovian models to capture the two alternating adjustment process can be due to the low persistence nature of the series, due to the parameter sets that underlie the DGP, especially the innovation variance. For the rent series used, the results strongly suggest that the markets are characterised by rapid rent correction process as can be inferred from the estimated adjustment parameters (-0.8851 and -0.9077). Similar conclusion is portrayed by the concentrated out sample, denoted as sample two of the table. The adjustment parameters also point to strong reversion of shocks onto the threshold point. The likelihood ratio

¹³ Henceforth figures stated in this order places the associated standard errors in parenthesis.

¹⁴ Davies test is based on the phase-dispersion minimization method that takes into account the presence of nuisance parameters in the non-linear alternative. Hansen (1992) employs simulation technique.

and the associated Davies test also rightly supports that no non-linear or switching inter-market adjustments processes characterise the system.

Table 2: TAR and MS-EM Estimates for Simple Non-linear Relations (B)

| | <i>Linear Model</i> | B-TAR (Price Differentials) | | |
|-------------------------|---------------------|------------------------------------|----------------------------|-----------------|
| <i>Variable</i> | | <i>Regime 1</i> | <i>Regime 2</i> | <i>Regime 3</i> |
| Thresh. point | | $R_1 \leq -1.80$ | $-2.06 \geq R_1 \geq 1.90$ | $R_1 \geq 1.90$ |
| Const | -0.1361(0.016) | -0.9908 (0.282) | 0.0418 (0.066) | 0.9282 (0.227) |
| R(t-1) | 0.0267 (0.036) | -0.4023 (0.078) | -0.1804 (0.065) | -0.3767 (0.064) |
| Reg Prob | 1.0000 | 0.2274 | 0.4611 | 0.3116 |
| Davies | | 29.94 (0.000)*** | | |
| | | MS-EM | | |
| Full series-Rent | | | | |
| Const | -0.0065 (0.046) | 0.0068 (0.045) | 0.0068 (0.045) | ----- |
| R(t-1) | -0.5766 (0.031) | -0.3384 (0.055) | -1.2346 (0.175) | ----- |
| Regime Prob | 1.0000 | 0.6441 | 0.3559 | |
| LR (Davies) | | 43.93 (0.000)*** | | |
| Sample 2-Rent | | | | |
| Const | 0.0052 (0.062) | 0.0052 (0.062) | 0.0052 (0.062) | ----- |
| R(t-1) | -0.1270 (0.049) | -0.1270 (0.195) | -0.5320 (0.149) | ----- |
| Regime Prob | 1.0000 | 0.4077 (0.672) | 0.592 (0.774) | ----- |
| Davies | | 2.145 (1.000) | | |

Source: Own Analysis with MSVAR 3.1: “***”, “**”, “*”, indicates significant levels (1,5 and 10 % significance levels)

The results from series B, presented in table 2, carry similar conclusions as implied by those in table 1 for the threshold model. The threshold model rightly captured the implied inter-market conditions and with relatively closer estimate of the threshold point (-1.80 and 1.90) compared to the true value of 2.2 in absolute terms.

The adjustment parameters for periods with trade also compare well to the true value of (-0.35). Slightly different results are however observed for the MS-EM. In this case since some sort of adjustment persistence holds for periods with trade, the model captured the two alternating

adjustment processes that are implied by the threshold effects. Again, the estimates for the concentrated out sample did not indicate any form of non-linearity.

The analysis above shows that when relatively higher noise and less persistence structure pertained, the MSAH(2) failed to detect the two alternating adjustment processes implied by threshold effects, while the TAR models captured the true dynamics. This is not surprising since the DGP of the series comes from TAR structure. It also implies that the MS-EM is more likely to capture real inter-market anomalies since these tend to show persistence over time. While both trade flow binary and transactions cost levels were known, they do not enter into the TAR/SETAR estimation. The presence of two distinct adjustment processes and the fact that increased number of threshold regimes did not alter the general results, indicate that the markets are perfectly integrated in the case of series A, and take some time to revert shocks to the TC levels in case of series B within threshold structure. Even though no direct test for testing for the type of threshold effects (b-TAR or Eq-TAR) is considered, the significance difference between the estimated regimes intercepts correctly imply b-TAR structure. Thus falling on the theoretical insights raised on major sources of non-linearities in inter-markets equilibrium systems in previous chapters, we have reduced the test for MI to testing for regime shifts in the arbitrage forces.

Since the null of linearity could not be rejected in the second stage test from the MS-EM results we conclude that the markets are integrated and arbitrage opportunities are efficiently responded to, especially with series A, once TC is exceeded. If linearity had been rejected in the second stage then, trade flow binary and price transmission analysis would be combined to distinguish between perfect/imperfect integration and segmented equilibria or disequilibria periods in a general six/four-state regime switching specification as in PBM setting (decomposed in (5.05)).

From equation (5.05) therefore, only one inter-market condition is interested for policy and strategic decisions; that is, whether trade occurs within the threshold band. With trade flow data, and the fact that two alternative adjustment processes ensue, the MS-VEM with four-state structure is imposed, following proposition (5.05). Since it is assumed under b-TAR models that the size of price differential at time t , determines tradability at time $t+1$ or t , given the underlying equilibrium condition of course, we are interested to checking whether periods with strong rent

persistence drove trade. For illustration purpose we present results for series B in table 3, since it appears from regime two (-0.1804 (0.065)) of table 2 that some activities occurred within the threshold band as (ρ_0) *does differ significantly from zero*. The estimated results of the vector formulation model (MS-VEM)- with Rent- are presented in table 3 below.

Table 3: MS-EM Results for MI with Tradability Implications (Series B)

| Variable | Regime 1 | | Regime 2 | | Regime 3 | | Regime 4 | |
|-------------|---------------------|---------|----------|---------|----------|---------|----------|---------|
| | Rent | Trade | Rent | Trade | Rent | Trade | Rent | Trade |
| Constant | -0.0549 | 0.0183 | -0.0549 | 0.0183 | -0.0549 | 0.0183 | -0.0549 | 0.0183 |
| Std Errors | (0.041) | (0.014) | (0.041) | (0.014) | (0.041) | (0.014) | (0.041) | (0.014) |
| Rent_1 | -1.2008 | 0.0010 | -0.4580 | 0.0368 | -0.3587 | 0.7192 | -0.3740 | 1.5335 |
| Std Errors | (0.111) | (0.015) | (0.128) | (0.027) | (0.066) | (0.024) | (0.120) | (0.060) |
| Trade | 0.0406 | 0.0038 | -0.6647 | 0.9303 | 0.1235 | 0.4060 | 0.5649 | 0.3301 |
| Std Errors | (0.086) | (0.024) | (0.115) | (0.041) | (0.061) | (0.020) | (0.196) | (0.054) |
| LR (Davies) | 581.273 (0.0000)*** | | | | | | | |

Source: Own Analysis with MSVAR

Expectedly, rent adjustment parameter (ρ) (-1.2008 (0.111)) for regime one shows a persistence that is linked to threshold effects. No trading activities are associated with this regime as indicated by the insignificant dependence between trade and lag rent (see regime one from table 3 above), correctly implying that the markets are well functioning.

Again, regimes three and four that characterise the major trading activities have strong link with trade as portrayed by highly significant rent coefficient ρ_1 (0.7192(0.024) and 1.5335 (0.060)), indicating that trade is highly dependent on the size of rent in the previous period. A third tradability regime was captured, where no significant trade dependence on rent was observed but otherwise. Regime two in effect seems to have captured periods of transition from tradability point into the threshold band (consider rent coefficient of (0.0368) and (-0.6647) of trade from regime two of the table). We also fixed a three and five state structure to the system, but no new economic outcome emerged. The number of significant trade regimes increased while others remained in the five-state case. The three-state model on the other hand combined regimes three

and four. In all the cases, the null of linearity was strongly rejected by the Davies test. The flexibility of MS-VEM along the theoretical propositions developed has been shown by the simple non-linear synthesised market data. The results show that the non-linear processes assumed for the concentrated-out samples were strongly and correctly rejected since no such mixture dynamics existed.

5.3.1.2 Results from PBM (series A and B)

In this section, the standard PBM is applied to the same basic non-linear data set used under the time series specifications above. We formulate the model along spatial competitive equilibrium conditions of ESTJ framework discussed under equations (2.01) to (2.02) of chapter two in mixture distribution function (5.06) and (5.07).

$$L = \prod_{t=1}^T \left[\lambda_1 f_t^1 + \lambda_2 f_t^2 + (1 - \lambda_1 - \lambda_2) f_t^3 \right] \quad 5.06$$

$$L = \prod_{t=1}^T \left(A_{jit} \cdot \left[\lambda_1 f_{jit}^{1Td} + \lambda_3 f_{jit}^{2Td} + \lambda_5 f_{jit}^{3Td} \right] + (1 - A_{jit}) \cdot \left[\lambda_2 f_{jit}^{1nT} + \lambda_4 f_{jit}^{2nT} + \lambda_6 f_{jit}^{3nT} \right] \right) \quad 5.07$$

For comparative purpose, we have assumed that the series presented above follow an ideal inter-market rent series, such that at any point in time t , one of three possible rent levels ($R_t < 0$, $R_t = 0$ or $R_t > 0$) and its corresponding arbitrage dynamics or responses obtain. Thus, by defining the threshold process as a global stationary system (see Balke and Fomby 1999), all rent levels that are less than zero ($R_t < 0$) at expectation are considered as disincentive to trade while those that exceed zero ($R_t > 0$) provide incentive to trade. This rationalisation is adopted to ensure that both the dynamic and static models can be subjected to same interpretation from the models estimates.

The models are therefore interpreted under the assumption of bi-directional tradability with symmetric TC constraints. In practical settings, as indicated already, rent to trade at any given time t , is measured by the absolute value of price differentials less inter-market TC at time t . If information on direction specific TC and trade flow volumes exist, direction specific rent can be constructed for each time point and estimated within the same modelling set up but results

interpreted accordingly. For our case we do not implement direction specificity aspects of the theory, since our focus is to evaluate how PBM will behave in complex non-linear settings when time series features hold on one hand; and how MS-VEM can be used to infer the very insights PBM generates.

The rationale behind this modelling framework is that when tradability holds, any indication of returns to inter-market trade ($R_t \neq 0$), signaling failed or foregone arbitrage opportunities, means that the markets are characterized by imperfectly competitive equilibrium. Hence, absolute higher rent levels that arise from threshold effects (rent levels greater than zero at expectation but less than TC in absolute terms ($\tau_t > R_t > 0$)) do not violate perfect competitive market equilibrium condition or inter-market integration if no trade is observed in the former. In effect, price differentials that fall below normal threshold implied TC constitute “normal” negative incentive to trade while those that exceed such TC levels imply arbitrage failure. It follows that negative incentives to trade ($R_t < 0$) that signify market segmentation of policy concerns are defined by woefully higher cost of trading (as may be caused by governments trade restrictions, or other physical barriers) beyond the usual TC that results in threshold effects. This means that normal price differentials that trigger trade do not exceed cost of trade under segmented equilibrium periods of no market integration to motivate trade. In effect, once a fair approximate of normal TC is determined, real rent to trade can be constructed. This follows in that in PBM framework one has to reconstruct the b-TAR series to reduce all arbitrated points to global equilibrium $R_t = 0$ (zero at expectation), thereby rolling back the observations within the TAR band as un-attended rent/loss depending on how far they deviated from the threshold bounds.

To implement the model, all price differentials were reduced by their respective TC (constant under series under consideration); and the data frequencies reduced in line with the practice in the literature. Thus, the data frequency is reduced to break the dependence structures imposed by the time series characteristics¹⁵ (see Barrett and Li, 2002 and Park et al. 2002). Though only one lag structure was imposed, we reduced the data frequency by three for series B and two for series A. Since information on trade flow and TC data were available, the complete PBM

¹⁵ It is worth noting that this does not remove the normal transitory shocks, since they do not occur at a deterministic interval. Lowering the data frequency breaks the dependence structure.

structure which assumes a six regime market conditions was imposed on the system.¹⁶ The regime probabilities of the unrestricted model are presented in table 4 below.

To infer the relative insights of our proposed Markovian regime switching specifications and the PBM with particular attention to possible DGP and rent construction procedures, two rent constructions were considered. In addition to our assumed structure, we also estimated results from the conventional data construction settings, where absolute price differentials minus TC are used. Row two of the table holds the observed regime probabilities. The series contain two components, namely periods with and without trading activities. We however, expect the models to distinguish between rent levels that differed significantly from the transactions cost component in absolute terms. As expected, the results from the conventional rent construction shown in row three of the table indicate strong evidence of inter-market segmentation and inefficiencies as indicated by regime probabilities for regimes three (0.1327), four (0.0644), five (0.1978) and six (0.6050) for series A in particular. Given the fact that, 34 percent of the series coincided with trade, the estimated (0.000) for regime one indicate under-estimation of the perfect integration regime, since rent fully cleared with trade.

Table 4: PBM Estimates for Simple Non-linear Inter-market Relations

| Data | Regime Probabilities | | | | | | LR-statistic |
|---------------------------|----------------------|----------|----------|----------|----------|----------|--------------|
| | Regime 1 | Regime 2 | Regime 3 | Regime 4 | Regime 5 | Regime 6 | |
| Observed Prob | | | | | | | |
| Series (A) | 0.3305 | 0.6695 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | ----- |
| Series (B) | 0.4600 | 0.5400 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | ----- |
| Conventional- Rent | | | | | | | |
| Series (A) | 0.0000 | 0.0000 | 0.1327 | 0.0644 | 0.1978 | 0.6050 | 0.812 |
| Series (B) | 0.0895 | 0.5394 | 0.3710 | 0.0000 | 0.0000 | 0.0000 | 1.179 |
| Constructed- Rent | | | | | | | |
| Series (A) | 0.3305 | 0.6694 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.812 |
| Series (B) | 0.3146 | 0.5394 | 0.0000 | 0.0000 | 0.1450 | 0.0000 | 1.367 |

Source: Own estimation ¹⁷ ; Probabilities may not sum up to one due to run-ups.

¹⁶ Direction specificity implications suggest more than six-regime structure for all possible equilibrium conditions.

¹⁷ Due to highly instability of the PBM structure with complex layers 'multiple initial values' in the maximization process were used; numerical score problems encountered.

It must be noted that the intrinsic structure of the PBM technique, makes it very sensitive to the clusters formed within the system and more importantly as demonstrated by its proponents, the model fails when the series is leptokurtic. Thus, since constant TC is applied, reducing absolute of all rent by the TC creates asymmetric structure (see figure 6 below).

Similar conclusion holds for series B, but in this case the cluster is formed around the perfect integration regimes. Again 37.10 percent of the 46 percent traded points were categorised as imperfect integration outcomes. These results are not surprising since as demonstrated by Barrett and Li (2002), the model is very sensitive to the distributional assumptions that underpin the model structure.

The series under our stated assumption also point to the fact that though periods of trade resulted in full adjustments, the model clearly underestimates regime one for series B in particular. As will become obvious in the next section, the PBM works well when clear component of the assumed normal distribution for regimes one and two hold; or obvious periods of deviations from the system obtain. To link the relative strength of our proposed Markovian specifications, we used same data set that was used in the estimation of the PBM.

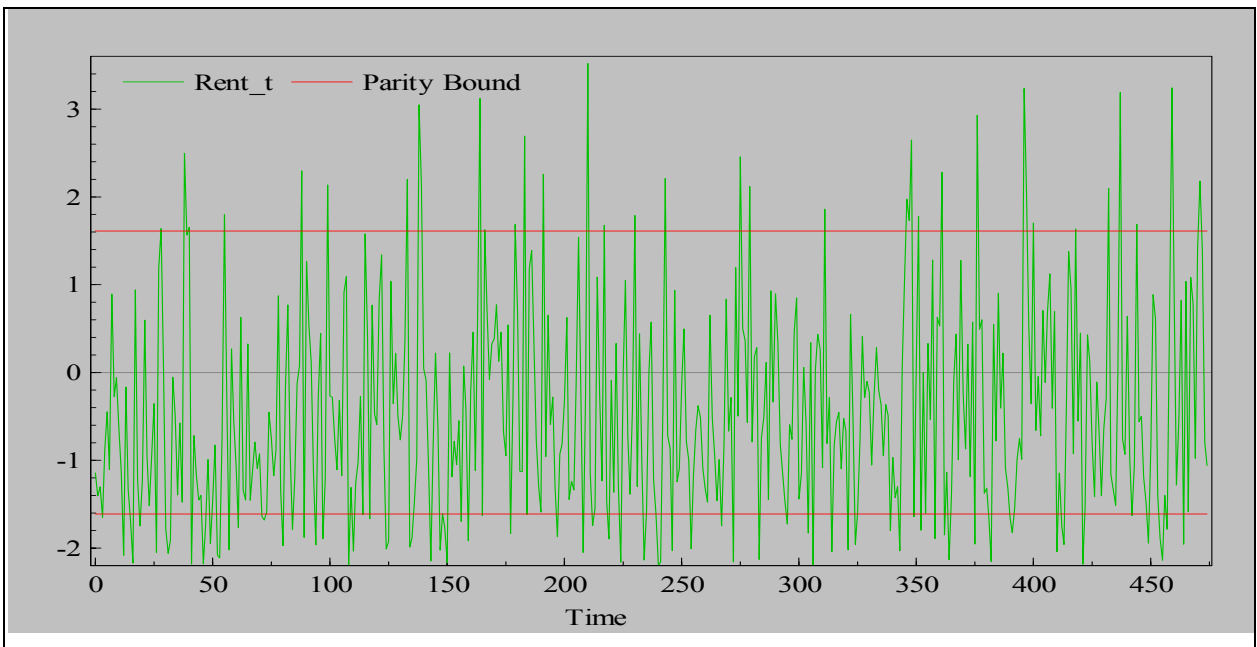


Figure 6: Conventional construction of Rent

From our theoretical position and as demonstrated in Krolzig (1998), a mixture error component model such as the PBM can sufficiently be captured by MS model with regime switching on the mean and or the variance of the assumed normally distributed error component, with respect to the number and structure of the differing components assumed. As noted already, the basic rationale for mixture distribution assumption behind the PBM is to capture three varying rent levels ($R=0$, $R<0$ or $R>0$). We in effect impose regime switching on the mean, implying that at any given time t , either rent is equal to zero or otherwise. For illustration, we use data set A, and model it within MS-(V)EM(0).

Table 5 used only rent information to capture the various inter-market conditions. In this case three regime structure is imposed as discussed in the previous section. Three distinct rent levels were correctly identified. The symmetric nature of the series is also reflected by the mean values for the outer regimes (-1.3886 and 1.3589). Regime two which correspond to zero rent at expectation has insignificant rent levels (-0.0388 (0.1314)). Regimes one and three however show that rent levels significantly differ from zero (-1.3886 (0.0875) and 1.3589 (0.0829) for the two regimes respectively). Given the data set under consideration, the associated regime probabilities can directly be linked to those in the PBM.

Table 5: MS (3)-EM(0) Results for MI with Simple non-linear series (A)

| Variable | Regime 1 | Regime 2 | Regime 3 |
|--------------------|--------------------|----------|----------|
| Rent (Mean) | -1.3886 | -0.0330 | 1.3589 |
| Std Err. | (0.0875) | (0.1314) | (0.0829) |
| Reg Prob | 0.3228 | 0.3285 | 0.3488 |
| PBM Prob. | (0.3285) | (0.3488) | (0.3228) |
| LR (Davies) | 54.7895 (0.000)*** | | |

Source: Own calculations

This is indicated in the table as (PBM Prob.) where regime one is linked to periods in which rent is fully cleared. This constituted (32.85 percent), while those in regime three and one with

positive and negative rent ($R_t \neq 0$) respectively cover about 67 percent of the inter-markets activities. Given that trade occurred 34 percent over the period, while TC constraints held for the remaining periods, the MS(3)-EM(0) correctly captured the regime dynamics implied by the DGP. It must be noted however that since threshold point was fixed at 2.2 in absolute terms, the captured ($R_t \neq 0$) levels must be interpreted with care in referring real inter-markets anomalies.

Figure 7 below shows the regime probabilities produced by the three-state model. The upper panel of the figure shows the rent levels (in red), and the estimated means or the fitted rent (in blue). While this model well describes the data, MI implications as to segmentation or imperfections are limited since no adjustment dynamics or tradability can directly be inferred. To verify whether regime two captured all the periods where trade was observed, we impose a six regime structure as in the case of the PBM. A vector specification is adopted in which rent and trade flow data are used. The estimation results are presented in table 6.

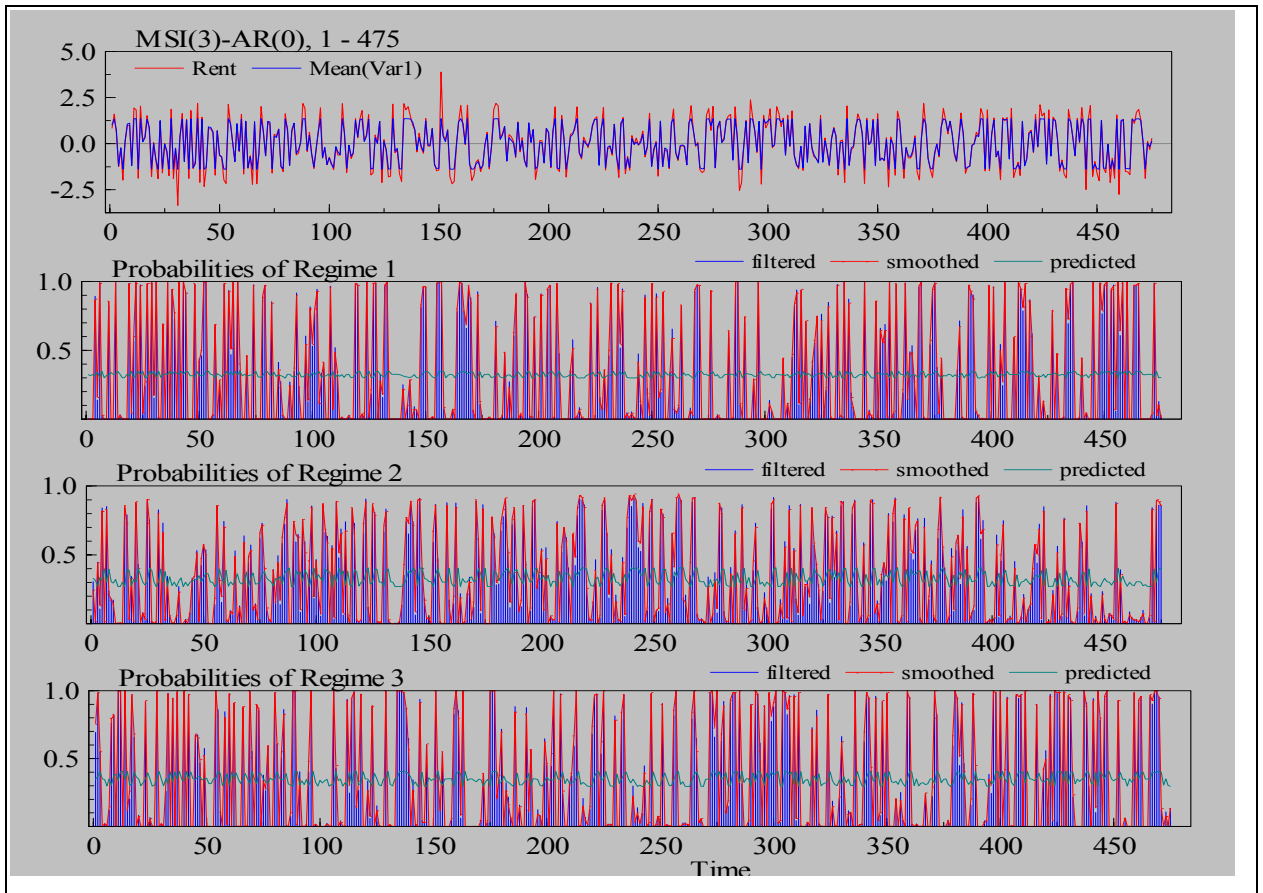


Figure 7: Regime Probabilities for MS(0)-EM (Series A)

The trade and rent levels show that the MS-EM identified the market conditions implied by the data set used. The various regime implications from the MS-VEM(0) within market equilibrium conditions are directly linked to the regime interpretations of the PBM. This is indicated as (PBM Prob.) in the table. Regimes one and six correspond to regime one of the parity bound model. In these regimes as can be seen from the table, trade volumes are significantly different from zero (-0.7229(0.010) and 0.7943(0.014)) with zero rent ($R_t = 0$) (0.0306 (0.099) and 0.1199(0.129) for regimes one and six respectively. Thus, when trade occurs under perfect competitive market system rent clears to zero, implying that no imperfect integration (regimes three and five under the PBM) holds. Three significant rent levels were captured that did not correspond to trade. From table 6, regimes two and three captured segmented equilibria conditions (negative rent levels- $R_t < 0$, (-1.1516 (0.22) and -0.8711 (0.202)), while regime five 1.0501 (0.182) correspond to segmented dis-equilibrium under our stated symmetric assumption; that is, regime four of the PBM.

Table 6: MS-VEM Results for MI with Tradability Implications

| Variable | Regime 1 | Regime 2 | Regime 3 | Regime 4 | Regime 5 | Regime 6 |
|---------------------|------------|------------|------------|-----------|-----------|----------|
| Rent (Mean) | 0.0306 | -1.1516 | -0.8711 | 0.4044 | 1.0501 | 0.1199 |
| Std Err. | (0.099) | (0.221) | (0.202) | (0.384) | (0.182) | (0.129) |
| Trade (Mean) | -0.7229 | -1.33e-007 | -2.90e-007 | 4.08e-007 | 1.91e-007 | 0.7943 |
| Std Err. | (0.010) | (0.014) | (0.011) | (0.017) | (0.009) | (0.014) |
| Reg Prob | 0.2076 | 0.1252 | 0.1962 | 0.0847 | 0.2613 | 0.1250 |
| PBM Prob. | 0.3321 | 0.0847 | 0.0000 | 0.2613 | 0.0000 | 0.3214 |
| LR | 658.106 | | | | | |
| Davies | (0.000)*** | | | | | |

Source: Own Analysis with MSVAR

Regime four from the Markovian model captured periods with zero rent (0.4044 (0.384)) and no trade (4.08e-007 (0.017)), which represent perfect competitive and integrated market conditions of regime two. Given our knowledge about the TC effects we can conclude that the MS-VEM has rightly identified the market structure. In this case the autarky conditions can be attributed to

normal TC constraints¹⁸. The low persistence nature of such segmented regimes can be seen from figure 8 below.

Within the MS-VEM, the four-regime market conditions system was directly supported by the available information criteria; (4.4078; 4.4595; and 4.5393 for three state specification. 3.2678; 3.3471 and 3.4694 for four state formulation: 3.2764; 3.3901; and 3.5656 for five state model; and 3.3170; 3.4721 and 3.7114 for the six state structure) given (4.5341; 4.5513 and 4.5779); for the null model, from AIC, HQ and SC criteria respectively.

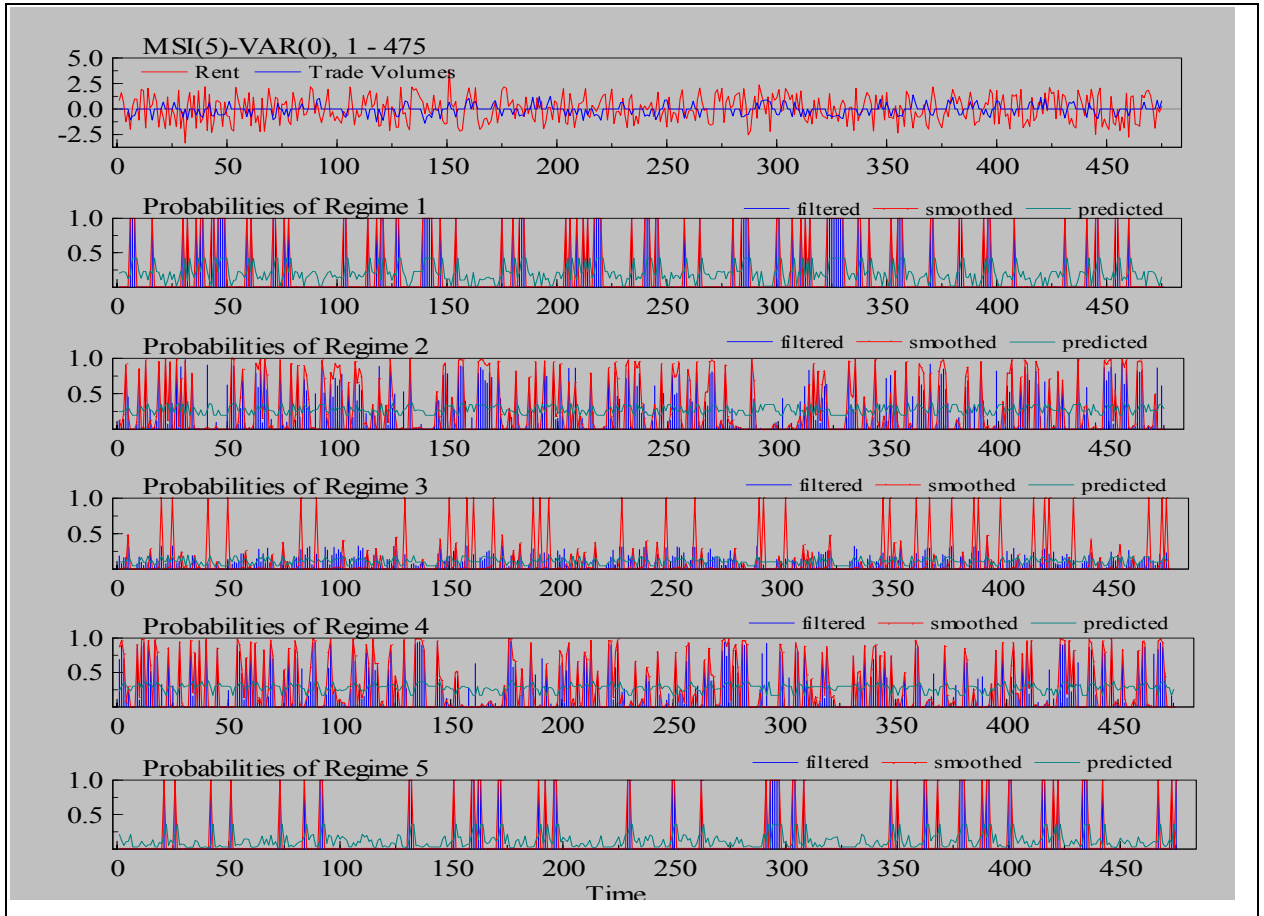


Figure 8: Regime Probabilities for MS-VEM(0) (Series A)

Since the true data generating processes used in the series analysed represented perfect competitive market and integrated system with transactions cost effect, the PBM model has produced mixed results. The results from series A indicate that the various cluster points formed

¹⁸ Similar conclusions hold for the other forms of rent construction used in table 2.

by the random walk nature of the rent series seem to influence the outcomes of the PBM, even though about half of the series were thrown away. While the strength of the PBM strongly depends on the underlying distributional assumptions, the flexibility and well established computational algorithm for the Markovian version allows it to capture variety of regime switching processes once appropriate theoretical foundation is developed.

In this section, the strengths and weaknesses of the PBM as have been already noted in the literature have been highlighted by the analysis of two simple nonlinear equilibrium-based synthesised data sets. Attempts have been made to pinning possible causes of the shortfalls of the model. Since the analysis in particular suggests that once distinct inter-market conditions exist expected results are generated for corresponding regimes, when perfect integration pertains under TC-based threshold systems, wrong conclusions can be drawn if no test for threshold effects are considered. Again, it must be noted that the PBM model is very unstable and sensitive to initial parameter values set. Given these, and the complexities that threshold effects can produce in both static and dynamic frameworks under current limited theoretical insights on implications of various forms of TAR structures (b-TAR and Eq-TAR) in MI studies, MI analysts need to take extra care when results from PBM are being interpreted for policy conclusions.

Based on the system dynamics, we have followed same theoretical assumptions that guide PBM and TAR models in a Markovian framework to capturing same market insights carried by the former models. It must be noted that while MSA(M)-AR(p) can be used to capture switching adjustment parameters, the TAR model that directly searches through an endogenous state variable, as it is already known, can capture threshold dynamics when low persistence structures exist than the MS alternative as occurred under the analysis of series A. Krolzig (1997) suggest endogenous selection MS model (EMSM) as an alternative model to STAR (smooth transition autoregressive model) and SETAR model. However, if the switching process does not follow threshold structure as real inter-markets anomalies imply, the TAR models would not be appropriate (see next section). From the settings of the PBM, we have demonstrated how the MS-(V)EM can be utilised to analyse market integration and implied efficiency based on the very assumptions and model structure of the former. This brings the PBM as a probabilistic

model and the time series models into a common equilibrium framework, which ensures that similar economic conclusions can be drawn from the two MI modeling techniques.

The above analyses of the simple non-linear set point to the fact that, while the two main lines of non-linear assessment tools for MI analysis have their particular strengths, the nature of the true underlying data generation process, thus whether inter-markets rent dynamics follow threshold effects as the model implies; or how the normal plus half-normal assumption fits into the system, can lead to different results and conclusions if they are not taken into account. For instance, while both MS-VEM and the classical TAR models were able to capture the non-linearity implied by TC-based threshold effects the PBM produced mixed results, based on the time dynamics and cluster the various distinct rent levels form.

Again, these simple non-linear series have shown that while MI analysis within equilibrium framework turns to be nothing more than arithmetic deductions, imposing a process-based description of the markets on the equilibrium structure provides both insights about arbitrage and competitiveness of the markets in question. Since TC-based threshold non-linearities do not in general imply market failure or inefficiency, how these models can assess market conditions in the presence of real market imperfections as may be attributed to market power or segmentations as defined by arbitrage failures or autarky conditions is worth consideration.

In the next section we add one more level of non-linear complication that introduces market segmentation and evaluate the models ability to sufficiently classifying them into various market equilibrium conditions that underpinned the true DGP

5.3.2 Switching Inter-market Conditions (Complex non-linear series)

In this section the same three major non-linear tools considered in the previous sections are used to analyse two different data sets that are characterised by relatively complex non-linear processes. The non-linearities reflect switches between inter-markets conditions within equilibrium structure as discussed in the previous sections. We again consider a data set from relatively static equilibria framework which does not contain time lags. In addition, regime changes that follow theoretical dynamics of arbitrage behaviours under imperfect or segmented

market equilibrium conditions are imposed. Like series B from the simple non-linear structure, we generate b-TAR series with switching inter-market conditions. These series are denoted as series C and D respectively in the diagram below (figure 9). Again a simplified structure of equation (5.05) was used. For series C, ρ_1 was set to negative one (beta=0) for tradability periods beyond the threshold point as in the perfect integration case considered above. We denote the series that contains the TC as price differentials (in blue) and those that the observed TC has been removed as rent (shown in green).

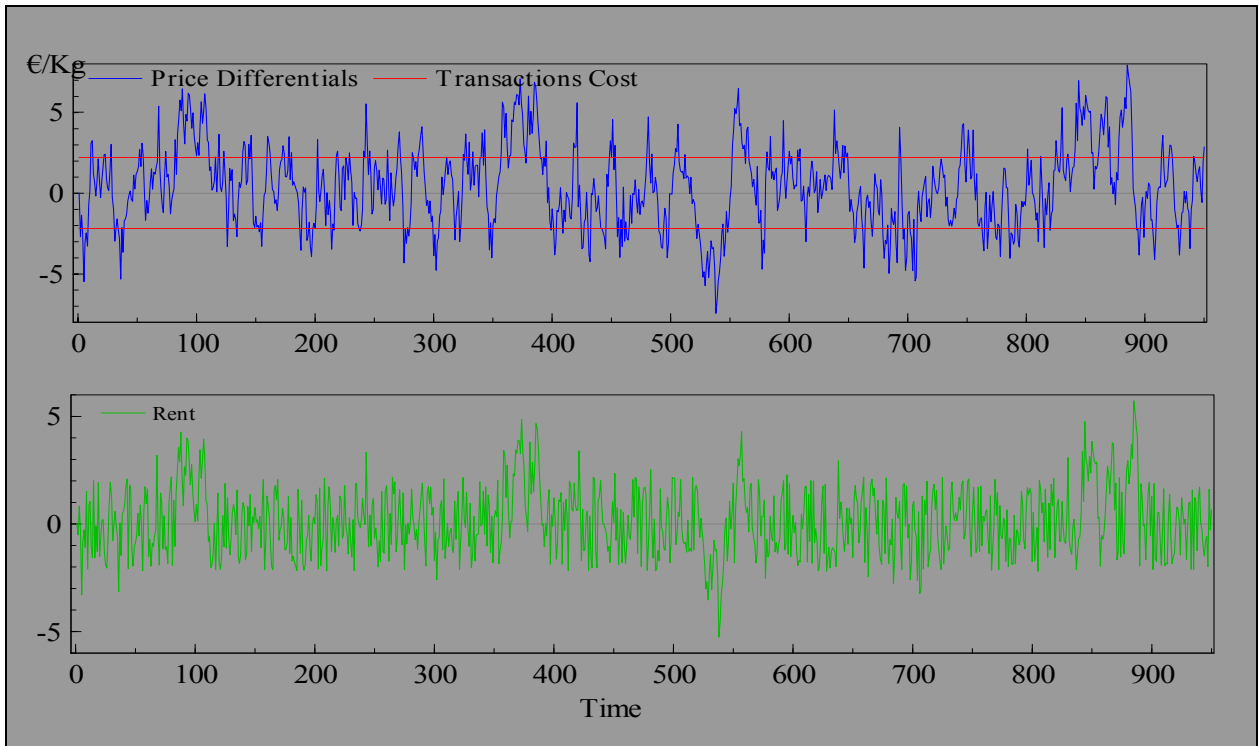


Figure 9: Complex Non-linear Series (Static structure-no time lags)

The innovation variance was set at 1.4 with same threshold point of 2.2. In addition 4 periods of “real” imperfect/segmented conditions were fixed around time points (71:115; 341:390; 516:560 and 831:885). In these periods ($\rho_0=0$) was implied to reflect inter-markets segmentation periods.¹⁹

¹⁹ Ox codes for all data sets -DGP- used are available from the author.

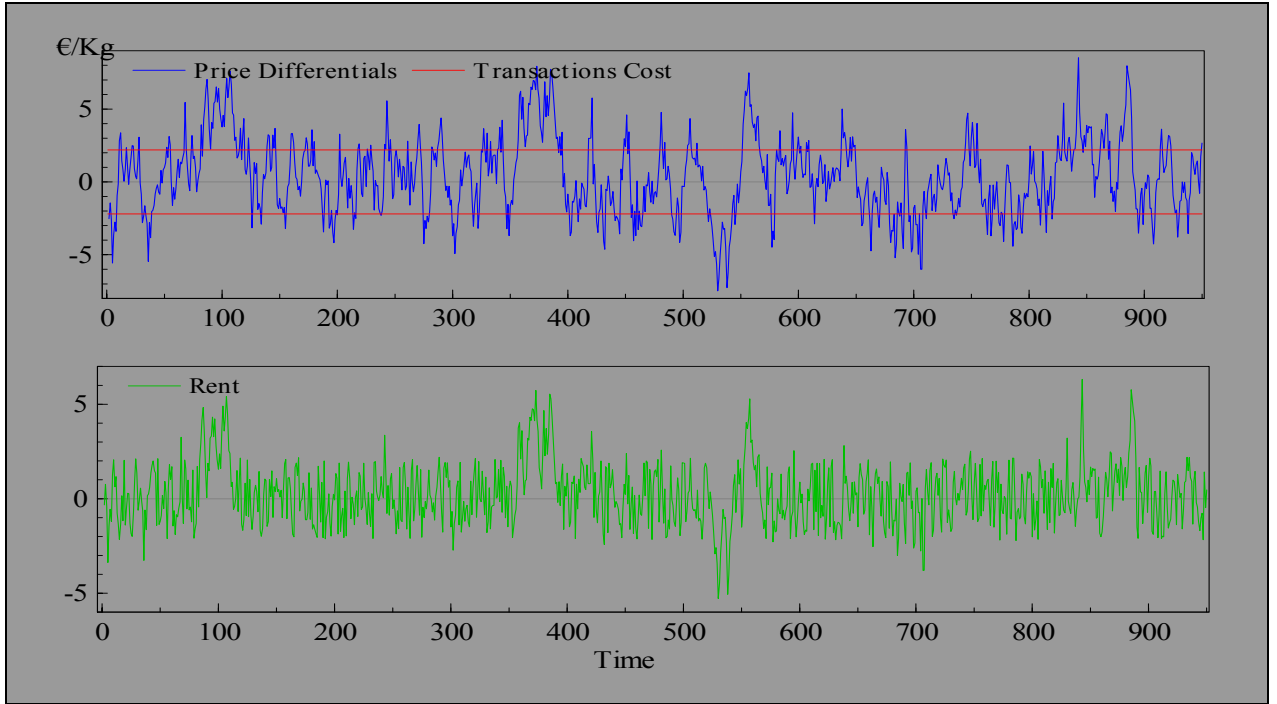


Figure 10: Complex Non-linear Series (Time lag structure)

With series D same time points were fixed but ($\rho_1 = -0.8$, with $\beta = 0.2$) with same threshold points and innovation variance of 1.4. Trade observations were not pre-assigned but were determined by the threshold constraint imposed on the innovations, in the general setting.

We also fixed true segmented periods in which neither trade nor rent adjustments occurred. In general, trade flow was allowed to vary within the imperfect integration regimes based on same threshold dynamics to imply indirect trade or information flow, rather than physical trade flow (price transmission or adjustments obtained but no physical trade observed).

5.3.2.1 Results from TAR models (series C and D)

Here, critical issues that associate analysis of relatively complex inter-markets processes with general TAR models as defined in (5.01) to (5.04)) are highlighted. We also demonstrate how such insights can be utilized in market integration analysis, when the DGP is characterised by mixture of threshold and switching market equilibrium conditions. Assuming that the series under consideration is characterised by non-linear equilibrium dynamics of inter-market margins due to say threshold effects in presence of significant amount of transactions cost, a b-TAR

specification is adopted. If the adjustment process is governed by threshold effects as a result of TC constraints, then ρ_1 from equation (5.03/4) should not differ significantly from zero, to reflect random walk nature of R_t within the threshold band (see section 5.2.1.1 above). The results from the two series are presented in table 7 and 8 respectively below. Again the estimated results from the MSVAR package on OX are presented. We also present the results of MS-EM (MSIAH(3)-and MSAH(2)-ARX(0) in same TAR formulation) in the lower half of the table.

Table 7: TAR and MS-EM Estimates for (Series C)

| | <i>Linear Model</i> | B-TAR | | |
|----------------------------------|---------------------|---------------------|-------------------------------|--------------------|
| Variable | | Regime 1 | Regime 2 | Regime 3 |
| <i>Price Diff. series</i> | | $R_{-1} \leq -2.77$ | $-2.77 \geq R_{-1} \geq 2.94$ | $R_{-1} \geq 2.94$ |
| Const | 0.0769 (0.043) | 0.5005 (0.637) | 0.1523 (0.074) | -0.5780 (0.553) |
| R(t-1) | -0.1860 (0.018) | -0.2033 (0.163) | -0.1647 (0.055) | -0.0588 (0.119) |
| Regime Prob | 1.000 | 0.2864 | 0.5695 | 0.1484 |
| LR (Davies) | | 42.00 (0.000)*** | | |
| | | MS-EM | | |
| <i>Price Diff. series</i> | | | | |
| Const | 0.0769 (0.043) | -0.7522 (0.303) | 0.0939 (0.168) | 2.0246 (0.843) |
| R(t-1) | -0.1860 (0.018) | -0.4669 (0.106) | -0.1425 (0.039) | -0.0551 (0.473) |
| Reg Prob | 1.000 | 0.2540 | 0.6745 | 0.0716 |
| LR (Davies) | | 26.62 (0.004)*** | | |
| <i>Full series-Rent</i> | | | | |
| Const | 0.0868 (0.037) | -0.0210 (0.045) | -0.0210 (0.045) | ----- |
| R(t-1) | -0.6223 (0.032) | -0.1320 (0.046) | -0.9531 (0.039) | ----- |
| Regime Prob | 1.0000 | 0.1738 | 0.8262 | |
| Davies | | 125.0 (0.000)*** | | |
| <i>Sample 2-Rent</i> | | | | |
| Const | 0.0816 (0.068) | -0.0339 (0.078) | -0.0339 (0.078) | ----- |
| R(t-1) | -0.2608 (0.037) | -0.1356 (0.058) | -0.9895 (0.077) | ----- |
| Regime Prob | 1.0000 | 0.4080 | 0.5920 | |
| LR (Davies) | | 47.31 (0.000)*** | | |

Source: Own Analysis with OX-MSVAR 3.1:

From table 7, like the simple data set considered in section 5.3.2, the impact of the strong rent correction and relatively large value of the innovation error in relation to the TC level tend to slightly overstate the threshold point (-2.77 and 2.94 compared to -2.2 and 2.2 levels fixed).²⁰ Analysis of the likelihood sequence also showed that an inter-mediate threshold regime, located around (-1.0999), was picked.

More importantly, unlike the *perfect* integration system considered above, the strong rent correction implied by periods of perfect integration (-0.2033 (0.163) and -0.0588 (0.119) for regimes one and three respectively) is blurred by the strong persistence that also characterise segmented inter-markets system that do not follow any threshold process. Nonetheless, test for threshold alternative against the null of linear process strongly favoured the former as can be seen from the likelihood ratio- LR (42.000) and highly significant p-value from the Davies test ((0.000)***).

The number of observations assigned to the respective three regimes also matched the observed probabilities that associated the series. For instance, the intermediate regime that correspond to periods of threshold effects was allocated 56.95 percent which is relatively close to the observed 62 percentage of the observations that fell under perfect integrated market condition. The results from series D also carry same conclusion. In this case, none of the adjustment parameters is statistically different from zero (-0.1419 (0.143); 0.0069(0.038) and -0.0377 (0.075)) for regimes one, two and three respectively, suggesting that the system is driven by purely random walk process. The adjustment parameter for the linear model (-0.1619 (0.017)) however, of low correction rate, differed significantly from zero, implying that at least the markets are characterised by some sort of imperfect integration than segmentation.

In general these results from the SETAR models do not point to exclusive conclusion for TC-based threshold effects, where rent correction parameter (ρ_1) values for regimes one and three are expected to be high in absolute terms. As demonstrated under the theoretical proposition of section 5.2.0, complete market integration implications alter the threshold space with an additional layer of non-linear complication. In this respect the three- state b-TAR model would not produce estimates that a pure threshold DGP will suggest. To capture the true inter-market

²⁰ In earlier version of the analysis where different errors were allowed for periods within and outside the TAR-band, the estimated threshold points were very close to the true value fixed.

dynamics in effect requires a regime structure of equation (5.05). As noted already, while system (5.05) is still characterised by same two major alternating adjustment dynamics (only that, same persistent adjustment dynamics do characterise some periods beyond the threshold bounds), the inherent b-TAR structure of system (5.05) does not allow an invariant mean but switching adjustment parameters as will hold for Eq-TAR structure.

The results of three-state mean/AR switching MS-EM as b-TAR alternative (see Krolzig 1997) are presented in rows five of tables (7 and 8) for series C and D respectively. In both cases regime one captures a relatively faster rent correction episodes ((-0.4669 (0.1062) and -0.3915 (0.103)) compared to those of the TAR models (-0.2033 (0.163) and -0.1419 (0.143)) for series C and D in that order. For series C, information criteria selected four-state model while five-state structure was selected for series D; but the overall conclusion of the presence of a mixture of two alternating adjustments structures remained. The symmetric nature of the system became rather obvious with additional states with different intercept terms.

With respect to inter-market analysis, if care is not taken, the model estimates can easily be taken as a process of asymmetry by degree of adjustments, especially under series C of table 7 above, where only one (-0.1647 (0.055)) of the three adjustment parameters appeared significant under the b-TAR specification.

While a general conclusion of a mixture of imperfect inter-market relation with some threshold effects can be drawn from the three-state MS-EM, the estimated intercept points (-0.7522 (0.303) and -0.6338 (0.303)) for regime one in both series C and D respectively, and their associated rent correction (-0.4669 (0.106) and -0.3914 (0.103) also imply a form of asymmetric structure with respect to those in regime three (2.0246 (0.843) and 1.4840 (1.255) for intercepts and -0.0551(0.473) -0.0178 (0.275) for the correction terms). In effect it was not surprising information criteria suggested higher level regimes. Thus, capturing the true inter-market relations within complex equilibrium system with relatively high noise component as one would expect in typical market phenomenon implies good understanding of the markets in question since increased number of regimes may be required to capture the real system dynamics as in our example. Nonetheless, the MS-EM is not affected by the mixed adjustment patterns once some sort of persistence exists with the periods of inter-market anomalies.

Table 8: TAR and MS-EM Estimates for Series D

| | <i>Linear Model</i> | B-TAR | | |
|-------------------------|---------------------|---------------------|-------------------------------|--------------------|
| <i>Variable</i> | | <i>Regime 1</i> | <i>Regime 2</i> | <i>Regime 3</i> |
| Price Diff | | $R_{-1} \leq -2.86$ | $-2.86 \geq R_{-1} \geq 2.44$ | $R_{-1} \geq 2.44$ |
| Const | 0.0718 (0.047) | 0.6415 (0.576) | 0.0474 (0.054) | -0.5568 (0.331) |
| R(t-1) | -0.1619 (0.017) | -0.1419 (0.143) | 0.0069 (0.038) | -0.0377 (0.075) |
| Reg Prob | 1.0000 | 0.1021 | 0.6874 | 0.2104 |
| Davies | | 39.0336 (0.000)*** | | |
| | | MS-EM | | |
| Price Diff | | | | |
| Const | 0.0718 (0.047) | -0.6338 (0.303) | 0.1607 (0.115) | 1.4840 (1.255) |
| R(t-1) | -0.1619 (0.017) | -0.3914 (0.103) | -0.1209 (0.038) | -0.0178 (0.275) |
| Reg Prob | 1.0000 | 0.2934 | 0.6310 | 0.0756 |
| LR (Davies) | | 18.6585 (0.0769)^ | | |
| Full series-Rent | | | | |
| Const | 0.1294 (0.047) | 0.0376 (0.044) | 0.0376 (0.044) | ----- |
| R(t-1) | -0.5186 (0.028) | -0.1351 (0.041) | -0.8582 (0.037) | ----- |
| Reg Prob | 1.0000 | 0.1879 | 0.8121 | |
| LR(Davies) | | 126.2348 (0.000)*** | | |
| Sample 2-Rent | | | | |
| Const | 0.0068 (0.064) | 0.0152 (0.078) | 0.0152 (0.078) | ----- |
| R(t-1) | -0.5385 (0.049) | -0.1132 (0.046) | -0.8058 (0.078) | ----- |
| Reg Prob | 1.0000 | 0.3893 | 0.6107 | ----- |
| LR (Davies) | | 34.8925 (0.000)*** | | |

Source: Own Analysis with MSVAR 3.1: ***, **, *, Indicates significance levels at 1, 5 and 10% levels.

^ Information criteria select five state model;

These complications suggest that when a mixture of TAR and switching inter-market conditions ensue, threshold models may miss the true inter-markets dynamics that govern the system. To focus on the analysis of market integration within equilibrium settings, and the fact that some other form of market data may be available imply that some of these complications can be reduced by making use of additional available market information.

With respect to theoretical insights highlighted above we use information on transactions cost to construct rent series as in the case of the PBM, to remove the complications imposed by TC-based threshold effects. In this case the rent series reduces to a particular form of Eq-TAR

structure. Estimates from row six of tables 7 and 8 (-0.1320 (0.046) and -0.9531(0.039) for regimes one and two for series C; and -0.1351 (0.041) and -0.8582 (0.037) for series D) represent the two state invariant mean, with switching adjustment parameters, for the reconstructed rent series-denoted as full series. These distinguishing adjustment parameters suggest the presence of either TC-based threshold effects or segmentation induced autarky conditions. To investigate the later which has market policy and strategic implications, the effect of the former must be isolated.

As noted earlier, information on TC levels were used to concentrate out all rent levels that exceeded the threshold point. Unlike the simple non-linear series considered in the previous section (series A and B), results from row seven (denoted as sample 2), show strong rejection of linearity in favour of the two-state switching equilibrium adjustments specification. The estimated adjustment parameters (-0.1132 (0.046) and -0.8058 (0.078) for regimes one and two respectively for series C; and in likewise order, -0.1356 (0.058) and -0.9895 (0.077) for series D) also indicate the existence of two differing adjustment processes beyond potential cost of trade. Again, the estimated parameters are very close to true values fixed ($\rho_1 = 1.0$ and $\rho_1 = 0.8$) for series C and D respectively in absolute terms. Here, the right conclusions that the markets are associated with strong irreversibility in some periods outside the threshold band can be seen as a complex mixture of perfect, imperfect or segmented inter-market conditions as conceptualised in (5.05).

5.3.2.2 Results from PBM (series C and D)

Similar to formulations in section 5.3.1.2, we use information on trade flows binary to specify a complete PBM structure on the data. Thus, a six regime market structure is assumed, such that segmented equilibrium or disequilibrium is distinguished from integrated market conditions by trade flow binary. Again it is assumed here that the series presented above constitute an ideal inter-market price differentials where observations that significantly differ from zero at expectation ($R_t \neq 0$) are categorized into imperfect integration or inter-market segmentation conditions. It follows that cost of trade varies with respect to the market condition that pertains at any given time t . In table 9, three results are presented; in addition to the “observed

probabilities”, which underlie the complete equilibrium conditions imposed, the rent series under our stated assumption and constructed rent from conventional setting are also shown.

As an alternative representation to the parity bound technique, the Markov switching equilibrium model is again formulated within same equilibrium structure and theoretical propositions that underpin the former. To evaluate how the MS-EM can capture the same market integration and equilibrium insights acclaimed by the PBM, as in the case of the simple non-linear series, we used same constructed and reduced frequency rent series that suits the later model for the MS-EM estimation. As noted earlier, since the object of arbitrage-based equilibrium measures is to focus on detecting differing rent levels with reference to $R_t = 0$, to show that the PBM model is observationally equivalent to MS-(V)EM(0), let's assume that the markets under consideration is of a perfect integration system as implied by the PBM. In this respect both models reduce to a condition where $R_t = 0$ at expectation.

Here the general form of MS-EM (p) becomes a linear model with normal error innovations if AR(p) is involved. Without any such dependence structure as assumed under the PBM, if expected profit levels equal $R_t = 0$, then MS-EM(0) becomes a zero-mean system with normally distributed errors. If R_t switches between $R_t = 0$ and $R_t \neq 0$ over time as ESTJ theory posits for periods of segmentations and imperfections, then MS(3)-EM(0) (which in effect is classical HMMs) ensues. Here the variance can be allowed to vary across regimes. In this case the two models, PBM and MS-EM, given $R_t = 0$, can be seen as a mixture of normal and non-normal profit margins over time (see highlights from Krolzig 1998 on the linkage between MS and other models).

For illustrational purpose only results from series C are presented for the MS-EM(0). Row two of the table contains the “observed classifications” and associated regime proportions of the series under analysis.

Table 9: PBM Estimates for Complex Non-linear Inter-markets Relations

| <i>Data</i> | Regime Probabilities | | | | | | <i>LR- statistic</i> |
|---------------------------|-----------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|--------------------------|
| | Regime 1 | Regime 2 | Regime 3 | Regime 4 | Regime 5 | Regime 6 | |
| Observed Prob. | | | | | | | |
| Series (C) | 0.2400 | 0.6100 | 0.0400 | 0.0800 | 0.0000 | 0.0300 | ----- |
| Series (D) | 0.2500 | 0.6010 | 0.0400 | 0.0800 | 0.0210 | 0.0080 | ----- |
| Constructed Rent | | | | | | | |
| Series (C) | 0.2399 | 0.6057 | 0.0296 | 0.1194 | 0.0001° | 0.0053 | 1.1759 |
| Series (D) | 0.1769 | 0.5414 | 0.0639 | 0.1027 | 0.0524 | 0.0624 | 0.6464 |
| Convnt Const. Rent | | | | | | | |
| Series (C) | 0.2432 | 0.6968 | 0.0262 | 0.0336 | 0.0000° | 0.0000° | 1.1547 |
| Series (D) | 0.2155 | 0.6464 | 0.0778 | 0.0601 | 0.0000° | 0.0000° | 1.0175 |
| MS- EM (C) | | | | | | | |
| MS (3)-EM(0) | 0.7917 | | 0.1051 | | 0.1032 | | 94.43*** |
| Mean Rent | -0.0123 | | 2.6253*** | | -1.1687 | | |
| PBM Prob | 0.8911 | | 0.1089 | | 0.0000 | | |
| With Trade | 0.2392 | | 0.0303 | | 0.0000 | | |
| No Trade | | 0.6484 | | 0.0785 | | 0.0000 | |

Source: Own calculations. Probabilities may not sum up to one, due to approximations. (***, ** and * indicate significant levels at 1, 5 and 10% for Mean and LR statistic). All Regime probabilities were significantly different from zero except otherwise marked “°”.

The results in row three are the constructed rent under our stated assumption (price differentials reduced by respective TC) and denoted as “Constructed Rent”. The third row of the table portrays the results from conventional construction of rent (absolute price differentials less TC). Unlike the simple non-linear set, here, the PBM estimates are very close to the “observed proportions” shown in row two of the table. For instance, from series C of row three the estimated 60.57 and 23.99 percentages for regimes two and one respectively, are close to the 62 and 23 percentages that are classified under perfect integration condition. Similar conclusion can be drawn for the series denoted “Convnt. Const. Rent”, even though there is slight overstatement for regime two (69.68 and 64.64) for series C and D respectively, compared to (60.57 and 62.0) in that order; the estimates for the remaining regimes are not unexpected given the asymmetric structure implied by the constant TC.

In all the cases the evidence for market segmentation and imperfections are maintained. Even though the simple likelihood ratio calculated (in column three) ²¹ did not support this complex inter-market relation against a single equilibrium system, based on our prior knowledge about the system under consideration however, we can conclude that the PBM has captured the presence of inter-markets segmentation and imperfections. This follows in that, the various distinct rent structures that exist in the series conform to a form of mixture distribution that underlies the PBM.

When MSI(3)-EM(0) was estimated in same PBM fashion similar results were produced (see item three of row five of the table). Thus, when the various implied market conditions ($R_t = 0$, $R_t < 0$ or $R_t > 0$) were considered based on regime specific mean and available information criteria (DIC, SC and HQ) two state structure (MSI(2)-EM(0)) was selected. This follows in that only three percent of the observations fall under rent regime three (regimes five and six that correspond to $R_t > 0$) as presented in row two of the table. Again when these rent levels were calibrated by their respective trade binaries the regime probabilities reflected the equilibrium settings of the underlying DGP, presented under items four and five of row five. While regime six in particular seems to be underestimated by above model formulations, this can be attributed to the inherent outcome-based structure of the PBM notion. Thus, given our proposition that, activities within the threshold band that do not correspond to trade do not violate MI or competitive equilibrium, the observed proportions were calculated from observations that fell outside the threshold band. Since the parity bound is grounded on a similar principle of the “*law of one price*”, even if the markets have no relations but their price differentials fall within an expected/estimated parity bound ($R_t = 0$), although no rent correction occurs irrespective of rent sizes, the markets will be wrongly denoted as perfectly integrated markets (since conditions under regime two are satisfied).

For instance while 20.5 percent of all activities were assigned to segmented/imperfect market conditions; segmented (15.3%) and imperfect regimes (5.3%), only 15% were recovered as the “observed series” since those that coincided with the threshold effects were not counted. This, further under-scores the weaknesses of the PBM conceptualization in general, as static

²¹ It must be noted that these are not standardised. Numerical instability of the PBM also affected those computed from simple bootstrap replication.

equilibrium model, in describing markets inter-relationships over time. Thus while classical time series models fail when perfectly integrated and well functioning markets do not exhibit price co-movements (rent adjustments due to TC constraints), the *outcome* based measures in the opposite manner incorrectly treat two economically separated markets periods that happen to have substantial time points of lower price differentials as perfectly integrated markets.

Another issue that arises from time dynamics on the inherent static structure of the model is the type of threshold effects that underlie the system. If TC imposes restrictions on arbitrage forces, then threshold effects follow; but as to whether b-TAR rather than Eq-TAR process is context specific outcome. In effect assuming for one, usually the former and reducing all price differences by estimated cost of trade may create some misrepresentations if a mixture process is the case, but no account is taken in the survey or TC estimation. Since the two TAR effects have same economic and efficiency implications on the trader in equilibrium analysis, there is strong likelihood that the two processes will alternate over time. Because when they are driven by TC, classical two-multiple equilibria structure is implied. This follows in that decision to trade onto the global equilibrium point ($R_t = 0$) or the TC bound when price differentials exceed TC is based on preference and or the type of market industry under consideration since profit margins cover cost of initiating trade onto the global equilibrium ($R_t = 0$).

The distinction between real segmentation and normal TC-based segmented equilibrium as implied by the DGP imposed is once again not evident. The results here and those from the simple non-linear set suggest that when b-TAR process ensues, the PBM classifies the observations within the threshold band into perfect integration regimes, based on the deepness of the threshold band vis-à-vis the error structure within the traded regimes as indicated above. This means that the strength of the model depends very much on the existence of right clusters formed by the various inter-market conditions. In this respect adopting MS-EM(0) with trade binary that provides further information about the estimated (R_t) under each regime can help understand the structure under each of the regimes identified.

The above issues indicate how complex inter-markets relations could be, when the complete equilibrating structure is to be represented. While the PBM seems to work well with both static

and dynamic systems when true non-linear structures obtain, they tend to produce mix results depending on the cluster formed within the traded regimes by the threshold effect.

5.3.2.3 Identifying inter-market conditions from MS-V/EM

Given the market efficiency and competitive equilibrium implications derived from the PBM on one hand, and the degree and extent to which such conditions are arrived as given by the time series models on the other, employing MS-VEM in the settings of (5.05) reveals the market and equilibrium conditions associated with a given inter-market system. Thus while regime-switching specification on both the rent levels (mean) and the adjustment parameters can directly be adopted to assess both rent adjustment dynamics and arbitrage conditions as suggested from the results above, relatively increased number of states are expected. Given other market information used in PBM, in addition to prices series or rent series that are classically used in time series models for MI analysis, we utilise additional market information available (TC-in particular) to employ sample-splitting technique to capture the regime dynamics that are dictated by arbitrage responsiveness. A vector extension is also adopted to accommodate trade flow volumes directly, when it is appropriate for further insights.

The procedure adopted in section 5.3.1.1 is followed here. In this sense, the test for MI is reduced to testing for tradability in both price transmissions (rate at which rent corrects) framework and trade flow changes. This follows in that, once TC constraints are accounted for in dynamic framework, arbitrage forces must correct any transitory shocks if market integration holds. Since inter-market segmentation condition implies stronger persistence in rent dynamics- in a form of random walk process- than imperfect integration may imply, the general tradability condition can also be imposed on the concentrated-out sample as a switching arbitrage dynamics. To this respect, one can distinguish between perfect integration points that remained in the sub-sample (that are due to transitory shocks), imperfect integration and segmented market equilibrium conditions by the differing strengths of their respective rent correction parameters in relation to the normal profit level ($R_t = 0$). Thus falling on the flexibility of MS-models, MI analysis within both time series and static equilibria structures is approached in a regression framework (along the rationale behind arranged autoregression strategy usually used

in TAR models). We present the results of MS (4)-VEM (1) in tables 10 and 11 below for the concentrated out samples for series C and D respectively. With reference to system (5.05), if symmetric adjustment is assumed then tradability imposes three differing rent correction parameters (ρ_s) on the general TAR structure defined in equation (5.01/2) for a mixture of perfect (ρ_1), imperfect (ρ_2) and segmented (ρ_0) conditions.

Table 10: MS-(V)EM Estimates for Switching Inter-market Conditions (C)

| Variable | Regime 1 | | Regime 2 | | Regime3 | | Regime 4 | |
|------------|-------------|---------|----------|----------|---------|---------|----------|---------|
| | Rent | Trade | Rent | Trade | Rent | Trade | Rent | Trade |
| Constant | 0.0438 | 0.0177 | 0.0438 | 0.0177 | 0.0438 | 0.0177 | 0.0438 | 0.0177 |
| Std Errors | (0.075) | (0.012) | (0.075) | (0.012) | (0.075) | (0.012) | (0.075) | (0.012) |
| Rent_1 | -0.1169 | -0.0023 | -0.1072 | -0.0041 | -0.2276 | 0.2381 | -0.9237 | -0.4100 |
| Std Errors | (0.074) | (0.011) | (0.073) | (0.012) | (0.194) | (0.032) | (0.081) | (0.013) |
| Trade_1 | -6.2018 | 1.9298 | -5.0542 | 2.1163 | 0.3364 | 0.3052 | -0.2237 | 0.3242 |
| Std Errors | (0.952) | (0.169) | (29.68) | (4.536) | (0.580) | (0.094) | (0.236) | (0.038) |
| Reg Prob | 0.1485 | | 0.1367 | | 0.0848 | | 0.6300 | |
| LR | (381.080) | | | | | | | |
| Davies | (0.0000)*** | | | | | | | |
| AIC | (3.4731) | 3.4076 | (3.4855) | (4.3512) | | | | |
| HQ | (3.6692) | 3.5514 | (3.5858) | (4.3904) | | | | |
| SC | (3.9659) | 3.7691 | (3.7374) | (4.4498) | | | | |

Source: Own estimation. Standard errors for parameters and information criteria for linear model in parentheses

For present objective we specify a four-state regime switching model, since two forms of adjustment parameters are expected, as produced by the MS(2)-EM(1) under tables 7 and 8 respectively for series C and D above (-0.1132 (0.046) and -0.8058 (0.078) for regimes one and two in same order for series C; and in likewise, -0.1356 (0.058) and -0.9895 (0.077) for series D). Series C and D respectively in the tables represent the concentrated out rent samples from the series used under the PBM and the TAR models above.

While the choice for the four-state model is based on inherent theoretical assumptions behind market equilibrium and arbitrage conceptualizations raised in chapter three and drawn from that

of the PBM (that is, rent correction/no correction with or without trade), systematic assessment of available information criteria (AIC, HQ and SC) for other formulations (3 or 5-state model in a given particular case) also supported the theoretical conclusions implied by the model estimates. The estimated regime specific adjustment parameters from series C and D strongly imply mixture of two different arbitrage dynamics over the period.

Table 11: MS-(V)EM Estimates for Switching Inter-market Conditions (D)

| Variable | Regime 1 | | Regime 2 | | Regime3 | | Regime 4 | |
|------------|-------------|---------|----------|----------|---------|---------|----------|---------|
| | Rent | Trade | Rent | Trade | Rent | Trade | Rent | Trade |
| Constant | -0.0149 | 0.0156 | -0.0149 | 0.0156 | -0.0149 | 0.0156 | -0.0149 | 0.0156 |
| Std Errors | (0.072) | (0.012) | (0.072) | (0.012) | (0.072) | (0.012) | (0.072) | (0.012) |
| Rent_1 | -0.1087 | -0.0018 | -0.0961 | -0.0028 | -0.3015 | 0.2095 | -0.7405 | 0.3896 |
| Std Errors | (0.076) | (0.011) | (0.072) | (0.011) | (0.182) | (0.030) | (0.079) | (0.013) |
| Trade_1 | -5.2286 | 1.8896 | -1.0133 | 1.2337 | 0.7283 | 0.3418 | -0.1943 | 0.3115 |
| Std Errors | (2.629) | (0.423) | (2.498) | (0.423) | (0.578) | (0.094) | (0.213) | (0.035) |
| Reg Prob | 0.1355 | | 0.1274 | | 0.0833 | | 0.6538 | |
| LR | (382.584) | | | | | | | |
| Davies | (0.0000)*** | | | | | | | |
| AIC | (3.4476) | 3.3889 | (3.3431) | (4.2811) | | | | |
| HQ | (3.6347) | 3.5261 | (3.4387) | (4.3185) | | | | |
| SC | (3.9189) | 3.7344 | (3.5840) | (4.3753) | | | | |

Source: Own estimation. Standard errors for parameters and information criteria for other specifications in parentheses

For instance, as expected, regime one which seems to capture periods of inter-markets segmentation conditions is characterised by virtually no rent corrections (-0.1169 (0.074)) and (-0.1087 (0.076)) for series C and D respectively. Regime two also identifies same persistent adjustment process. Indeed, when three-state model was fixed under series D for instance, as suggested by the information criteria, the two regimes merged up (see figure 11 below).

While same holds for series C, the three-state model in addition combines that of regime three and as rightly suggested by the order of the three information criteria (see row three of the table

10) the true picture of the system distorts. These are shown in the order; five, four, three and the one-state models, with no parenthesis on the values for the presented model. In addition the adjustment parameters and the residual analysis support the four-state model for series C. The categorising factor from the estimates is the rent coefficient on trade.

Thus since trade is manifestation of arbitrage, we expect that where rent levels were high trade must associate the rent correction in the subsequence time point. As can be seen from the table, regimes one and two do not indicate any significant arbitrage activities as indicated by $(-0.0023(0.011))$ and $-0.0041(0.012)$ for regimes one and two under series C. Similar conclusion holds for series D(see table 11 above). Although lag trade was included in the estimation as in classical vector representations, it does not have any direct impact or meaning on the rent series in this example. In addition to near random walk process $(-0.2276(0.194))$ and $-0.3015(0.183)$, regime three has significant trade activities $(0.2381(0.032))$ and $0.2095(0.030)$ for series C and D respectively. As can be seen from the graphs below (figures 11/12), state three rightly captures the period where trade was fixed. In this particular case we did not allow rent corrections as may apply under price control economies.

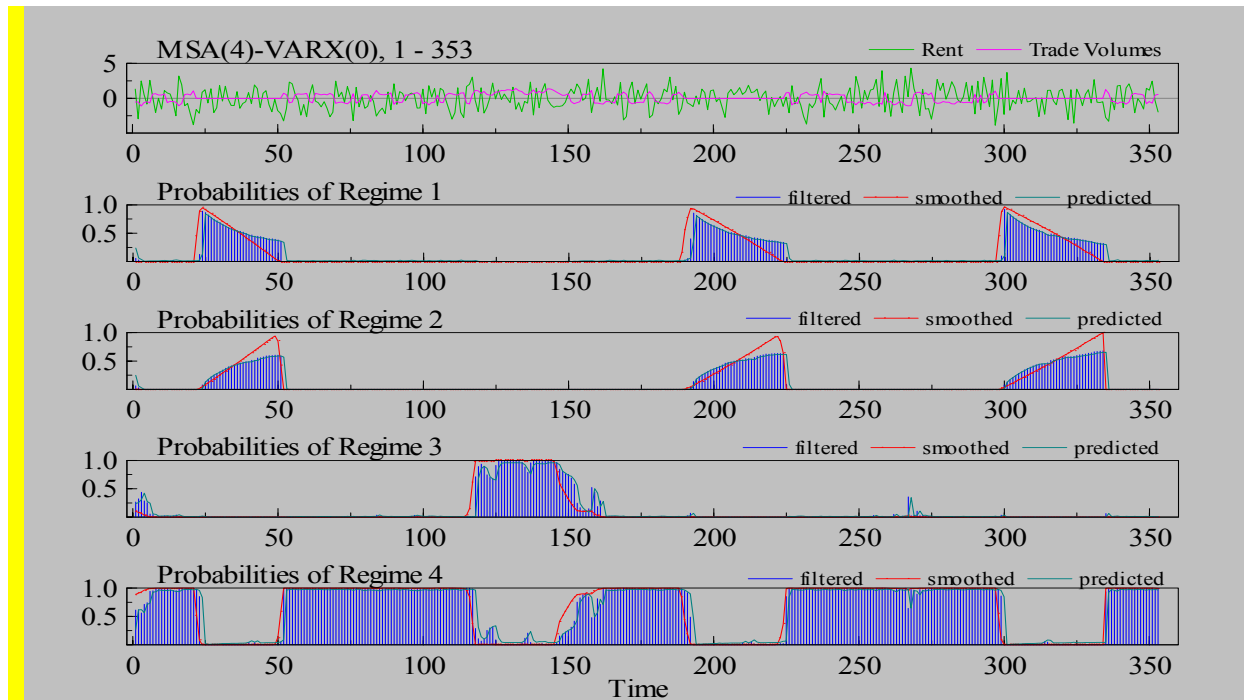


Figure 11: Regime Probabilities for Series C

Again, as expected, regime four indeed captures lagged elements of the normal transient shocks that come from perfect integration conditions. This is characterised by rapid rent corrections (-0.9237 (0.081) and -0.7405 (0.079)) for series C and D respectively. Tradability as expected from significant values for rent coefficients (-0.4100 (0.013) and 0.3878 (0.013)) on trade is confirmed.

Moreover, the true market conditions imposed on the inter-market processes for the two series are identified by the MS-VEM. The associated rent adjustment parameters also indicate that perfect integration existed in A (-0.9237). The foregoing analysis has shown that, with information usually available to MI tools, the MS-VEM can be adopted on same theoretical frame to combine the various insights they offer.

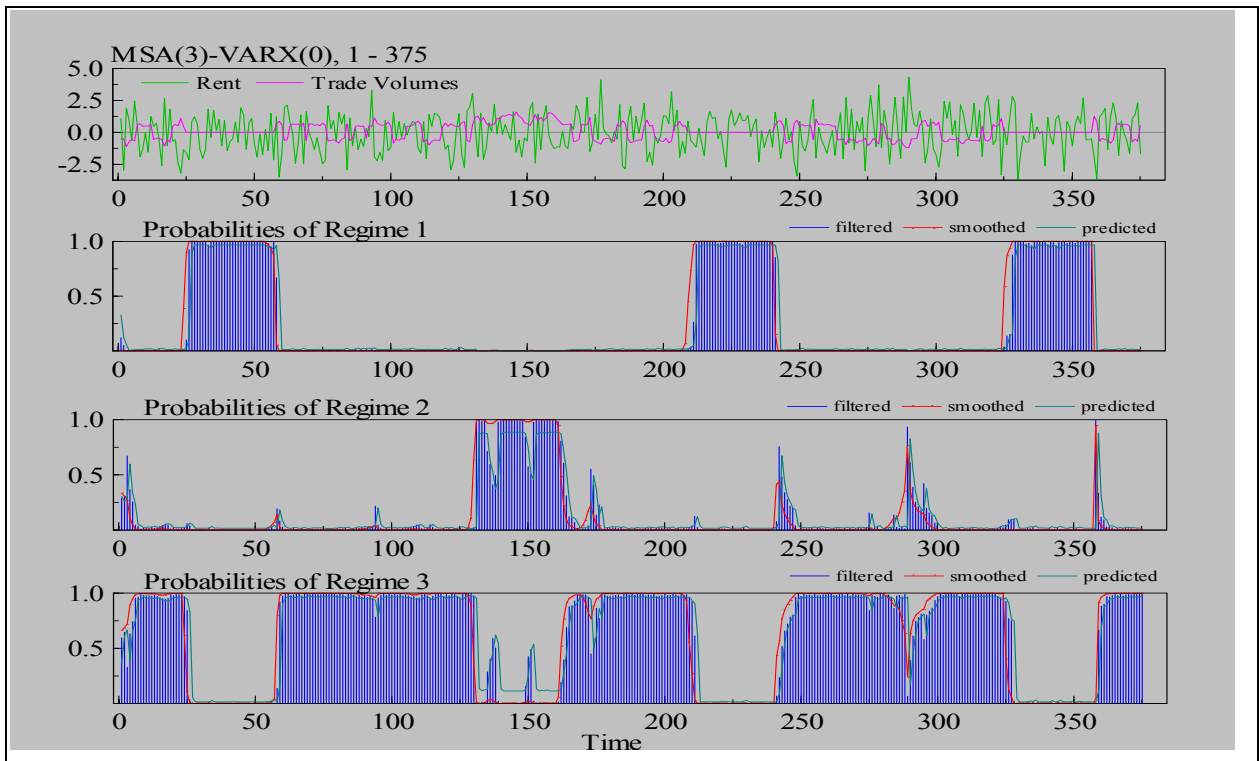


Figure 12: Regime Probabilities of Series D

Finally, as already noted above the need for MIA that include time dynamics (price transmission) or equilibrium correction is seen from the concept of tradability. If tradability holds by information flow or indirect physical trade flows through other markets, price transmission and in effect rent adjustments follow. While in time dynamics such adjustments

processes can be captured by price transmission analysis; this will tend to over-state the segmented regimes in the static formulation as under PBM. To illustrate this position we re-adjusted the trade flow volumes with respect to series C, such that the markets are characterised by two adjustment processes when the threshold point is exceeded. In addition to the instantaneous one time correction (ρ_1), we set ρ_2 to -0.55 in four distinct periods such that some persistence is created. This implies switches between perfect and an imperfect market conditions. In two cases physical trade is observed while rent gradually corrects towards the TC bound. Here we follow the proposition that price transmission occurs throughout the inter-market process once tradability holds (i.e. threshold point is exceeded). The trade binary is superimposed on the extracted rent series in figure 13 below. For periods around 71-120 and 511-560 as can be seen from the figure trade is observed with rent adjustments (ρ_2). In time points 341-390 and 831-885 however, no physical trade flows but rent adjustments hold. Since both sets have rent levels greater than the normal inter-market margins ($R_t = 0$), the use of trade binary unduly associate later periods into segmented market condition under PBM where no information on time dynamics is reflected.

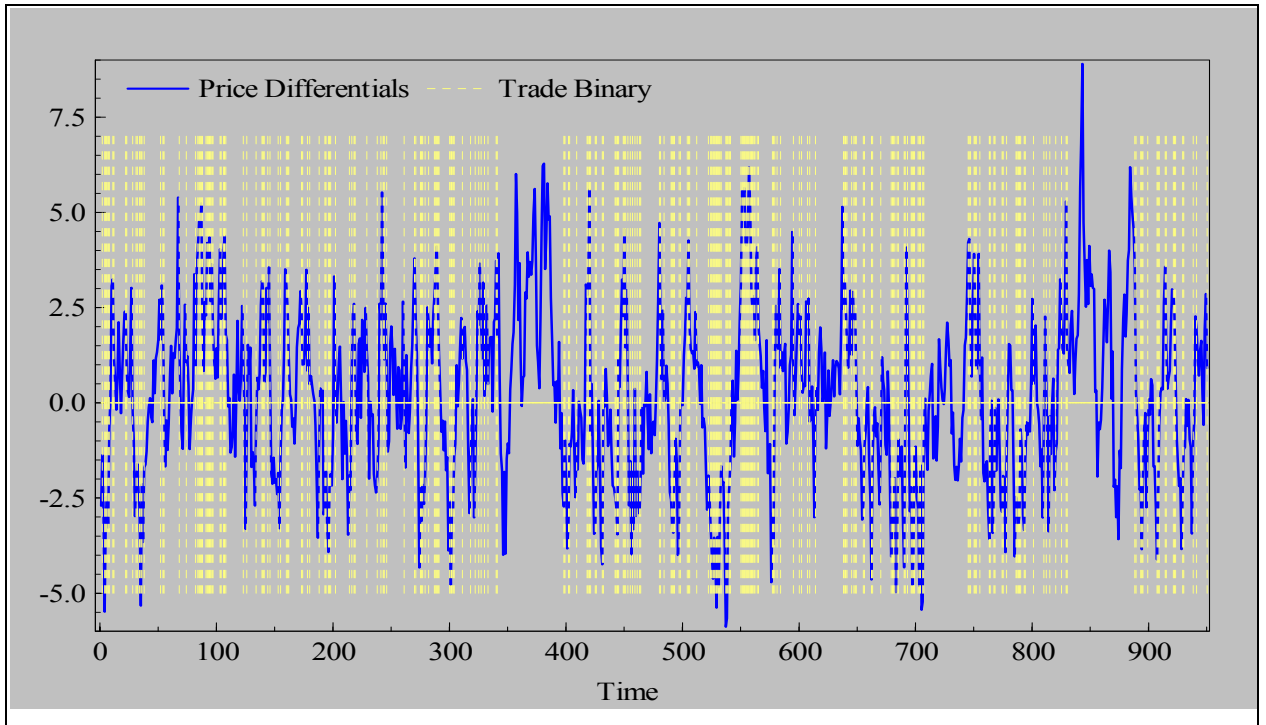


Figure 13: Series C with Modified Trade Patterns (full series)

In table 12 below we estimate MS-(V)EM for the series, where trade flow series, in volumes were included in the estimation. Even though regime one show relative persistence (-0.2799 (0.081)) than regime two (-0.5166 (0.164)) as a results of the vector specification, looking at their respective standard errors they are very close. In addition as expected regime three captured the perfect integration cases. Clearly, the results do not support evidence for market segmentation as opposed to the results from the PBM. In the later settings, imperfect integration periods that do not coincide with physical trade are taken as segmented regimes, thereby understating imperfect integration episodes. For the series under consideration, the PBM allocated (0.2669, 0.6527, 0.0000, 0.0000, 0.0173 and 0.0631) regime probabilities for state one through six in that order. About 06 percent of all activities were considered to have occurred under inter-market segmentation. As can be seen from table 12 below, even though four-state model is implied by tradability, three-regime model is suggested by the information criteria, since all traded periods correspond to some sort of rent adjustments or price transmission holds across all the regimes.

Table 12: MS-VEM Estimates for MI with Tradability Implications

| Variable | Regime 1 | | Regime 2 | | Regime 3 | |
|-----------------|-----------------------------------|---------|----------|---------|----------|---------|
| <u>Series A</u> | Rent | Trade | Rent | Trade | Rent | Trade |
| Constant | 0.0079 | 0.0095 | 0.0079 | 0.0095 | 0.0079 | 0.0095 |
| Std Errors | (0.081) | (0.013) | (0.081) | (0.013) | (0.081) | (0.013) |
| Rent_1 | -0.2799 | -0.0022 | -0.5166 | 0.2880 | -0.9843 | 0.4377 |
| Std Errors | (0.081) | (0.013) | (0.164) | (0.027) | (0.103) | (0.020) |
| Trade_1 | -4.7255 | 1.7992 | 0.5568 | 0.3147 | -0.2560 | 0.3345 |
| Std Errors | (2.087) | (0.358) | (0.451) | (0.020) | (0.292) | (0.044) |
| Reg Prob | 0.1863 | | 0.2049 | | 0.6123 | |
| LR | (308.29) | | | | | |
| Davies | (0.0000)*** | | | | | |
| AIC | (3.5885) 3.5420 (3.6264) (4.2839) | | | | | |
| HQ | (3.7440) 3.6504 (3.6971) (4.3263) | | | | | |
| SC | (3.9780) 3.8134 (3.8034) (4.3901) | | | | | |

Standard errors for parameters and information criteria for linear model in parentheses; ***, **, * indicate significant levels at 1, 5 and 10 percentage levels.

Similar conclusion is drawn from a two state model that do not utilise trade volumes. In this case, the adjustment parameters point to strong (-0.9794) (0.0989) and a weaker (-0.2936) (0.0856) rent correction regimes. Evaluation of a three-state model by information criteria and the associated estimated parameter values for the MS-EM strongly and rightly favoured the two-state specification.

The graphical representation of the regime probabilities portrayed in figure 14 shows that the two imperfect integration periods fixed without trade were rightly identified by both MS-EM and MS-VEM (see figures 14 and 15) below, as imperfect conditions.

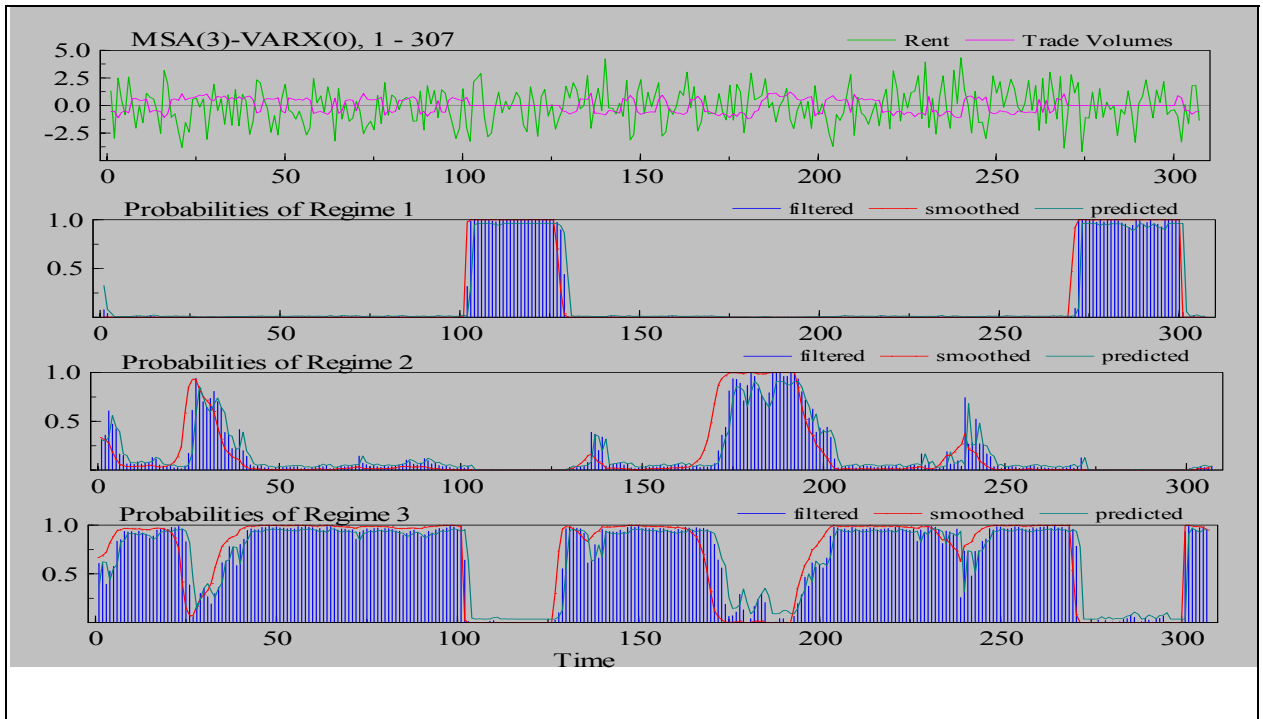


Figure 14: Regime Probabilities for New Series C (with trade)

The Markovian specifications presented above have demonstrated strong flexibility in the analysis of complex inter-market relations within market equilibrium framework, and within same theoretical conditions that guide both PBM and general TAR models.

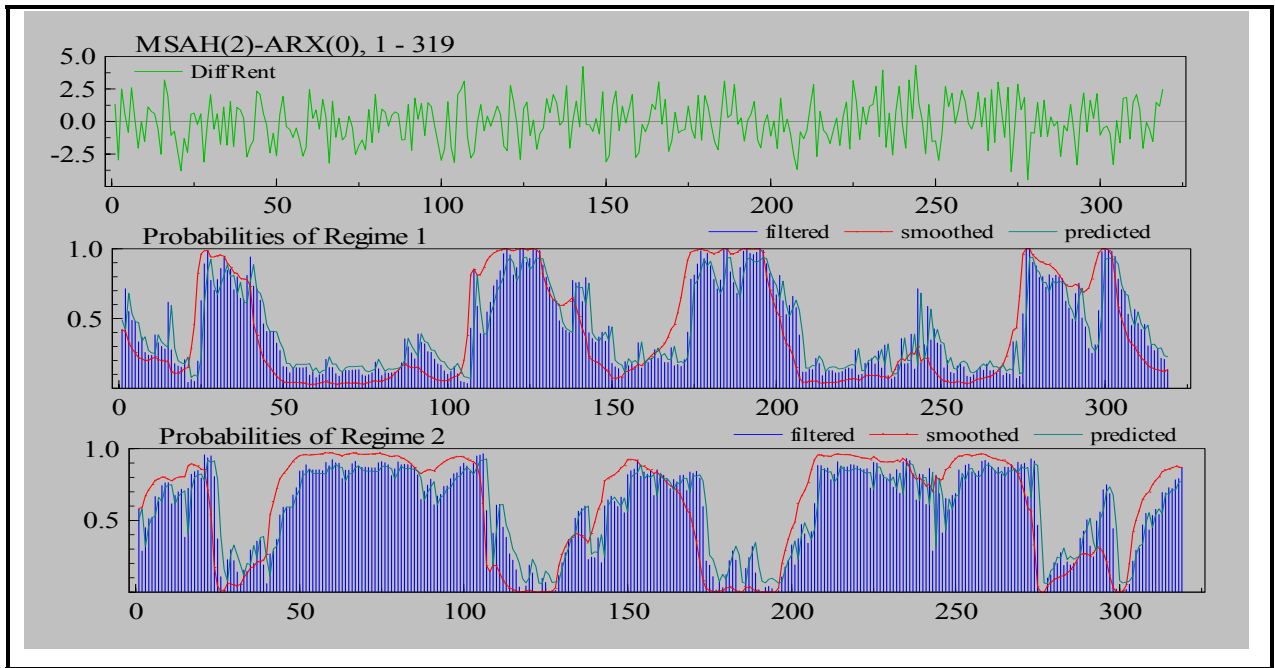


Figure 15: Regime Probabilities for New Series C (No trade)

5.4.0 Summary and Concluding Remarks

Synthesized market data have been used to demonstrate the implications of theoretical conceptualizations developed in previous sections. These major issues have been analysed within market equilibrium framework. Consequences of representing the true data generation process with different model specification assumptions on market integration conclusions underpinned the application of two different sets. When inter-market relations propagate on equilibrium structures for series with less dependencies, the PBM and HMM models work well with trade flow binary to identifying the various market conditions. The PBM suffers much when the system is characterised by normal transitory shocks within threshold induced clusters. Similarly, while trade flow information (binary) plays important role in market integration studies, they tend to over-state market segmentation when tradability is driven by both information and physical flow of goods. In this case price transmission or application of time-series models that condition arbitrage responsiveness on switching adjustment parameters appropriately capture the true inter-market conditions, once transactions cost complications are accounted for. Complications that result from imposing crucial static or dynamic equilibrium structures assumptions, that have driven the two methodological lines of MIA have also been highlighted and narrowed into a unified regime switching framework.

Methodologically, we have demonstrated that though when complex inter-market conditions characterise the equilibrium process traditional time-series models may fail to capture the true market conditions as the proponents of PBM claim, our proposed MS-(V)EM works well under such cases. Shortfalls and strengths of the various models have been demonstrated under specific inter-markets equilibrium conditions. More importantly, we have shown that within dynamic equilibrium structure, MS-VEM can be formulated within b-TAR framework to capture crucial complex non-linear systems that are implied by mixture of different inter-market conditions even if trade and TC data are not available. If information on TC is available or can be fairly estimated from b-TAR models, then sample-splitting along the idea behind the PBM (isolating the TC effects) that reduces the complexities imposed by the system noise and threshold effects on the real inter-market anomalies can be adopted in MS-VEM settings.

In sum, the chapter has highlighted and clarified intricacies that various theoretical propositions of MI concept impose on the two methodological lines through data application. We have systematically sliced up the concept of MI along both arbitrage *outcomes* and *processes* in time space. Possible economic implications of each complexity- trade flow, normal TC based threshold effects, static or dynamic structures and their combinations- have been raised. Suggested steps have been demonstrated through data reconstructions, decoupling or sample splitting; based on the complication at hand. The flexibility of the MS-(V)EM allows it to be formulated within both dynamic and static systems, which in effect stands superior to both SETAR and PBM techniques. The broadness of the concept implies that each market analysis with respect to methods and data must be supported by institutional analysis as a guide to attaching economic significance to significant econometric results.

SECTION SIX

6.0 SUMMARY OF FINDINGS AND CONCLUSION OF STUDY

The major findings of the study are summarised in this section, reflecting the main study objectives defined under the introductory chapter. Conclusions from the findings based on market integration and equilibrium concepts are drawn in the light of MI measurement techniques and associated policy implications. The contributions and limitations of the study, and future directions for improvements are also discussed in this final section.

6.1 Major findings of the study

The basic aim of the thesis focused on improving existing MI tools in the light of theoretical diversities that underpin the MI concept; or suggestion of an alternative econometric tool to the analysis of market integration within equilibrium dynamics. The worthiness of the challenge has been viewed from the important role market integration findings play in trade and market policies. In order to propose an alternative tool to the existing ones, theoretical investigation was launched in sections two and three of the thesis to diagnose the sources of limitations that are associated with MI techniques. Basic market integration concepts and the two major non-linear methods used in MI studies were assessed within market equilibrium structures. The modeling and policy connotations for over generalising basic but crucial concepts in market theory were systematically and contextually explained. By unearthing the basic forms and sources of inter-markets non-linear structures, a theoretical foundation was built that links the structure of Markov switching models to equilibrium representation of inter-markets relations over time.

Theoretically, we have re-defined market integration as a *process* and *outcome* instead of “*process or outcome*” that has driven existing methodologies. While the former is involving, it consistently fits the MI concept into the theory of multiple equilibria in time-space. Consequences of ignoring the time series dimension of the inter-market processes on modeling estimates and how these can lead to a loss of insightful policy information have been

theoretically explained and demonstrated in a synthesized data analysis. Specifically in the theoretical frame the research has;

- i) *linked the conceptual complexities that are inherent within perfect market integration conditions to the two main (Eq/b-TAR) TC-based threshold effects that characterise commodity markets. Chapter two in particular explained the implications and appropriateness of tradability assumption within ESTJ spatial equilibrium theory under Walrasian transfer. While trade flow information is crucial in commodity markets analysis in general, the role of information flow in price transmission or arbitrage dynamisms, especially in present information age, calls for extra care in categorizing inter-markets conditions based on physical trade binary. For instance, when tradability holds without physical trade flow, inferring tradability by only physical trade flows in instances of imperfect integration can result in misleading conclusions of MI and ME.*
- ii) *also asserted in section two that consistently analyzing inter-markets relations over time depends on the suitability of MI notion (outcome or process-based) assumed on the equilibrium system. The relationships between the law of one price, competitive spatial market equilibrium and implied efficiency, nature of arbitrage dynamics; and market integration within each modeling framework carry different economic implications. Thus depending on the nature of trade policy environment, distortions that characterise the markets and transactions costs involved in conducting trade, price series may behave in various ways of relationships. In price transmission and competitive market equilibrium modeling for instance, while transactions cost deter arbitrage to a certain threshold of price/rent variations between two market points, the behaviour of the long-run relationship between same markets in the face of economic uncertainties, policy changes or trade barriers may indeed be far from constant or linearity as may be implied by threshold processes.*
- iii) *by noting the failures of existing models in specific market conditions, in section three developed a conceptual foundation within theoretical implications of ESTJ spatial equilibrium model and time dynamics of the equilibrating process. On the grounds that market integration can be assessed by arbitrage conditions (outcomes), i.e. no arbitrage, arbitrage failure or autarky ruling, a particular form (time-space) of multiple equilibria is*

imposed on the equilibrating structure to represent the non-linearity defined by changes in arbitrage conditions over time. Since changes in such arbitrage conditions can be viewed in time dynamics from degree of rent irreversibility, it is in effect claimed that markets inter-relations fall within a complex dynamical system.

In order to infer inter-markets conditions from both the adjustment processes and the underlying data generation mechanism defined by arbitrage conditions (outcomes), we have proposed a Markovian forms of regime switching techniques, namely; MS/MC-VEM based on data availability. Methodologically the research has,

- i) in chapter four, argued from the basic structures of HMMs representations and existing regime switching models of MI measurement that MS-(V)EM framework provide a basic unified front for MI analysis. This is based on the degree of dependence and nonlinearity of markets inter-relations defined by spatial market equilibrium and tradability concepts. Put differently, since market integration in its complete structure is assessed by arbitrage conditions (outcomes) we have theoretically imposed adjustment dynamics that correspond to each outcome-based equilibrium condition. This allows us to discriminate between real inter-market anomalies and those that are due to normal TC constraints, the so-called threshold effects, by considering neighbourhood conditions in time frame. This dwells on implicit rationale behind PTE, where basic representation of market integration is described by the adjustment parameters (especially of the ECT); and PBM, which considers how rent at time t differs from zero at expectation.*
- ii) in chapter five, demonstrated the strengths and flexibility of the proposed MS-VEM along conventional tools with a synthesized market data. Theoretical conceptualisations developed were reflected in the analysis. The basic b-TAR was taken as the basic frame of MI analysis within equilibrium settings. When the inter-market relations propagate on equilibrium structures with less dynamic adjustments, the PBM and HMM models work well with trade flow data (binary) to identifying the various market conditions. However, while trade flow information plays important role in market integration studies and in the vector representation, they tend to over-state market segmentation when tradability is not only driven by physical trade flow between the markets under consideration. In this case application of PTE models that condition arbitrage responsiveness on switching adjustment parameters and trade information in a pair-wise structure seems more appropriate to*

capturing the true inter-market relations. Complications that can result from imposing crucial assumptions about b-TAR or Eq-TAR on the equilibrium structures that have driven the two non-linear methodological lines of MI analysis were also explained.

6.2 Concluding Remarks

Based on theoretical position that MI dwells not just on whether two markets prices series are inter-related, but more importantly how they differ conditional on expected cost of trade (transactions cost component), the research has operationalised a working definition for MI analysis as both outcome and a process. This allowed us to directly formulate equilibrium version of the classical threshold model in Markov switching modeling frame, in the form of MSAH-AR(p). MS-VEM attempts to bridge the gap between conventional PTE and PBM techniques by incorporating the basic equilibrium rationales behind the later two. Our choice for Markovian framework is based on the fact that, they are flexible and more importantly, the concept of MI can be complex.

From methodological standpoint, the synthesised exercise has shown the flexibility of Markovian models in handling non-linear processes without ignoring important features that may be present in the inter-markets process. We have thus, demonstrated that at least the Markovian regime models perform well as the PBM under same modelling assumptions, when even lesser information is available in the former case. While traditional time-series models usually applied in market integration analysis fail when the dynamic features are broken- trade reverses and discontinuities among others hold- the HMM version do not suffer under such conditions. Again, the study has revealed that threshold models may miss important non-linear structures that have serious policy implications if both threshold and other non-linear processes that are due to market insanity prevail. In the like manner MSAH(2)-AR(p) may fail to capture or well distinguish between threshold conditions if the system has less persistence and noisy structures with relatively short TAR band. More importantly, our sampling splitting technique suggests that b-TAR and MS-VEM can be combined to detect crucial complex non-linear systems that are implied by mixture of different inter-market conditions even if trade and TC data are not available (further investigation required).

Finally we have demonstrated that once arbitrage conceptualisations are adopted to impose adjustment structure on the equilibrating system (measure of irreversibility), consistent statistical testing of non-linearity in MI can be applied in stepwise. We have suggested sample-splitting styles along arranged autoregression idea, since the threshold effects imposed by TC do not influence adjustments processes outside the TAR bound. If the TC levels are taken as known, then existing testing tools can be combined to handle such complex non-linearities imposed by various market equilibrium conditions. It is however, obvious as it is always in economic issues that the conclusions from MI models are more or less specific given ones knowledge and underlying assumptions about the markets in question. The broadness of the concept implies that each market analysis with respect to a particular method and information used must be supported by other non-quantitative institutional insights in order to distinguish between economic and econometric significance of results.

Although, the flexibility of MS-(V)EM allows it to be formulated within both dynamics and static systems, which in effects stands superior to both traditional TAR and PBMs, our proposed models can be seen as a benchmark for integrated and robust tools for MI analysis. The basic models, as defined above can be extended to take into accounts all sorts of conceptually consistent notions of market integration- asymmetry and particularly imposing non-constant restrictions on the threshold parameter in order to account for policy effects and market efficiency over time.

The study did not consider short run adjustment structures that have dominated PTE models. However, following Krolzig et al. (2002) and Brümmer et al. (2005), same theoretical propositions raised here can be imposed on the error correction term (ECT) within the MS-VECM. More importantly, the study did not formulate formal hypothesis test for identifying number of market equilibrium conditions that pertain within a given time frame. This limitation is stemmed from current computational challenges that exist for standardising non-linear switching models and in Markovian framework in particular. Instead sample splitting technique was suggested to narrowing down the non-linear complications that arise from threshold effects after which our theoretical assertion was verified by combining standard information criteria and significance of expected state variables in both statistical and proportions of observations they represent. Another limitation of the exercise is concerned with the TC component and the basic

b-TAR model assumed. If TC is not available and non-constant over time, a more complicated model would have applied.

With the flexibility of the Markovian models, future work should attempt to directly model the PBM structure in HMM frame where the distributional assumptions are maintained. Thus, given the inherent similarity between the PBM and the MS-VEM, future research should incorporate dynamic adjustments within the basic model structure of the former. Since time and computational requirements did not allow us to demonstrate the proposed multi-chain Markov version, future work should attempt it, since it holds much promise when tradability switches significantly by information flow.

Because many market policies are oriented to improving markets functionality with respect to resource allocation and or correcting market imperfections, we recommend that MI studies be conducted within equilibrium framework in order to appropriately distinguish between perfect, imperfect and segmented market integration conditions. Again, given the fact that different trade and market policy strategies are required for each market condition, we suggest that systematic evaluation of market conditions that reflect both arbitrage processes and outcomes should underlie market integration policies.

In a nutshell, the study has highlighted and clarified complexities that various theoretical propositions of the MI concept impose on the two major methodological lines. Attempts have been made to classifying and linking each of the major non-linear complications to specific market economic theory. By dissecting the MI concept in this wise, we have made explicit the roles of various market data and methodological claims within both static and dynamic modelling structures. This allowed us to propose a switching model in a form of changing arbitrage behaviours over time, which at least performs well as the alternative TAR and PBM.

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APPENDICES

Appendix A- Conditional Independence

In this appendix the notations and definitions remain the same as those presented and defined in the text.

In general, if C_t is a random variable, (takes on values with certain probabilities), it might still or not have the ability to influence each other. Such a notion is quantified by statistical independence. Two random variables, $X(C_{t+1})$ and $Y(C_0, \dots, C_{t-1})$ to correspond our representation in the text, are said to be (marginally) statistically independent if and only if $p(X = x; Y = y) = P(X = x)P(Y = y)$ for every value of x and y . Formally written as $X \perp\!\!\!\perp Y$. The implication here is that regardless of the outcome of one random variable, the probabilities of the outcomes of the other random variable is not affected. In Markov process it is the knowledge of a third random variable $Z(C_t$ in this case) that determines whether X and Y might or might not be independent of each other, a concept captured by conditional independence. The conditional independence concept postulates that a random variable X is conditionally independent of a different random variable Y given a third random variable Z under a given probability distribution $P(\cdot)$, if the following relation holds:

$$P(X = x; Y = y \mid Z = z) = P(X = x \mid Z = z)P(Y = y \mid Z = z) \quad \text{AA.01}$$

for all x, y , and z . This is written $X \perp\!\!\!\perp Y \mid Z$ and it is said that “ X is independent of Y given Z under $P(\cdot)$ ”. It can be seen that given the representation structure of our Markov variables, they do not strictly fit into the flexibility of the above three-variable exposition, although same logic is followed. An equivalent definition and direct representation of Markov process is $p(X = x \mid Y = y; Z = z) = P(X = x \mid Z = z)$. The conditional independence of X and Y given Z in this sense carries the interpretation that if knowledge of Z is available, then knowledge of Y does not change one's knowledge of X and vice versa. Conditional independence is different from unconditional (or marginal) independence. Therefore, it might be true that $X \perp\!\!\!\perp Y$ but not true that $X \perp\!\!\!\perp Y \mid Z$. One valuable property of conditional independence follows: if $X_A \perp\!\!\!\perp Y_B \mid Z_C$, and subsets $A_0 \subseteq A$ and $B_0 \subseteq B$ are formed, then it follows that $X_{A_0} \perp\!\!\!\perp Y_{B_0} \mid Z_C$. Conditional independence is a

powerful concept and when such assumptions are made, a statistical model can undergo enormous simplifications (Bilmes 2002).

Appendix B: Statistical Estimation of hmm

Computing $P(\mathbb{R} | \Theta)$ from HMM implies that one wants to calculate the probability of realising the observed sequence for a given set of observations, which means computing $\sum_{c_{1:T}} P(R_{1:T}, c_{1:T})$. This is because to sample from each $P(R_{1:T})$ requires all possible samples from $P(c_{1:T})$ that produce $(c_1, c_2, c_3, \dots, c_T)$. That is, to realise each sample from $(R_{1:T})$ demands a new and different sample of $(C_{1:T})$. Consequently, an HMM observation sample is obtained using the marginal distribution $P(R_{1:T}) = \sum_{c_{1:T}} P(R_{1:T}, c_{1:T})$ and not the conditional distribution $P(R_{1:T} | c_{1:t})$ for some fixed hidden variable assignment $c_{1:T}$. To find solution to problem one therefore reduces to computing:

$$P(R_{1:T}) = \sum_{c_{1:T}} P(R_{1:T}, c_{1:T})$$

$$P(R_{1:T}) = \sum_{c_{1:T}} P(c_1) \prod_{t=2}^T P(c_t | c_{t-1}) \prod_{t=1}^T P(R_t | c_t)$$
AB.01

This assertion can also be seen from the following facts;

$$P(\mathbb{R} | C, \Theta) = \prod_{t=1}^T P(R_t | c_t, c_{t+1}, \Theta)$$

$$= bc_1(R_1) \dots bc_1 c_2(R_2) \dots bc_{T-1} c_T(R_T)$$
AB.02

in lines with the symbols from the questions in the text; and with the state transition:

$$P(C | \Theta) = \prod_{t=1}^T P(c_t | c_{t+1}, \Theta)$$

$$= \pi * ac_1 c_2 \dots ac_2 c_3 \dots ac_{T-1} c_T$$
AB.03

which implies that the joint process;

$$P(\mathbb{R}, C | \Theta) = P(\mathbb{R} | C, \Theta)P(C | \Theta) \quad \text{AB.04}$$

Equation (AB.04) is equivalent to (AB.01) above and hence marginalising either $P(\mathbb{R} | C, \Theta)$ or $P(C | \Theta)$ out, (AB.04/01) results in the other, as follows;

$$\begin{aligned} P(\mathbb{R} | \Theta) &= \sum_{C=c_{1:T}} P(\mathbb{R} | C, \Theta)P(C | \Theta) \\ &= \sum_{c_{1:T}} \pi c_1 \prod_{t=2}^T P(c_t | c_{t-1}) \prod_{t=1}^T P(R_t | c_t) \end{aligned} \quad \text{AB.05}$$

From (AB.05) it seems quite straightforward to compute for $P(\mathbb{R} | C, \Theta)$ by summing the observation probabilities for each of the possible state sequence. Direct attempt to computing $\sum_{c_{1:T}} P(R_{1:T}, c_{1:T})$ is however intractable. As the length of T of the sequence grows, the computation grows exponentially. This calculation involves the sum of M^T multiplications, each being a multiplication of $2T$ terms. The total number of operations is on the order of $2TM^T$. Fortunately, the conditional independence properties allow for an efficient computation of this quantity, which is indeed the *forward backward algorithm* (see below).

To address the second issues, that is, given the observed sequence $\mathbb{R} = \{R_1, R_2, \dots, R_T\}$ of outputs, the objective is to compute efficiently a state sequence $C=(c_1, c_2, c_3, \dots, c_T)$ that has the highest conditional probability given \mathbb{R} . In other words, we want to find a C that makes $P[C | \mathbb{R}]$ maximal. There is the possibility that there may be several optimal C for $P[C | \mathbb{R}]$ maximal. One may decide to choose the states that are individually most likely at each time t . For each time t ; $1 \leq t \leq T+1$, the probability variable below is computed:

$$\mu_{(t)i} = P(c_t = i | \mathbb{R}, \Theta) = \frac{P(c_t = i, \mathbb{R} | \Theta)}{P(\mathbb{R} | \Theta)} = \frac{\alpha_i \beta_i}{\sum_{j=1}^N \alpha_{tj} \beta_{tj}} \quad \text{AB.06}$$

where as detailed below α and β defines forward and backward probabilities respectively. If we define the most likely individual state sequence C^* , then expression (AB.07) holds.

$$C^* = \arg \max_{1 \leq i \leq M} \mu(t)i, \quad 1 \leq t \leq T+1, \quad 1 \leq i \leq M \quad \text{AB.07}$$

While the above quantity (AB.07) maximizes the expected number of correct states, it may generate an unlikely state sequence. This is due to the fact that it does not take into accounts the state transition probabilities, which for instance, if at some point we have zero transition probability $a_{ij} = 0$, the optimal state sequence found may be invalid. To avoid this situation there is a more efficient algorithm, -*Viterbi algorithm*- a dynamic programming technique, that finds a best state sequence. The Viterbi algorithm follows the induction procedure, similar to the forward-backward algorithm, except that while the forward-backward algorithm uses summation over previous states, the Viterbi algorithm uses maximization procedure to find the best state sequence $C^* = (c_1, c_2, c_3, \dots, c_T)$ given the observation sequence $\mathbb{R} = \{R_1, R_2, \dots, R_T\}$. For the details of Viterbi algorithm see below.

The third question, parameter estimation, represents the most challenging task about HMMs. With a given observation sequence, $\mathbb{R} = \{R_1, R_2, \dots, R_T\}$, what model parameters set, $\Theta = (\Pi, A, B)$, that best explains the observation sequence. In this case if we take \mathbb{R} as given, then the problem can be reformulated as to finding the parameters that maximize the probability:

$$\arg \max_{\Theta} P(\mathbb{R} | \Theta) \quad \text{AB.08}$$

Which implies that the model parameters, Θ are adjusting to maximize the likelihood of the observed sequence \mathbb{R} . The Θ cannot be directly picked such that $P(\mathbb{R} | \Theta)$ is maximized, but a local maximization algorithm can be employed to find the highest probability. This is the so-called Baum-Welch algorithm which is a special case of the Expectation Maximization (EM). Thus, instead of calculating the required frequencies directly from the observation sequence, it works iteratively to improve the likelihood of $P(\mathbb{R} | \Theta)$. On each of the iterations, the probability of observing \mathbb{R} from the model is improved and continues till some probability limit is reached. To implement the process, an additional intermediate variable in addition to μ_i above is defined:

the probability of being at state i at time t , and at state j at time $t + 1$, given the model Θ and the observation \mathbb{R} , (v_{ij}) . Where

$$\begin{aligned}
 v_{t(ij)} &= P(c_t = i, c_{t+1} = j | \mathbb{R}, \Theta) \\
 &= \frac{P(c_t = i, c_{t+1} = j, \mathbb{R} | \Theta)}{P(\mathbb{R} | \Theta)} \\
 &= \frac{a_{ij}\alpha_{(t)i}b_{ij}(R_t)\beta_{(t+1)j}}{\sum_{k=1}^M \alpha_{(t)k}\beta_{(t)k}} = \frac{a_{ij}\alpha_{(t)i}b_{ij}(R_t)\beta_{(t+1)j}}{\sum_{k=1}^M \sum_{l=1}^M a_{kl}\alpha_{(t)i}b_{kl}R_t\beta_{(t+1)l}}
 \end{aligned} \tag{AB.09}$$

and μ_i is in effect

$$\begin{aligned}
 \mu_{(t)i} &= P(c_t = i | \mathbb{R}, \Theta) \\
 &= \sum_{j=1}^M P(c_t = i, c_{t+1} = j | \mathbb{R}, \Theta) \\
 &= \sum_{j=1}^M v_{t(i,j)}
 \end{aligned} \tag{AB.10}$$

The above equation holds because $\mu_{(t)i}$ is the expected number of transition from state i and $v_{t(i,j)}$ is the expected number of transitions from state i to j . With above variables, we recall the model as defined by Θ in equation (AB.01) for the complete data likelihood, whose three parameter sets $[A, B, \Pi]$ are re-estimated along $[V_{ij}, \mu_i]$ as follows:

$$\begin{aligned}
 \pi'_i &= \mu_{i(1)} = \text{the probability of being at state } i \text{ at time } t = 1 \\
 a'_{ij} &= \frac{\sum_{t=1}^T v_{t(i,j)}}{\sum_{t=1}^T \mu_{t(i)}} = \frac{\text{expected number of transitions from state } i \text{ to } j}{\text{expected number of transitions from state } i} \\
 b'_{ijk} &= \frac{\sum_{t: R_t=k, 1 \leq t \leq T} v_{t(i,j)}}{\sum_{t=1}^T v_{t(i,j)}} = \frac{\text{expected number of transitions from } i \text{ to } j \text{ with } k \text{ observed}}{\text{expected number of transitions from } i \text{ to } j}
 \end{aligned}$$

The forward backward procedure

If we define the forward variable $\alpha_i(t)$ as;

$$\alpha_{(t)i} = P(\mathbb{R}_1 = R_1, \mathbb{R}_2 = R_2, \dots, \mathbb{R}_{t-1} = R_{t-1}, c_t = i | \Theta) \quad \text{AB.11}$$

where $i = 1, \dots, M$ and $t = 1, \dots, T$. The $\alpha_{i(t)}$ stores the total probability of ending up in state c_i at time t , given the observation sequence $\{R_1, R_2, \dots, R_{T-1}\}$. That is;

$$\alpha_{(1)i} = P(R_1, \dots, R_t, c_t = C_i) \quad \text{AB.12}$$

hence,

$$\begin{aligned} \alpha_{(1)i} &= P(R_1, c_1 = C_i) = P(R_1 | c_1 = i)P(c_1 = i) \\ &= \pi_i b_{1i}(R_1) \quad 1 \leq i \leq M \end{aligned} \quad \text{AB.13}$$

$$\begin{aligned} \alpha_{(2)i} &= P(R_1, R_2, c_2 = i) = P(R_1, R_2, c_2 = i) \\ &= \sum_{j=1}^N P(R_1, R_2, c_1 = j, c_2 = i) \\ &= \sum_{j=1}^N P(R_1, R_2 | c_1 = j, c_2 = i)P(c_1 = j, c_2 = i) \\ &= \sum_{j=1}^N P(R_1 | c_1 = j)P(R_2 | c_2 = i)P(c_2 = i | c_1 = j)P(c_1 = j) \\ &= \sum_{j=1}^N \alpha_{1j} a_{ji} b_{2i} \\ &= \sum_{j=1}^N \pi_j b_{1j} a_{ji} b_{2i} \end{aligned} \quad \text{AB.14}$$

$$\text{which follows that;} \quad (\alpha_{21}, \dots, \alpha_{2m}) = (\pi_i D_1 A D_2) \quad \text{AB.15}$$

where A and $D_i = \text{diag}(b_1(R_i)b_2(R_i)b_3(R_i)\dots b_N(R_i))$ are state and diagonal of observation transition matrixes, as defined above under the questions respectively. Hence the whole forward algorithm over observation length t , can be represented as

$$(\alpha_{t1}\dots\alpha_{tm}) = (\pi i D_1 A D_2 \dots A D_t) \quad \text{AB.16}$$

Alternatively, the backward algorithm could be used to answer the question by backward probabilities;

$$\beta_{ij} = P(R_{t+1} = R_{t+1} \dots R_T = R_T \mid c_i = C_i = i) \quad \text{AB.17}$$

where $i = 1, \dots, N$ and $t = 1, \dots, T-1$. Again, by induction through (AB.07) the $\beta(t)i$'s can be calculated. The process starts with the value $t = T - 1$, then for the value $t = T - 2$, and onwards, working back finally to $t = 1$. Thus, we start the algorithm by setting:

$$(\beta_{T1}\dots\beta_{TN}) = (1 \dots 1)_{1 \times N}$$

$$\begin{aligned} \beta_{(T-1)i} &= P(R_T \mid c_{T-1} = i) \\ &= P(R_T, c_{T-1} = i) / P(c_{T-1} = i) \\ &= \sum_{j=1}^N P(R_T, c_{T-1} = i, c_T = j) / P(c_{T-1} = i) \\ &= \sum_{j=1}^N P(R_T \mid c_{T-1} = i, c_T = j) P(c_{T-1} = i, c_T = j) / P(c_{T-1} = i) \\ &= \sum_{j=1}^N P(R_T \mid c_T = j) P(c_T \mid c_{T-1} = i) \end{aligned} \quad \text{AB.18}$$

implying $(\beta_{(T-1)1}\dots\beta_{(T-1)N})' = A D_{T-1} 1'$; and hence $(\beta_{t1}\dots\beta_{tN})' = (A D_{t+1})(A D_{t+2})\dots(A D_T) 1'$.

Thus, given the parameter estimates of the model Θ , the $(T \times N)$ matrices $\Lambda = (\alpha_{ti})$ and $\Gamma = (\beta_{ti})$ can be estimated in a recursive manner.

The Viterbi Algorithm:

Here, we first define, for arbitrary t and i , ℓ_i to be the maximum probability of all ways to end in state C_i at time t , with observed sequence $\mathbb{R} = \{R_1, R_2, \dots, R_T\}$.

2. Initialization

$$\begin{aligned}\ell_{i(1)} &= \pi_i b_{i(R_1)}, & 1 \leq i \leq N \\ \psi_{i(1)} &= 0, & 1 \leq i \leq N\end{aligned}\tag{AB.19}$$

3. Induction

$$\begin{aligned}\ell_{j(t)} &= b_{ij(R_t)} \max_{1 \leq i \leq N} \ell_{i(t-1)} a_{ij} \\ \psi_{j(t)} &= \arg \max_{1 \leq i \leq N} [\ell_{i(t-1)} a_{ij}]\end{aligned}\tag{AB.20}$$

4. Update time

$$t = (t + 1)\tag{AB.21}$$

Return to step two if $t \leq T$ else terminate the algorithm

5. Terminating the algorithm

$$\begin{aligned}P^* &= \max_{1 \leq i \leq N} [\ell_{i(T)}] \\ s_T^* &= \arg \max_{1 \leq i \leq N} [\ell_{i(T)}]\end{aligned}\tag{AB.22}$$

5. Read out

$$s_t^* = \psi_{t+1}(s_{t+1}^*)\tag{AB.23}$$

CURRICULUM VITAE

PERSONAL DATA

| | | |
|----------------|---|-------------------------------------|
| Name | : | Isaac Abunyuwah |
| Date of Birth | : | 13 th September 1973 |
| Place of Birth | : | Asuafu-Nsuta Ashanti-Ghana |
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