Theoretic Essays in Public Economics

Profit Shifting in General Oligopolistic Equilibrium and the Role of Information in Career Concern Models

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Abstract

This dissertation combines four theoretic models in public economics. The first two chapters consider welfare and distributional effects within a general equilibrium trade model with oligopoly focusing on corporate taxation and profit shifting. The second part analyses the efficiency of elections depending on communication between voters and regional structures affecting voters' information.

The first part of this dissertation investigates asymmetries between countries in a general oligopolistic equilibrium framework. The first chapter proposes a trade model with two countries, which allows oligopoly in segmented markets. When countries or their policies diverge, oligopolistic behaviour has macro-level effects on welfare and distribution. Through firms' strategic behaviour, country asymmetries can lead to deviations from the law of one price, which affects the countries' terms of trade and thus leads to shifts in consumption and welfare. This framework is utilised in the second chapter to analyse company taxation and tax-motivated transfer pricing. Tax rate differentials increase the income in the high-tax country leading to higher consumption. Profit shifting through transfer pricing partly reverses this income effect. This leads to an increase in welfare in the low-tax country. Transfer pricing also creates an additional incentive to export and expand production, which affects wages in such a way that real wages increase in both countries.

In the second part, communication and regional structures are analysed within a career concern framework of elections. The third chapter shows that voters' information becomes more precise if they communicate with others, even though it also leads to duplicated information within the electorate and thus more synchronous voting decisions. Better information of voters leads to more incentives for the incumbent to exert effort before the election, increasing the accountability property of the election. It also improves the electorate's selection of the more competent politician. The fourth chapter incorporates regional structures in the career concern framework to discuss their effects on accountability. Other regions serve as yardsticks to better assess one's own incumbent, thus improving accountability. If, however, the number of voters in a federation is fixed, there exists a trade-off between yardstick competition and smaller electorates. In a parliament of directly elected politicians, increasing the number of constituencies has a dual effect. With more parliamentarians, each has less influence individually leading to less effort. However, voters' information becomes more precise which positively affects effort.

Keywords: Accountability, communication, elections, general oligopolistic equilibrium, information, labour share, income distribution, international policy transmission, international trade, parliament, profit shifting, strategic trade, tax evasion, transfer pricing, yardstick competition.

Zusammenfassung

Diese Dissertation präsentiert vier theoretische Modelle im Bereich der Finanzwissenschaft. Die ersten beiden Kapitel stellen Wohlfarts- und Verteilungseffekte in einem allgemeinem Gleichgewichtsmodell mit Handel zwischen zwei Ländern und oligopolistischen Märkten vor. Der Fokus liegt hierbei auf der Unternehmensbesteuerung und der Verrechnungspreisgestaltung. Der zweite Teil behandelt die Effizienz von Wahlen und wie diese von Kommunikation zwischen Wählern und regionalen Strukturen beeinflusst wird.

Zunächst analysiert die Dissertation Asymmetrien zwischen zwei Ländern im allgemeinen oligopolistischen Greichgewichtsmodell. Das erste Kapitel führt sie in das Modell ein, was die Untersuchung von oligopolistischem Verhalten in segmentierten Märkten erlaubt. Unterschiede zwischen den Ländern können wegen des strategischen Verhaltens der Firmen zu unterschiedlichen Preisen in den Ländern führen. Dies verändert das reale Austauschverhältnis und kann somit zu Änderungen der Wohlfahrt führen. Dieses Modell wird im zweiten Kapitel genutzt um die Auswirkungen von Unternehmensbesteuerung und Verrechnungspreis-Manipulationen zu analysieren. Das Hochsteuerland profitiert von Steuersatz-Unterschieden, da das dortige Einkommen ansteigt und so der Konsum steigt. Gewinnverlagerung hebt diesen Effekt teilweise auf. Das Niedrigsteuerland profitiert, wenn Firmen ihre Verrechnungspreise manipulieren um Steuern zu sparen. Zugleich führt es zu einem weiteren Anreiz, Exporte und die Produktion insgesamt zu erhöhen. Dies hat Verteilungseffekte, da die Reallöhne in beiden Ländern ansteigen.

Der zweite Teil der Dissertation widmet sich den Auswirkungen von Kommunikation und regionalen Strukturen innerhalb des "career concern"-Modells. Das dritte Kapitel zeigt, dass Kommunikation zwischen Wählern zu besser informierten Entscheidungen führt, obwohl die Wähler einheitlicher abstimmen. Besser informierte Wähler erhöhen den Anreiz für Amtsinhaber, sich vor der Wahl anzustrengen. Zudem steigert es die Wahrscheinlichkeit, dass der bessere Politiker gewählt wird. Im selben Modell werden in Kapitel vier die Auswirkungen von regionalen Strukturen auf die Anstrengung von Amtsinhabern vor Wahlen untersucht. Der eigene Politiker kann mit den anderen verglichen und besser eingeschätzt werden. Dadurch entsteht ein Anreiz für Politiker, sich mehr anzustrengen. Wenn jedoch die Größe der Wählerschaft fest vorgegeben ist, gibt es einen trade-off bei der Einrichtung einer zusätzlichen Region zwischen dem verstärkten Maßstabswettbewerb und der kleineren Wählerschaft in jeder Region. Werden Politikmaßnahmen in einem Parlament direkt gewählter Abgeordneter beschlossen, hat die Vergrößerung des Parlaments zwei Effekte. Zum einen hat jeder Abgeordneter geringeren Einfluss, sodass der Anreiz für Anstrengung sinkt. Zum anderen sind Wähler besser informiert, sodass sich Anstrengung für Politiker mehr lohnt.

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Introduction

The field of public economics subsumes several strands of research that study the impact of government policies on economic efficiency and distribution. More recently, two main questions developed that Hindriks and Myles (2013, p. 3) summarises:

> "Public economics attempts to understand both *how* the government makes decisions and *what* decisions it should make."

On the one hand, public economists are interested in the outcomes of economic policies enacted by governments – both in terms of economic efficiency and distribution. These insights can then be used to give advice to decision-makers. Governments, on the other hand, may not always act in accordance with a society's wishes, but out of self-interest or for the advancements of certain sub-groups. Public economists, therefore, also study politicians' decision-making and the conditions under which they align with the interests of the public. Both of these aspects are integral to understand the government's influence on the economy and society.

In this dissertation I contribute to both of these strands of public economics research. The first part of the thesis in chapters one and two presents and applies a theoretical model on oligopoly in general equilibrium with trade between two differentiated countries based on a framework by Neary (2016). This model allows to analyse the effects of public policies on welfare in two countries but also shows distributional effects of such policies within these countries. Thus, it contributes to the literature on understanding policies that may ultimately lead to advice on *what* governments should do.

The third and fourth chapter form the second part of this dissertation, which focuses on elections and thereby on *how* government decisions come about. I utilise the career concern framework by Persson and Tabellini (2002) to theoretically discuss variations in the availability of information to voters. In this setting, elections determine which politician gets into office and thus fulfil the dual purpose of choosing good politicians and incentivising good administrative work before the vote.

All four chapters of this dissertation are microeconomic models whose results are based on characterising the actions taken by individual units, for instance firms, voters or politicians. Individuals' decisions follow the constraints and incentives they face which leads to aggregate outcomes – be it consumption in the general equilibrium in the first part or re-election probabilities in the second part of the thesis. Such models provide insights on the outcomes that differing or even contradicting incentives can produce. Rubinstein (2006, p. 881) compares theoretic models to fables:

> "A good model in economic theory, like a good fable, identifies a number of themes and elucidates them. We perform thought exercises that are only loosely connected to reality and that have been stripped of most of their real-life characteristics. However, in a good model, as in a good fable, something significant remains."

The research in this dissertation aims to provide relevant arguments that are not only valid in these models but in reality. It shows that profit shifting has real effects beyond mere tax avoidance by affecting supply decisions. Similarly, yardstick competition can be beneficial for an electorate, but the size of regions has to be considered as well. In this way I add to the research in public economics to understand the effects of policies and how they come about.

The first and second chapter form the first part of this thesis, which extends the general oligopolistic equilibrium (GOLE) introduced by Neary (2016) and incorporates tax-motivated transfer pricing by firms. In recent years it has become apparent that market concentration increases (e.g. Autor et al., 2020). Especially international trade is affected by a few large firms which can exploit their market power (e.g. Head and Spencer, 2017). Firms' strategic incentives that result from such oligopolistic markets cannot be captured adequately in models with competitive or monopolistic settings. Therefore, Neary (2016) developed the GOLE framework that allows to model oligopolistic sectors in a general equilibrium with two countries. I introduce fundamental asymmetry between countries into this framework which allows to analyse (possibly asymmetric) public policies in two countries.

The first chapter presents the asymmetric general oligopolistic equilibrium model with segmented markets in two countries. It shows how asymmetries between countries have macro-level effects by changing oligopolists' incentives. Firms' strategic behaviour adjusts to differentiated countries in such a way that country asymmetries can induce deviations from the law of one price, which gives rise to terms-of-trade based international shifts in consumption. Additionally, it is possible to determine distributional effects within countries by comparing wage and profit changes. This extension of the GOLE framework by Neary (2016) allows the study of the interplay between a broad range of asymmetric policies and oligopolistic market structures in general equilibrium. I demonstrate the underlying mechanisms and the procedure for solving the model by analysing asymmetric labour market policies.

Within this framework, chapter two studies the effects of corporate taxation and profit shifting by manipulating transfer prices. Asymmetric tax rates lead to tax exporting in this framework, whereby the burden of the higher tax falls on the low-tax country. Higher tax revenues increase the demand in the high-tax country so all firms shift supply from the low- to the high-tax country. Tax-motivated transfer pricing is used by firms to reduce their tax payments, which now shifts demand back towards the low-tax country increasing welfare there. At the same time, firms have an incentive to increase exports – and production – as with each exported unit they can save taxes. This leads to higher wages which affects the income distribution within countries. While real wages increase in both countries, the labour-profit ratio increases in the low-tax country.

The second part of this dissertation turns toward the question how politicians are elected into office and which circumstances allow for election outcomes that are beneficial for the electorate. I use the career concern framework developed by Persson and Tabellini (2002) to investigate the effect of communication and a regional setting on elections' efficiency. With the rise of social media and instant messaging services it became easier to exchange information with people living in other cities or even countries. Such communication impacts how people assess their politicians (e.g. Bode, 2016). Elections fulfil a dual purpose in the career concern framework. On the one hand, the prospect of re-election incentivises the incumbent politician to exert effort before the election to increase her chances of another term. This is the accountability property of elections. On the other hand, voters try to select the more competent politician – the selection property. I follow the idea of a large electorate with individual voters introduced by Aytimur and Bruns (2019) rather than a representative voter. It allows to integrate communication between individuals before the election and to properly analyse the effect of regional structures as the number of voters overall has an important positive effect on accountability and selection. This needs to be taken into account especially when considering the effect of decentralisation.

The third chapter shows that communication between voters improves the outcome of elections for the electorate. Firstly, each voter has better information that determines their voting decision. Voters have a more precise signal about the incumbent's competence, which generally improves both accountability and selection. Secondly, however, information exchange leads to more correlated information which results in a tendency of synchronous voting decisions, as voters develop a more uniform estimate of politicians' competence. It hinders the incumbent to predict the election outcome and makes their effort less profitable. I show that the first effect outweighs the second, leading to better accountability and selection overall. I further extend the type of communication by introducing a grouped electorate. If voters only communicate with others within their group it improves the election outcome, because the uniformity of information remains low for the electorate overall. However, if voters take their own signal as more important than the information they receive from others, this can reduce an election's efficiency.

In chapter four, I assume that there is a regional structure with politicians in each of these regions and consider the accountability effect of such structures. Firstly, voters can observe the situation not only in their own but also in other regions. Voters use the additional information to get a better idea of their own politician's competence, which leads to higher accountability if the number of regions increases. However, if the number of voters is fixed overall and the addition of new regions leads to smaller electorates, there is a trade-off between this yardstick competition and smaller regions. Secondly, I consider elections for a parliament in which each parliamentarian is elected directly by their constituents. With a larger parliament, each politician has less influence on the outcome, which makes effort less profitable. Yet, the information voters get becomes more precise when there are more constituencies and leads to more effort. Thus, increasing a parliament only improves accountability as long as politicians' influence does not decrease strongly.

Chapter 1

Asymmetric General Oligopolistic Equilibrium*

1.1 Introduction

In the last four decades we have observed a large increase in the share of pure profits in gross value added, while the labour share has declined strongly. Industry data suggests that this could be driven by rising market concentration.[1] Accordingly, modeling strategic interactions among large firms is of growing importance. While general equilibrium models of monopolistic competition do not allow for such strategic behaviour, partial equilibrium models of oligopoly cannot reflect the

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[1] See for example Barkai (2020), De Loecker et al. (2020), Autor et al. (2020) and Shepotylo and Vakhitov (2020).

interactions between product and labour markets, that are highly relevant in many contexts. In a series of papers, J. Peter Neary proposed a trade model that allows to analyse oligopoly in general equilibrium (GOLE).[2] We contribute to this literature by showing and explaining novel effects of asymmetric policies that arise in general oligopolistic equilibrium with segmented markets.

We explore the effects of fundamental asymmetries between countries[3] in this framework. Segmented markets allow for strategic considerations by individual firms with respect to market-specific supply decisions. Thereby, we analyse reciprocal dumping as described by Brander and P. Krugman (1983) in general oligopolistic equilibrium. Demand and price asymmetries across markets have implications for firms' strategic supply decisions. The model allows to explore the welfare and distributional effects of asymmetric policies and differing country characteristics under oligopoly. We demonstrate the economic mechanisms that lead to cross-country welfare effects of asymmetric labour market policies in a stylized model variant, which rules out welfare effects from comparative advantage by assuming identical productivity across sectors and countries. Thereby we isolate the welfare shifting properties of asymmetric policies from standard comparative advantage effects.

The GOLE framework allows to analyse the effects of market power in a global setting. However, many classical questions in international economics are concerned with the effects of cross-country differences in endowments and with the implications of national policies in an international setting. The GOLE literature

[2] See Neary (2007), Eckel and Neary (2010), Neary (2003b), Neary (2016), Neary (2003a) and Neary (2010).

[3] Countries are fundamentally asymmetric whenever differences in their characteristics lead to differences in their marginal utilities of income.

has mainly focused on fundamentally symmetric countries[4], because this keeps the countries' marginal utilities of income symmetric even when markets are allowed to be segmented. We study asymmetries, which affect firms' country-specific supply decisions once we stop imposing the assumption of a fully integrated world market. As a result, asymmetric policies induce novel welfare and distributional effects due to firm-level strategic behaviour, which to the best of our knowledge is a new mechanism. Our main contribution is to point out its existence, to explain the underlying economic mechanisms and the resulting implications, as well as to demonstrate how the model can be applied to analyse asymmetric policies.

The macro-level effects of national policies and other country asymmetries are shaped by firms' oligopolistic behaviour in segmented markets. Country asymmetries impact marginal costs and marginal revenues, which determine firms' supply decisions. As oligopolists treat both markets as segmented, they can encounter different demand levels in the two countries, which manifests in marginal revenue differences for sales in the two markets. Likewise, marginal costs might be directly impacted by asymmetric policies. The revenue and the cost channel together determine the overall supply decisions, which in turn affect prices. Due to their effect on strategic firm behaviour, country asymmetries can induce deviations from the law of one price, which gives rise to terms of trade based international shifts in consumption and welfare. This cannot arise in a standard GOLE framework with a world market approach.

Our application of asymmetric labour market policies shows that, if one country's labour supply becomes larger, it starts to produce more at lower wages. This raises

[4]The only asymmetries allowed are those which keep the countries' marginal utility of income equal, e.g. a sectoral productivity distribution with symmetric moments.

its real income per capita and, thus, demand, while reducing marginal costs. However, firms find it optimal to supply a part of the additional production to the other country at reduced prices in order to inflate domestic prices artificially until marginal revenues equal marginal costs again in both markets. Therefore, oligopolistic competition shifts consumption to the other country that now benefits from the labour market liberalization abroad due to a change in the terms of trade. The country with higher labour supply would be better off under autarky in our simple setting that does not consider technological differences across countries and thus precludes Ricardian gains from trade. It is important to consider the strategic incentives on the firm level when analysing the welfare and distributional effects of asymmetries in countries' characteristics and policies.

Non-GOLE approaches to study oligopoly in general equilibrium have drawbacks (Neary, 2016). If firms have economy-wide influence, it is natural to assume that they use their monopsonistic power in factor markets. Additionally, the objective of firms can become unclear as they can maximize profits or the utility of their owner. Dixit and Grossman (1986), Cordella and Gabszewicz (1997), Chao and Yu (1997) as well as Dierker and Grodal (1998) introduce oligopolistic aspects into general equilibrium models. For example, Azar and Vives (2021) allow firms to be large with respect to the economy without causing normalization problems by assuming that they maximize a share-weighted average of shareholder utilities. The GOLE model (Neary, 2016) avoids normalization issues by assuming a continuum of sectors, such that firms are small relative to the economy. There is a growing literature using the GOLE model (Neary (2007), Bastos and Kreickemeier (2009), Eckel and Neary (2010), H. Egger and Etzel (2012), Beladi et al. (2013), Kreickemeier and Meland (2013), Fujiwara and Kamei (2018), Beladi and Chakrabarti (2019)).[5] Although some asymmetries have been introduced, countries remain fundamentally symmetric in these models. However, once we allow for fundamental asymmetries, it matters whether markets are assumed to be integrated or segmented. H. Egger and Etzel (2014) implement an asymmetric degree of labour union centralization in a general oligopolistic equilibrium model, which is a fundamental asymmetry between countries as it manifests in differing marginal utilities of income. However, due to their assumption of an integrated world market, firms have no country-specific supplies they could adjust to these differences in countries' marginal utilities of income.

If integrated markets are assumed, a single world market is solved to derive the equilibrium. This allows to consider only the world aggregate marginal utility of income, which improves tractability, but comes at the cost of reduced strategic leeway for firms in deciding on their supplies in the two countries. In order to incorporate reciprocal dumping-type trade as in Brander and P. Krugman (1983) in general oligopolistic equilibrium, firms need to be able to discriminate between the countries they serve. Therefore, we make the assumption of segmented markets, which has been introduced to the GOLE framework by Bastos and Kreickemeier (2009) under fundamental symmetry. This also enables us to determine exported quantities of individual firms and on the aggregate level.

Segmented markets allow to explain the empirical observation of pricing-to-market (see e.g. Fitzgerald and Haller (2014)), although in our Cournot-framework firms optimise their supplied quantities separately for different markets, which only

[5] For a detailed survey of the GOLE literature see Colacicco (2015).

then results in differing prices in the two countries.[6] Head and Spencer (2017) provide anecdotal justification for segmented markets by noting that contracts often include conditions that restrict sales to other countries, such that arbitrage is not possible. Ben-Zvi and Helpman (1988) point out that partial equilibrium oligopoly with segmented markets coincides with a single integrated market in the absence of trade costs. Analogously, the symmetric country equilibrium in our model corresponds to an integrated world market, but country asymmetries give rise to differences in market outcomes even in the absence of trade costs. Markusen (2013) finds that in trade models with imperfect competition non-homothetic preferences can explain higher price levels in richer countries. This is in line with the empirical observation of a positive per-capita-income coefficient in gravity equations. Our model with quadratic preferences in oligopolistic competition exhibits this mechanism. Brander (1981) was the first to stress that strategic interactions can give rise to two-way trade in identical commodities, which is the sole rationale for trade in our model version without technological differences.

In Section 1.2, we introduce the basic model structure including the Cournot equilibrium. We turn to the general equilibrium in Section 1.3. Section 1.4 conducts comparative static analyses for the case of asymmetric labour market policies. The mechanism that determines how asymmetries affect micro-level behaviour and as a result macro-level outcomes is discussed in Section 1.5. Section 1.6 concludes.

[6] By assuming that firms can treat markets separately, they can strategically decide on market-specific supplied quantities. Depending on market-specific demand, the equilibrium price can be different as well. Consequently, firms' prices are market-specific.

1.2 Model Setup

In this section, we describe the underlying elements of the model. We assume oligopolistic competition within each sector of a two-country economy and bring the sectors together in general equilibrium. To this end, we adapt the model of international trade in general oligopolistic equilibrium developed by Neary (2016), but assume that markets are segmented. We usually present expressions for the Home country only, if expressions for Foreign are analogous.

1.2.1 Demand

Each country is inhabited by a representative consumer, whose preferences over the consumed goods from different sectors $z \in [0,1]$ are additively separable. In each sector there are n firms producing their good in Home and n^* firms from Foreign. Following Beladi et al. (2015) we use continuum-quadratic preferences with product differentiation:[7]

$$U[\{{y_h}^1(z),...,{y_h}^n(z), y_h^{*1}(z),...,y_h^{*n^*}(z)\}] = \int_0^1 u[{y_h}^1(z),...,{y_h}^n(z), y_h^{*1}(z),...,y_h^{*n^*}(z)]dz,$$

where

$$u[{y_h}^1(z),...,{y_h}^n(z), y_h^{*1}(z),...,y_h^{*n^*}(z)] = a\left(\sum_{\ell=1}^{n} {y_h}^\ell(z) + \sum_{k=1}^{n^*} y_h^{*k}(z)\right) - \frac{1}{2}b\left(\sum_{\ell=1}^{n} {y_h}^\ell(z)^2 + \sum_{k=1}^{n^*} y_h^{*k}(z)^2\right.$$

[7] Consumption in country $i \in \{h, f\}$ of a good produced in Home by firm ℓ is ${y_i}^\ell$. Consumption of Foreign firm k's goods in country i is marked with an asterisk: y_i^{*k}.

$$+\,\eta \left[\sum_{\ell=1}^{n} \sum_{\substack{k=1\\k\neq\ell}}^{n} y_h{}^{\ell}(z)\; y_h{}^{k}(z) + \sum_{\ell=1}^{n^*} \sum_{\substack{k=1\\k\neq\ell}}^{n^*} y_h^{*\ell}(z)\; y_h^{*k}(z) + 2 \sum_{\ell=1}^{n} \sum_{k=1}^{n^*} y_h{}^{\ell}(z)\; y_h^{*k}(z) \right] \Bigg).$$

The parameters $a, b > 0$ are exogenous with $a > by(z)$ to ensure non-satiation. The degree of product differentiation is given by $\eta \in [0, 1]$, where $\eta = 1$ corresponds to homogeneous goods in each sector.

The representative consumer inelastically supplies L units of labour to a perfectly competitive labour market. The nominal wage rate w is determined in general equilibrium and results in a wage income of $w \cdot L$. Additionally, aggregate profits (Π) of all firms producing in Home are disbursed to the representative consumer. Therefore, income is given by

$$I = w\; L + \Pi. \tag{1.1}$$

With price $p_h{}^{\ell}(z)$ per unit of the good bought in Home from Home firm $\ell \in \{1, ..., n\}$ in sector z, the budget constraint is[8]

$$\int_0^1 \left(\sum_{\ell=1}^{n} p_h{}^{\ell}(z)\; y_h{}^{\ell}(z) + \sum_{k=1}^{n^*} p_h^{*k}(z)\; y_h^{*k}(z) \right) dz \leq I. \tag{1.2}$$

Utility maximization yields the inverse Frisch demands for goods in sector z from Home firm ℓ and Foreign firm k which are given by

$$\begin{aligned} p_h{}^{\ell}(z) &= {}^1\!/\!_\lambda \left[a - (1-\eta)\; b\; y_h{}^{\ell}(z) - \eta\; b\; y(z)\right] \qquad (1.3) \\ p_h^{*k}(z) &= {}^1\!/\!_\lambda \left[a - (1-\eta)\; b\; y_h^{*k}(z) - \eta\; b\; y(z)\right], \end{aligned}$$

where $y(z)$ is the overall consumption of goods in sector z and λ the marginal

[8] Analogously, $p_h^{*k}(z)$ is the price of the good in sector z bought in Home from Foreign firm $k \in \{1, ..., n^*\}$.

utility of income in Home. Frisch demands specify a relation between the price, the quantity demanded and the marginal utility of income λ, which acts as a demand aggregator, where a higher value indicates a lower demand for goods in every sector. The value of λ will be determined in general equilibrium.

1.2.2 Supply

The producers aim to maximize their profits. They perceive the demand they face in the two separate markets as well as the wage rates as given. Analogously to Neary (2016), firms are assumed to have market power in their respective markets. However, they do not have direct influence on aggregate economic factors, as many sectors z exist and only jointly determine these aggregates. This especially includes the demand aggregator λ, but also the wage rate w, which all firms take as given.

As firms do not affect economy-wide variables, it is natural to assume that they simply maximize their own profit. Alternative objectives such as the overall welfare of their owner – the representative consumer – are not appropriate as firms are not able to influence economy-wide parameters and each sector's individual influence on the overall welfare is negligible. Gabszewicz and Vial (1972) argue that firms are myopic in such a context.

We assume that there are neither fixed costs of production nor trade costs.[9] The firms play a static one-stage game where they compete à la Cournot over supply in the Home and the Foreign market. Irrespective of the functional form of λ they perceive the inverse demand as linear.

Production occurs with constant returns to scale and common technology in each sector, such that costs in sector z are linear in output. Labour is the only

[9] See Rudsinske (2023) for the introduction of asymmetric tariffs in such a framework.

factor of production and moves freely across sectors within a country but not across national borders. The unit-cost function for sector z is given by $c(z) = w\ \gamma(z)$, where $\gamma(z) > 0$ is the sector-specific unit-labour requirement and w is the wage rate. Following Neary (2016) we assume that these labour inputs per unit of output are continuous in z in both countries. As in the original GOLE model, the moments of the technology distributions over the sectors in the two countries are the same with[10]

$$\alpha = \int_0^1 \gamma(z)dz = \int_0^1 \gamma^*(z)dz \quad \text{and} \quad \beta = \int_0^1 \gamma(z)^2 dz = \int_0^1 \gamma^*(z)^2 dz.$$

Additionally, we define $\Delta \equiv \int_0^1 \gamma(z) \cdot \gamma^*(z)\ dz$. Due to the symmetry of both countries' technology distributions $\Delta \leq \beta$ holds.

Companies supply their output to both countries, where the underlying demand may differ. The profit of firm ℓ in sector z is given by

$$\pi^\ell(z) \quad = \quad ({p_h}^\ell(z) - \gamma(z) \cdot w)\ {y_h}^\ell(z) + ({p_f}^\ell(z) - \gamma(z) \cdot w)\ {y_f}^\ell(z), \qquad (1.4)$$

where ${p_i}^\ell(z)$ are the inverse Frisch demands defined in equation (1.3) for the Home market and analogously for Foreign.

1.2.3 Cournot Equilibrium

In this subsection we determine the supplies of the firms in one of the sectors. We therefore drop the index z in this subsection. Firm ℓ maximizes its profit by choosing the amount of goods to produce and sell given the demand and the other

[10] The technology distribution in Foreign is marked with an asterisk: $\gamma^*(z)$.

companies' supply. In equilibrium, firms supply their good in both markets until marginal revenues and marginal costs are equalized. As companies producing in the same country are symmetric, they supply the same quantity, ${y_i}^{\ell} = y_i$, in market $i = h, f$, which accordingly also holds for Foreign firms.[11] The first order conditions can be transformed into reaction functions depending on the supply of Foreign companies.

$$y_h = \frac{a - \eta\ b\ n^*\ y_h^* - \lambda\gamma w}{b(2 + (n-1)\eta)} \tag{1.5}$$

$$y_f = \frac{a - \eta\ b\ n^*\ y_f^* - \lambda^*\gamma w}{b(2 + (n-1)\eta)} \tag{1.6}$$

For firms from Foreign, these reaction functions are defined analogously. We obtain the Cournot-equilibrium supply of each firm in the two countries by combining the reaction functions. The equilibrium supply of each individual company producing in Home is

$$y_h = \frac{\lambda}{b\ \rho}\left\{\frac{a}{\lambda} - \gamma\ w + \frac{\eta}{2-\eta} n^*(\gamma^*\ w^* - \gamma\ w)\right\} \tag{1.7}$$

$$y_f = \frac{\lambda^*}{b\ \rho}\left\{\frac{a}{\lambda^*} - \gamma\ w + \frac{\eta}{2-\eta} n^*(\gamma^*\ w^* - \gamma\ w)\right\}, \tag{1.8}$$

where $\rho = 2 + (n + n^* - 1)\eta$. The supplied quantities by Foreign firms are analogous.

The number of firms in either country has the usual competitive effects on the supplied quantities. A new entrant leads to reduced marginal revenues resulting in reduced supply by incumbent firms. A fall in product differentiation firstly reduces the demand for each individual firm's variety. Thus, supplies are negatively affected.

[11] As a result, prices are also equal for all firms producing in the same country: ${p_i^{(*)}}^{\ell} = p_i^{(*)}$ for $i = h, f$.

Due to higher substitutability, firms are secondly faced with higher competition, which reinforces firms' cost based (dis-)advantages leading to overall ambiguous effects in partial Cournot equilibrium. Changes in unit costs through wage rates or the labour requirement have two effects in the Cournot equilibrium. Higher unit costs reduce the supply by companies, which bear the rising production costs. In reaction, firms from the other country expand their supply as they benefit from a cost-based advantage.

1.3 Asymmetric General Oligopolistic Equilibrium

In general equilibrium we need to determine the wage rates and the demand aggregators in both countries. We first solve the labour market equilibrium before turning to our solution strategy for the marginal utilities of income in Home and Foreign.

1.3.1 Labour Market

The representative consumer inelastically supplies L units of labour. The labour demand depends on the equilibrium production in a country. Each Home company in sector z supplies $\bar{y}(z) = y_h(z) + y_f(z)$. The total labour demand is given by $L^D = \int_0^1 n\ \gamma(z)\ \bar{y}(z)\ dz$. In equilibrium, demand has to equal supply.

$$L = \int_0^1 n\ \gamma(z)\ \bar{y}(z)\ dz = \frac{n}{b\ \rho} \left\{ 2\ a\ \alpha - \bar{\lambda} \left(\frac{\eta}{2-\eta} n^* + 1 \right) \beta\ w + \frac{\eta}{2-\eta} \bar{\lambda}\ n^*\ \Delta\ w^* \right\}, \tag{1.9}$$

where $\bar{\lambda} \equiv \lambda + \lambda^*$. This relation defines the labour market equilibrium in Home. In combination with the analogously defined equilibrium on the Foreign labour market, the wage rate in Home is given by

$$w = \frac{1}{\bar{\lambda}}\;\Gamma\;\left\{\frac{2\;a\;\alpha}{\rho}\left(2-\eta+\left(n+\frac{\Delta}{\beta}\;n^*\right)\eta\right)-b\left[(2+(n-1)\eta)\frac{L}{n}+\frac{\Delta}{\beta}\eta L^*\right]\right\} \tag{1.10}$$

with

$$\Gamma = \left[\beta + \frac{\eta^2}{2-\eta}\frac{nn^*}{\rho}\left(\beta-\frac{\Delta^2}{\beta}\right)\right]^{-1}.$$

The Foreign wage rate w^* is defined analogously. We can already observe that for constant $\bar{\lambda}$ an increase in any country's labour supply reduces the wage rate in both countries.

1.3.2 Balance of Payments Equilibrium

The model is characterized by eleven equations in the general equilibrium. The Cournot equilibrium quantities determine the supply of each multinational company to each country given the wages and the marginal utilities of income. The labour market clearing in each country determines the wage rates given the produced quantities in the respective country. The prices are given by the representative consumers' inverse Frisch demand functions.

The last equation results from the budget constraint of the representative consumer in either country. We use the budget constraint to attain an implicit definition of the marginal utilities of income in equilibrium.[12] The budget constraint in Home is given by

$$\int_0^1 \left(p_h(z)\;n\;y_h(z) + p_h^*\;n^*\;y_h^*(z)\right)\;dz \quad = \quad w\;L + \int_0^1 n\;\pi(z)\;dz$$

[12] Neary (2016) uses the same relationship to determine the marginal utility of income, see his footnote 13.

This can be rearranged using the definition of $\pi(z)$ in equation (1.4) to obtain a straightforward relationship that has to hold in equilibrium:

$$n \int_0^1 p_f(z)\ y_f(z)\ dz = n^* \int_0^1 p_h^*(z)\ y_h^*(z)\ dz \tag{1.11}$$

To close the model, we need a balance of payments equilibrium. In our simple setting, the capital balance is zero. Thus, the respective values of trade over all sectors have to be equal in general equilibrium. The value of exports from Home to Foreign on the left has to be equal to the value of imports from Foreign to Home on the right. The equality of these values does not imply that the traded quantities of goods are equal. If prices or the number of firms differ, one country might import more units of a good than it exports.

To attain the general equilibrium values of the endogenous variables and to solve the system of equations, we need to determine the marginal utilities of income. The foundation of our model is given by Neary (2016), but in his case this step is less difficult. Most importantly, in his case the two countries are symmetric such that $\bar{\lambda} = 2\lambda$, which significantly reduces the complexity of the equilibrium. In our case, we cannot reduce the result that way, as countries are not necessarily symmetric.

The marginal utilities of income affect firms' relative supply decisions across markets, which can have real distributional effects between countries. Thus, the difference between λ and λ^* is of high relevance in our model. As we have eleven equations for the ten endogenous variables, we can normalize the aggregate marginal utility of income to one, i.e. $\bar{\lambda} = 1$. Hence, the aggregate marginal utility of income is used as numéraire. This translates into the relationship between λ and λ^* that

$\lambda^* = 1 - \lambda$, which allows us to substitute all λ^*. Because both marginal utilities of income have to be positive, it follows that both lie between zero and one.

We express the remaining endogenous variables, such that they only depend on the marginal utility of income in Home. These formulations can then be used in the balance of payments condition, where we can show that for the admissible range $\lambda \in (0, 1)$ there is only one solution to this equation.

Proposition 1.1 (Existence and Uniqueness)**.** *There exists a unique solution to the condition of an even balance of payments in* $\lambda \in (0, 1)$.

Proof. See appendix A.1.1. □

The equilibrium marginal utility of income $\hat{\lambda}$ is defined as a root of a cubic polynomial with a positive discriminant. Therefore, it is possible to obtain the expression of $\hat{\lambda}$ in closed form and all solutions to the polynomial are real-valued. Only one of the roots is within the interval $(0, 1)$.[13]

However, in many cases it is not necessary to determine the exact value of $\hat{\lambda}$. Firstly, the derivative of the marginal utility of income in general equilibrium with respect to a parameter of interest can be derived from the balance of payments condition using implicit differentiation. Secondly, we know that $\hat{\lambda} \in (0, 1)$. Newton's method allows to determine stricter borders for $\hat{\lambda}$. This often suffices to determine signs of endogenous variables' derivatives with respect to a parameter of interest, given the sign of the derivative of $\hat{\lambda}$ with respect to that parameter. Hence, qualitative results can often be shown even in extensions of the model that are too complex to derive $\hat{\lambda}$ in closed form.

[13]However, two of the three roots are complex. For the proof of Proposition 1.2 below a derivation is needed and provided in the supplement S1 with mathematical software.

1.3.3 Graphical Representation

A defining feature of the model are the supply decisions by the oligopolists. To give the reader some intuition about the mechanisms that determine the supply in general equilibrium, we develop a graphical representation of their supply decisions. For these representations we set $\gamma(z) = \gamma^*(z) = 1$ in all sectors, thus precluding Ricardian gains from trade. Trade only occurs due to strategic interactions. This allows us to drop the z, as well as the labour input requirement.

We can reduce the system down into two fundamental conditions. This way we can focus on the firms' strategic considerations. The consumption indifference condition (CI) states that under utility maximization the representative consumer in either country has to be indifferent between all goods in each sector. The market indifference condition (MI) states that in equilibrium all firms have to be indifferent between selling the marginal unit in Home or in Foreign. The MI line represents all Cournot-equilibria for all possible demands firms face. We can plot these two conditions in a box diagram in Figure 1.1.

In the box diagram, the supply decision of one firm from Home and of one firm from Foreign are depicted. As all firms producing in the same country are symmetric, this still provides useful insights not only into each firm's decision but also the overall supply.

The CI line is derived from the budget constraint.

$$CI: \qquad y_h = \frac{I/p_h}{n} - \frac{n^*}{n}\frac{p_h^*}{p_h} y_h^*$$

It gives us a function with substitutability between the goods of a Home and a Foreign firm from the consumer's perspective, for whom income I as well as prices

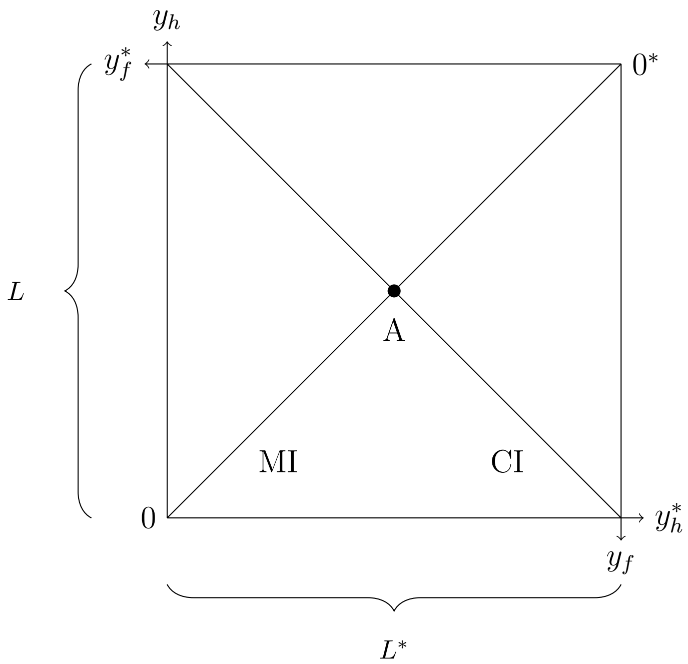

Figure 1.1: General Equilibrium with Symmetric Countries

($L = L^*$ and $n = n^* = 1$)

p_h and p_h^* are exogenous. If all Foreign firms increase their supply, Home firms need to reduce theirs such that the budget is equalized. An increase in income in Home shifts the CI line towards the upper-right corner. In symmetric equilibrium ($L = L^*$) with homogeneous goods ($\eta = 1$), the intercept is exactly in the upper left corner of the graph. In this case both countries consume the same quantities. The same can analogously be done for the Foreign representative consumer giving us exactly the same line in the graph.

The MI line follows from the profit maximization as marginal revenues need to be equal across countries in equilibrium for each firm. The same relation needs to hold for all Foreign firms. As $y_h^{(*)} + y_f^{(*)} = {}^{L^{(*)}}/_{n^{(*)}}$, we obtain a relation between the supply to Home by one Home firm and one Foreign firm, which has to hold in the Cournot-equilibrium.

$$MI: \qquad y_h = \lambda\left(\frac{L}{n} - \frac{L^*}{n^*}\right) + y_h^*. \tag{1.12}$$

This results in a line with the slope +1. The intercept consists of two parts. Firstly, within the brackets we have the Home production per firm minus the Foreign production per firm. If this is positive, Home firms have a larger quantity to supply than Foreign firms. Secondly, the marginal utility of income λ gives the share of this excess production that is allocated to the Home market, whereas $1 - \lambda = \lambda^*$ is the share allocated to Foreign. Figure 1.1 shows how the equilibrium supplies are determined at the intersection of the MI line and the CI line.

1.4 Comparative Static Application

The model allows to analyse a wide variety of economic aspects, for example asymmetries in market concentration[14], trade policies or taxation.[15] Such analyses can be structured into three main steps. Firstly, the other endogenous variables need to be determined depending on λ and λ^*. Secondly, existence and uniqueness – or the conditions thereof – need to be established. Both of these steps have been described above.

In this section we illustrate a procedure for the third step of conducting comparative static analyses in such models. The aim is to obtain meaningful results for

[14] For $L = L^*$, it is easy to show the effects of changes in n. The domestic labour share falls with rising concentration in a country, while the other country is unaffected. With $L \neq L^*$ this becomes computationally more demanding.

[15] See Rudsinske (2023) for an analysis of asymmetric tariffs and Quint and Rudsinske (2020) for an analysis of asymmetric taxation and tax-motivated transfer pricing.

endogenous variables such as exports or consumption but also interesting insights into distributional effects. In most cases, the exact value of the marginal utility of income in equilibrium is not necessary to derive qualitative effects resulting from changes in exogenous variables. In these cases, we first narrow the possible range of $\hat{\lambda}$, before determining its derivative with respect to the exogenous variable of interest as well as the derivative's sign. These elements often suffice to determine the sign of other derivatives of interest. Sometimes the equilibrium value of $\hat{\lambda}$, although not particularly illuminating by itself, is needed to determine certain comparative static results. It can be obtained with mathematical software as we show in the supplement.

To present this procedure, we analyse a straightforward change in the labour market regime in one of the countries. Changes in labour endowment L in our setting may be interpreted as labour market policies that alter the average working hours in a country, such as changes in vacation entitlements or reductions in the workdays per week. Such changes in L do not increase the population as given by the number of consumers, which are characterized by the representative consumer. This follows from the structure of the utility function of a country's representative consumer, which does not change with L. An increase in the population would increase the labour endowment and the number of consumers equally, such that it should not affect consumption per capita and the marginal utility of consumption. In our application, only working hours per capita are rising, while the number of consumers is unaffected, so consumption per capita rises and the marginal utility of consumption falls. Such differences in labour endowment are not comparable to the technology differences in Neary (2016), where labour measured in efficiency units can only be different within the sectors but not on the aggregate level due to the symmetry assumptions on the technology distributions in the two countries. We discuss changes in L in the Home country.

In order to focus solely on the effect of strategic interactions, we disregard technological differences across countries in this section. By setting $\gamma(z) = \gamma^*(z) = 1 \; \forall z$ we only consider the case of identical technology across sectors and countries so we can drop z throughout. However, we retain the assumption of a continuum of sectors even though they are symmetrical. Thus, the model does not capture a Ricardian-style technological comparative advantage anymore. The reasons for trade in this setting are the strategic considerations among firms in Cournot competition described in Brander and P. Krugman (1983). As companies remain small in the large, they still do not take their effect on wages into account when maximizing their profits. For simplicity, we assume $n = n^* = 1$ and homogeneous goods in each sector ($\eta = 1$).[16] Assuming homogeneous goods eliminates the within sector differentiation between firms' goods such that all firms set an equal price for their good in a country: $p_h = p_h^* = p$ and $p_f = p_f^* = p^*$. Additionally, we set $b(L + L^*) < a$ to ensure a positive marginal utility of consumption in any case. This also assures positive wage rates in both countries.

Firms in a substantially smaller country possibly supply to the larger country only but nothing to the country in which they produce, as demand and hence the marginal revenues are smaller there. To ensure an interior solution we assume that the countries' labour endowments are sufficiently similar to obtain positive supplies

[16] Increasing product differentiation (reducing η) leads to a rising labour share for symmetric countries. With higher differentiation willingness to pay increases such that firms want to raise their production, which leads to higher wages. Higher prices are exactly offset by higher wages such that profits remain unchanged.

to all countries by all firms with $2/5 < L/L^* < 5/2$.[17] These restrictions reduce the possible values $\hat{\lambda}$ may take. In full symmetry with $L = L^*$, the marginal utility of income is one half in equilibrium in both countries. Using Newton's method we can show that $1/4 < \hat{\lambda} < 3/4$.[18]

To analyse the changes in general equilibrium, we need the reaction of $\hat{\lambda}$ to changes in the exogenous parameter of interest (L). We determine the expression for this derivative depending on $\hat{\lambda}$ and show the sign of this derivative.

Lemma 1.1 (Effect of L on $\hat{\lambda}$)**.** *An increase in L reduces the marginal utility of income in Home in general equilibrium ($\hat{\lambda}$).*

Proof. See appendix A.1.2. □

When average working hours L increase, Home production rises. Home firms maximize their profits and thus set marginal revenues equal to marginal costs, which are identical for supplies to both markets. As a consequence, it is not optimal for Home firms to supply all newly produced goods to Home. They can increase their average mark-ups by supplying more to both markets. Thus, they do not only increase their domestic supply, but also increase their exports. Consequently, as long as prices do not react, Home runs a trade balance surplus. This is impossible in equilibrium. Therefore, the balance of trade needs to adjust. On the one hand, the price ratio p/p^*, which is also the Foreign terms of trade, increases. On the other hand, supply decisions adjust, which does not reverse the initial impetus. Because both the traded quantity $y_f + y_h^*$ and the Foreign terms of trade increase, it follows that Foreign consumption rises as well. Therefore, Home's production increase

[17] This is not an overly restrictive assumption. According to OECD (2021), the ratios of average weekly working hours per full-time employee clearly lie within our allowed space.

[18] See appendix A.2 for the derivations.

after liberalization is not fully consumed there. Home loses from international trade if $L > L^*$ and would prefer autarky in this case, where consumption is equal to production.

Proposition 1.2 (Consumption and Trade)**.** *Liberalizing the labour market, i.e. increasing the average working hours, in one of the countries increases consumption in both countries and raises the total traded quantity. International trade under oligopoly causes the more liberal country to be worse off compared to autarky.*

Proof. See appendix A.1.3. □

Figure 1.2 graphically illustrates the effects. ΔL plots the rise in Home labour supply. Under autarky, the additional production would be fully consumed in Home, such that we would arrive at point C on $\mathrm{CI_A}$. However, the CI line shifts less than the initial rise in the labour endowment under free trade. This captures that less than 100 percent of the additional production is consumed in Home, i.e. real consumption (the CI line intercept I/p) rises less than real production (L). The related rise of consumption in Foreign can be described as a spillover effect. Also the MI line shifts less than the rise in labour supply[19], because all firms have an incentive to increase exports. Foreign firms want to increase their exports to the Home market as demand increases. Home firms export more as their production increases. However, which curve shifts more is ambiguous. We depict the case where the CI shifts more, which is always true if $L^* \leq 2L$ and depends on the parameters a and b otherwise.

Additionally, the general equilibrium makes distributional effects of labour market policies visible. In Home, real wages decrease, whereas in Foreign the real

[19] With the intercept $\lambda(L - L^*)$, this is because λ falls in L counteracting the increasing labour supply difference.

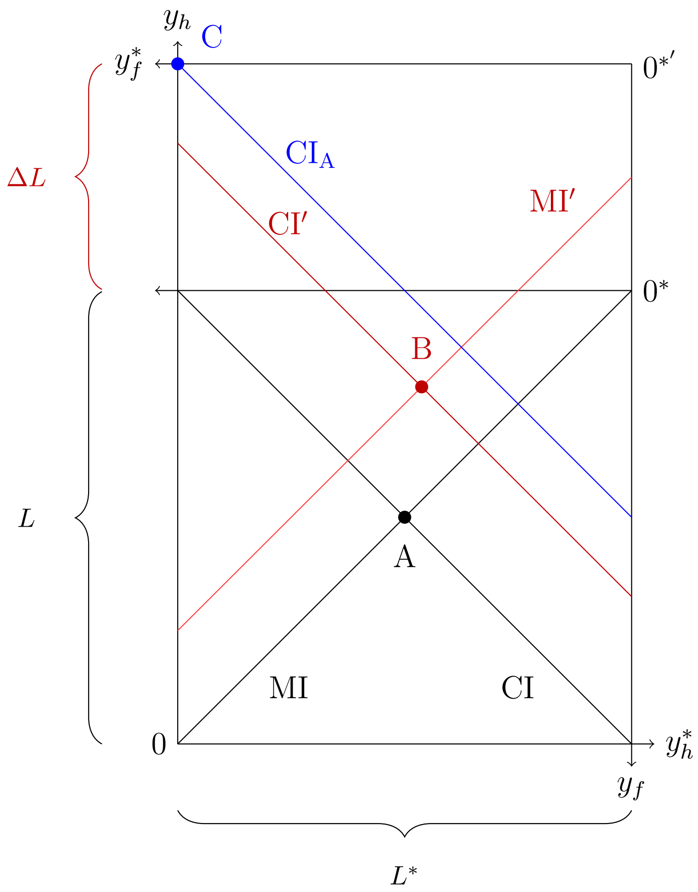

Figure 1.2: Home Labour Market Liberalization

$(n = n^* = 1)$

wage change is ambiguous with declining nominal wages and decreasing prices. If Home labour market policies were more liberal already ($L > L^*$), further liberalization reduces the labour share in Foreign.

Proposition 1.3 (Income and Distribution)**.** *Liberalizing the labour market, i.e. increasing the average working hours, in one country causes its real wages to fall. Whenever that country's labour market has been more liberal already, i.e. the average working hours have been larger, real profits rise and the labour share declines in the* *other country.*

Proof. See appendix A.1.4. □

With the growing labour supply on a perfect labour market and oligopolistic product markets, nominal wages fall in Home. At the same time, Home firms expand their production and supplies in both countries. This competitive pressure lets Foreign firms reduce their supply, thereby reducing their labour demand. As the labour supply is fixed at L^* the decline in labour demand leads to a reduction in the nominal wage rate in Foreign. With declining prices, however, the real wage effect in Foreign is ambiguous.[20] By contrast, the real wage rate unambiguously decreases in Home. Nominal wages decline due to the higher labour supply, while prices increase with the higher demand.

In equilibrium, Home firms' unit costs, i.e. the nominal wages, decline more strongly than those of Foreign firms. Home firms can improve their nominal profits, while the real profit reaction is ambiguous.[21] Foreign firms' nominal profits increase, if Home already had a more liberal labour market. Most importantly, their unit costs go down, which even outweighs potential revenue losses. Overall, the decline in nominal wages shifts Foreign income from workers to firms leading to the decrease in the labour share.

1.5 From Micro Behaviour to Macro Effects

Proposition 1.2 implies that country asymmetries induce a consumption spillover effect. One country can consume more than it produces, while the other country

[20] At least for $L = L^*$ Foreign real wages increase.
[21] At least for the initial increase at $L = L^*$ Home real profits increase.

produces more than it consumes. The simplified model with identical technology in all sectors does not include aggregate gains from trade. Accordingly, international trade under oligopoly causes a shift of consumption, and thus welfare, from one country to the other. This is a somewhat surprising result that requires further explanation.

The optimization behaviour of individual firms with respect to supply decisions on asymmetric segmented markets is the core micro-level element of the model. Each firm sets its supplies such that marginal revenues equal marginal costs in each market. Thus, country asymmetries can affect firms' decision-making via their impact on marginal costs and marginal revenues. This micro-level adjustment to country asymmetries has macro-level effects. This section aims to clarify the cost and the revenue channel for the case of asymmetric labour market policies.

A shift of consumption between the two countries requires differences in the amount of exported goods, i.e. the net export quantity of Home NXQ is not zero ($NXQ \equiv y_f - y_h^* \neq 0$). With an equalized balance of trade, this condition can only be fulfilled in general equilibrium with market segmentation, because a uniform world market price would imply that exported quantities are identical for both countries. Although this results endogenously from asymmetric policies in general equilibrium, the underlying transmission channels are already present in partial equilibrium models with oligopolistic competition and segmented markets like in Brander and P. Krugman (1983). Therefore, we first present the two channels in partial equilibrium, before turning to their interplay in general equilibrium.

For the partial equilibrium analysis, we stick to our previous notation, but consider the demand aggregators $\lambda^{(*)}$ and the wages $w^{(*)}$ to be exogenously given for a single oligopolistic sector. In partial equilibrium, the oligopolistic sector's trade can be imbalanced. We can combine the Cournot supplies to obtain

$$NXQ \equiv y_f - y_h^* = \frac{1}{3b}\left\{-\lambda^*(2w - w^*) + \lambda(2w^* - w)\right\},$$

such that we arrive at

$$NXQ \begin{cases} > 0 & \text{if } w^* \geq 2w \\ \gtrless 0 & \text{if } \frac{w^*}{2} < w < 2w^* \ \wedge \ \frac{\lambda}{\lambda^*} \gtrless \frac{2w-w^*}{2w^*-w} \\ < 0 & \text{if } w \geq 2w^*. \end{cases} \tag{1.13}$$

This shows that wage differences matter. When wages are not too different, as in the second case of equation (1.13), we have to compare countries' relative wages with their relative market sizes (demands). The exports of firms from different countries are only equalized if the expressions on the right of the second case are equal.

The above expression reveals two interrelated channels. For otherwise identical countries, a difference in marginal costs ($w \neq w^*$) or a difference in marginal revenues caused by demand asymmetries ($\lambda \neq \lambda^*$) is necessary for a net export quantity unequal to zero. By allowing countries to be exogenously asymmetric in one or both of these aspects in partial equilibrium, the resulting trade equilibrium can feature a non-zero net export quantity. However, in partial equilibrium this is not necessarily equivalent to a shift in welfare between the two countries because a sectoral trade imbalance can be compensated in other sectors.

The fundamental cause of the effect is the strategic behaviour of oligopolistic firms. When the two countries differ only in their marginal costs, as expressed in Figure 1.3, the high-cost country is net importer of the oligopolistic good in quantity terms. According to equation (1.13), this means that the demands are symmetric, but Foreign is the high-cost country and thus the foreign firm exports less than its counterpart from Home ($NXQ > 0$). The result is not surprising since with equal demands in both countries each country's firm supplies the same quantity in each market, in order to equalize marginal revenues. Because the low-cost country's firm finds it profitable to have larger equilibrium quantities in both markets, this country necessarily runs an export surplus in terms of traded quantity. With equal demand in both countries, prices are identical as well, such that the low-cost country runs a sectoral trade balance surplus.

When the two countries differ only in their aggregate demand and thus marginal revenues are no longer symmetric in both markets, as illustrated in Figure 1.4, oligopolists acknowledge that additional supply affects their marginal revenues stronger in a market, where they already sell more. Each oligopolist accepts a lower price in the smaller market to equalize marginal revenues across countries. If Foreign's demand is lower ($\lambda^* > \lambda$), it has a sectoral terms of trade advantage ($p > p^*$). At the same time, its firm exports more than the Home firm due to the demand asymmetry. Because the second case of equation (1.13) simplifies to $\lambda < \lambda^*$ if wages are equalized, the Home country, which has a higher demand, is the net importer in quantity terms. This is not surprising since its consumers demand more of the good.

In both of the described partial equilibrium cases trade imbalances emerge that are not compatible with a general equilibrium of symmetric sectors. Accordingly, prices and wages would have to adjust in order to restore balance of payments equilibrium. However, prices can still differ between the two countries in the new

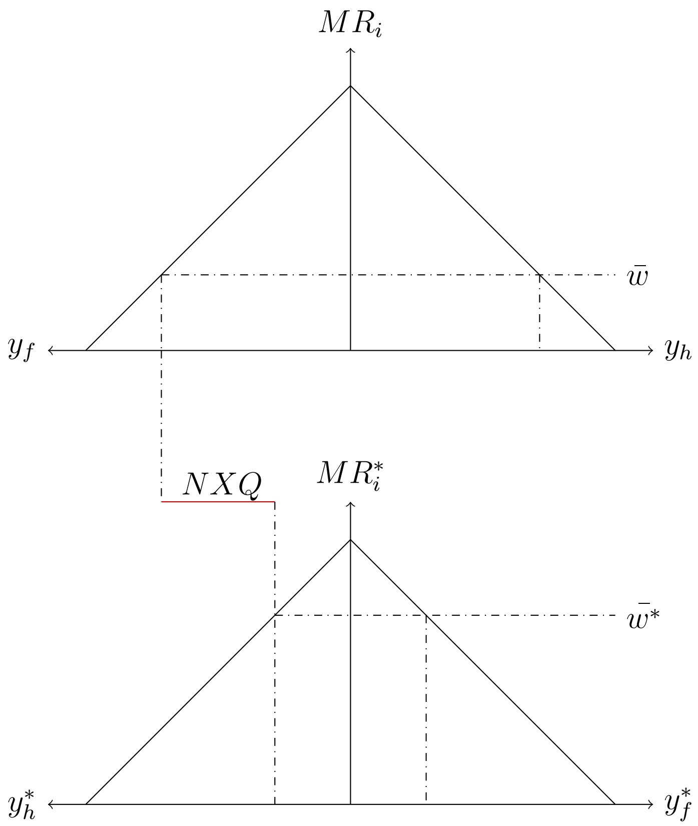

Figure 1.3: Marginal Cost Differences

equilibrium, such that one country is net quantity exporter in all sectors.

Compared to partial equilibrium, in general equilibrium the balanced trade condition has to hold, but also supplied quantities and labour supply are now connected. Consider a symmetric starting point. A growing labour supply in Home implies that Home firms have to increase their sales proportionally. The equilibrium production is fully determined by the exogenous labour supply. Wages and prices have to adjust, such that firms use all labour in equilibrium. When L becomes larger than L^*, this has two simultaneous effects. First, the Home wage falls and becomes lower than the foreign wage. Second, Home demand grows ($\lambda < \lambda^*$, such

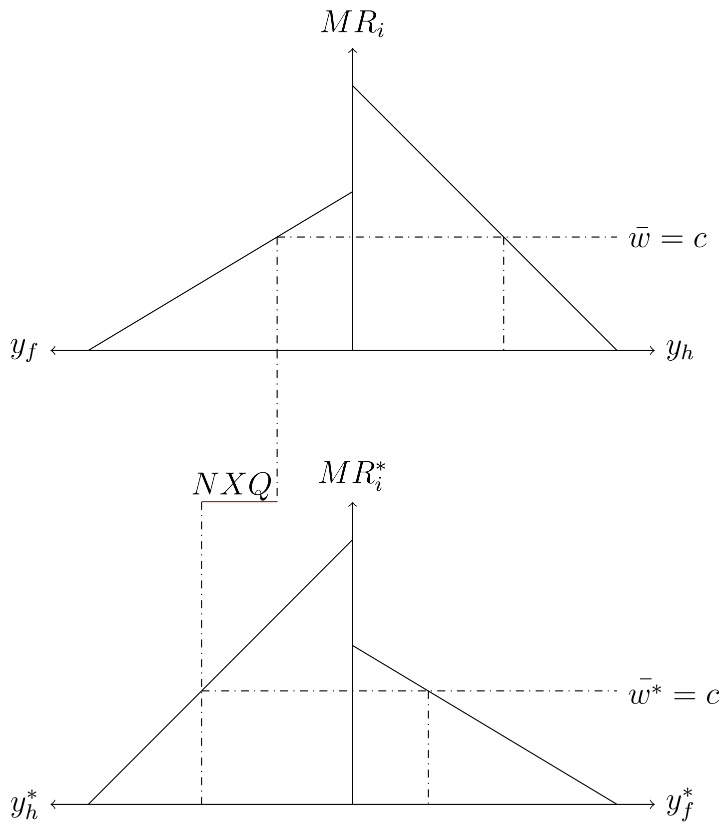

Figure 1.4: Marginal Revenue Differences

that also $p > p^*$).

As we have seen, the partial equilibrium marginal cost channel induces a positive net export quantity ($NXQ > 0$) of the low-cost country (Home in this case). The demand-side driven marginal revenue channel induces a negative net export quantity ($NXQ < 0$) for the larger market (also Home in this case). Proposition 1.2 shows that without running a trade balance deficit Foreign becomes a net importer in quantity terms and, thus, gains welfare at the cost of Home. This implies that the cost channel dominates the revenue channel for an asymmetric labour market policy in general oligopolistic equilibrium.

It is profitable for firms from both markets to supply more to the smaller Foreign market than they would in absence of strategic considerations, because this allows them to keep the price in the larger Home market, where they sell most of their output, artificially high. In a sense, firms from both countries are dumping on the smaller Foreign market. Additionally, Home production is larger, such that Home firms generally wish to supply more to both markets than Foreign firms, and thus to export relatively more, which again is associated with downward pressure on the Foreign price as compared to the Home price. This is beneficial for Foreign consumers. In our simple setting it follows from the balanced trade condition that the (Foreign) terms of trade $p/p^* = y_f/y_h^*$ are a sufficient statistic for the ratio of exported quantities.

The importance of the cost and the revenue channel for the firm-level adjustment to country asymmetries remains intact for other types of asymmetries. Under oligopoly, such asymmetries can induce deviations from the law of one price, which gives rise to terms-of-trade based international shifts in consumption and welfare. Accordingly, when firms have market power and countries are not symmetric, it is important to take micro-level firm behaviour into account as it affects macro-level outcomes.

1.6 Conclusion

We explore the effects of country asymmetries in a general oligopolistic equilibrium model with segmented markets. On the micro level, the supply behaviour of an individual firm in the two segmented markets is affected by country asymmetries via their impact on marginal costs and marginal revenues. These firm-level adjustments, in turn, have macro-level effects. Changes in a country's characteristics and policies can have international effects by inducing deviations from the law of one price. The resulting change in the terms of trade shifts consumption and welfare from one country to another. We argue that incorporating this mechanism is important when analysing asymmetric countries or policies in presence of firms with market power, because it can shape their welfare and distributional effects.

While we focus on the effects of asymmetric labour market policies in GOLE with otherwise symmetric and featureless economies to highlight this mechanism, it is important to also consider the industrial structure in the two countries as well as the degree of product differentiation for equally sized countries. If there are less active firms in each sector in a country, this leads to redistribution within this country as reduced competition leads to less labour demand and thus lower wages. The opposite effect occurs if goods become more differentiated. Firms want to increase their supply due to higher demand leading to increased wages and a higher labour share. The interplay of these elements is an interesting aspect for further research.

The model in this paper can be extended in various directions. Firstly, government policies like taxation or tariffs are a promising avenue to analyse within GOLE with asymmetric countries. Taxation of companies in an international context is appealing to discuss. Such taxes lead to redistribution of income from firms to governments across borders such that demand changes follow. In Quint and Rudsinske (2020) we consider such taxes but also tax-motivated transfer pricing. Another interesting tax measure to consider would be (asymmetric) value-added taxes, as it would reallocate income towards governments, but also directly influence firms' revenues and, thus, supply decisions. Tariffs reduce the profitability of exports and again affect government income as Rudsinske (2023) shows. In all of these cases, cross-country welfare effects emerge that are due to firms' strategic supply behaviour to asymmetric markets.

Further future applications include, for example, imperfect labour markets, cross-country ownership of firms, vertically related industries or unequally developed countries. Introducing some form of capital as a second factor of production might be of interest and would allow to analyse the effects of asymmetric capital stocks. Reintroducing differences in technology across countries could illuminate the interrelation of Ricardian gains from trade and the oligopolistic micro-level effects that we isolate in our application.

Chapter 2

International Trade and Tax-Motivated Transfer Pricing*

2.1 Introduction

In its "Transfer Pricing Guidelines" the OECD states that "[t]he role of multinational enterprises in world trade [...] increase[d] dramatically" (OECD, 2022, p. 11) over the last two decades. Indeed, Antras (2003) showed that at the turn of the millennium roughly one-third of world trade was intrafirm trade. This poses a challenge for firms but also governments as the taxation of multinationals needs to be viewed in its international setting. As trade within multinational firms does not take place on a market with independent participants, firms are free to set prices

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to maximize their profits. Therefore, some rules for transfer prices are necessary that are appropriate especially for tax purposes. The OECD sets out guidelines for such transfer prices, which should be set "at arm's length". Still, firms have the incentive and many possibilities to use the leeway in this framework in order to reduce their global tax bill by adjusting transfer prices.[1]

Tax avoidance through the use of transfer prices makes exporting more profitable. Empirically, multinationals' trading activity increases when tax rate differences become larger[2], which also affects aggregate country statistics.[3] Thus, the question naturally arises which welfare and distributional implications tax-motivated transfer pricing has for the countries involved.

We show in our model that the possibility of profit shifting via transfer price adjustments affects welfare and distribution through two channels. Firstly, income shifts from the high- to the low-tax country as firms reduce their tax payments. Secondly, exporting becomes more attractive for all firms, because reducing taxes is tied to the real activity of exporting. Therefore, firms do not only want to increase their intrafirm trade but also their overall production leading to higher wages. In general equilibrium, welfare increases in the low- and decreases in the high-tax country. Additionally, the labour share of income rises in the low-tax country.

We analyse corporate taxation and tax motivated transfer pricing in an asymmetric general oligopolistic equilibrium trade model based on Quint and Rudsinske

[1] For empirical evidence see e.g. Bernard et al. (2006) and Cristea and Nguyen (2016).

[2] P. Egger and Seidel (2013) find a 5.5% increase in intrafirm trade flows due to a 3.1% increase in the average host-country tax-gap to the United States.

[3] Carloni et al. (2019) estimate that the observed 13.1 percentage points decrease in the U.S. corporate tax rate in 2017 may have increased the U.S. trade balance by 9% through tax-motivated transfer pricing. Vicard (2015) and Casella et al. (2018) provide estimates of the substantial tax revenue losses attributable to transfer pricing. Additionally, see e.g. Clausing (2003), Clausing (2006), Liu et al. (2020).

(2023). The key element – first introduced in Neary (2016) – are firms that are modeled as “large in the small” sector they supply to but “small in the large” economy. We allow for segmented markets and fundamental country asymmetries. Trade does not occur due to technological differences, but firms’ strategic considerations matching the “reciprocal dumping”-argument in Brander and P. Krugman (1983).

Without profit shifting, an increasing profit tax rate shifts welfare towards the tax-increasing country as its income and hence demand increase while both decrease in the other country. The oligopolists’ optimization is affected as the marginal revenues in the two markets change. Firms respond to the demand changes by supplying more in the tax-increasing country, where the higher demand leads to higher prices. In the other country prices decrease due to the lower demand, so firms supply less. However, firms only re-adjust their supply between the countries and do not want to change their overall production. Hence, nominal wages remain unchanged and taken together with the price changes this leads to lower real wages in the country with the increased tax rate and higher real wages in the other country.

Transfer pricing generates an additional benefit from exporting for all firms because exports enable profit shifting. This incentivizes exporting with the result that companies not only shift their supplies between countries, but want to expand production and demand more labour inputs. Caused by this supply channel, nominal and real wages rise in both countries as the total labour supply is fixed. Due to shifting tax incomes, a cross-country demand channel relocates consumption – and thus welfare – from the high- to the low-tax country. In the low-tax country, real profits decrease and the labour share of income rises.

In our setting we analyse a situation in which all firms are already established and supply to both countries. We do not incorporate investment decisions and the extensive margin of exports into our model. While this undeniably affects the impact of profit shifting on welfare and distribution we are interested in the short-term outcomes of tax-motivated transfer pricing.[4] To this end we combine a simple transfer price decision with a general equilibrium framework that allows to include oligopolistic market structures and asymmetric countries.

Ramondo et al. (2016) find that intrafirm trade is concentrated among large affiliates within large multinational corporations, while Martin et al. (2020) show that tax avoidance even leads to increasing industry concentration because large firms can use it best. In line with their findings, we opt for an oligopolistic industry structure that accounts for firms' market power in their respective sector. Head and Spencer (2017) stress that oligopolistic firms need to be considered when assessing welfare effects of policies – especially when the allocation of profits across countries is involved. They also discuss the issue that oligopolistic firms might treat markets in each country as segmented rather than integrated and that it matters for policy analysis whether the market in only one country is affected or whether there exist linkages between markets. We account for both aspects by allowing firms to treat the markets as segmented and by explicitly modeling the labour market. While firms are able to set their quantities separately for both markets, their quantity reactions to policy changes in one of the markets will have repercussions on their supply in the other market via aggregate wage reactions in general equilibrium.

The general equilibrium is inherently necessary to capture the welfare, labour market and distribution effects. Most of the literature on profit shifting explores

[4] See for instance Davies and Eckel (2010) for a model of tax competition where firms decide on entry and production.

partial equilibria, only few models look at it within a general equilibrium framework. Existing general equilibrium approaches in that area usually focus on structurally different countries and on the influence of tax systems in presence of transfer pricing instead of the direct effects of transfer pricing for a given tax policy. For example, Krautheim and Schmidt-Eisenlohr (2011) analyse profit shifting between a large country and a tax haven in a general equilibrium monopolistic competition model. Eichner and Runkel (2011) compare different corporate tax systems in a general equilibrium model, but do not explicitly include transfer prices to determine the extent of profit shifting. Bond and Gresik (2020) compare different tax regimes in presence of transfer pricing from a welfare perspective, but assume that only the high-tax country can provide headquarter services that are necessary for the production of differentiated goods. In contrast, our model allows to look at two countries that are similar in their industrial structure and we can assess the welfare effects that directly stem from the mere existence of transfer pricing possibilities.

Before we analyse transfer pricing, we look at asymmetric profit taxation in the absence of transfer pricing. We find that a country benefits from unilaterally introducing a corporate tax because this relocates a part of foreign firms' profits towards domestic tax revenue, leading to tax exporting as foreigners bear the tax incidence (Krelove, 1992). Kohl and Richter (2021) utilize a heterogeneous firms monopolistic competition model that incorporates a fair wage-effort mechanism, and analyse a unilateral tax on operating profits. In contrast to our model without a transfer price scope, their tax distorts the companies' decisions. However, the tax is able to reduce inequality in the trading partner country, which is mainly in line with our results – though in our case the tax favours labour as compared to profit income in the taxing country as well.

The literature looks at transfer prices from different angles. Firstly, transfer prices are a device for decentralized decision-making within a company that establishes multiple divisions (see e.g. Hirshleifer (1956), Bond (1980) and Elitzur and Mintz (1996)). We assume centralized decision-making by the parent company, which from the firms' perspective might be the obvious organizational form to optimise profit shifting in presence of tax differences. Secondly, transfer prices matter when looking at intrafirm trade and profit-shifting possibilities. Early models included exogenous boundaries on transfer prices, which restricted profit shifting (Horst, 1971). Kant (1988) developed the concept of concealment costs that leads to endogenously determined transfer prices. Similar to Auerbach and Devereux (2018), we opt for the analytically more tractable way of setting exogenous boundaries on the transfer price decision. Much of the recent literature on transfer prices and taxation focuses on organizational and locational decisions and the incentives of tax-motivated transfer pricing (see e.g. Behrens et al. (2014), Peralta et al. (2006), Devereux and Keuschnigg (2013), Auerbach and Devereux (2018)). In these partial equilibrium models, governments may decide on tax rates and on different taxation systems such as differing transfer pricing benchmarks. In our general equilibrium approach, we do not include capital in the production process and assume that companies already made their locational decision, which they cannot change at a reasonable cost.

We first introduce the theoretical model and our solution strategy in Section 2.2. After showing the effects of unilateral taxation in Section 2.3, we analyse the effects of transfer pricing in Section 2.4. The final section concludes. All proofs are deferred to the appendix.

2.2 Transfer Pricing in Asymmetric General Oligopolistic Equilibrium

We integrate national corporate taxation and a scope in transfer price decisions that allows for profit shifting into a two-country model of international trade in asymmetric general oligopolistic equilibrium.

2.2.1 Model Components

In the following, we usually present expressions for the Home country only. Expressions for Foreign are analogous. Variables referring to Foreign are marked with an asterisk.

Each country is inhabited by one representative consumer, whose preferences are additively separable. He consumes goods from each of the sectors in the continuum $z \in [0, 1]$. The representative consumer inelastically supplies L units of labour to a perfectly competitive labour market. We assume continuum-quadratic preferences:

$$U[\{y(z)\}] = \int_0^1 u[y(z)]dz \quad \text{with} \quad \frac{\partial U}{\partial y(z)} > 0 \text{ and } \frac{\partial^2 U}{\partial y(z)^2} < 0$$
$$\text{where} \quad u[y(z)] = ay(z) - {}^1\!/\!{}_2\; by(z)^2.$$

Here, $y(z)$ is the amount of consumption of a homogeneous good produced in sector z and a as well as b are parameters with $a, b > 0$. We assume that the parameters are set such that marginal utilities of each good are positive.

The representative consumer's income consists of three elements. The wage income $w \cdot L$ is not taxed and the wage rate w is determined in general equilibrium. Additionally, aggregate after-tax profits (Π) of Home country companies and tax revenues (T) of the country are disbursed to consumers. Thus, companies are fully owned by the representative consumer in the parent's residence country. Therefore, the budget constraint of the representative consumer is given by

$$\int_0^1 p(z)y(z)dz \leq I = w \cdot L + \Pi + T, \tag{2.1}$$

where $p(z)$ is the price per unit of the good in sector z. Utility function and budget constraint lead to the utility maximization problem represented by the Lagrangian:

$$\max_{y(z),\forall z} \mathcal{L} = \int_0^1 \left(ay(z) - {}^1\!/_2\, by(z)^2\right) dz + \lambda \left(I - \int_0^1 p(z)y(z)dz\right)$$

The first order condition then gives $0 = a - by(z) - \lambda p(z)\ \forall z$ with λ being the Lagrange-parameter and therefore the marginal utility of income. The inverse Frisch demand follows straightforwardly and is given by

$$p(z) = \lambda^{-1} \frac{\partial u[y(z)]}{\partial y(z)} = {}^1\!/_\lambda [a - by(z)]\ \forall z. \tag{2.2}$$

The inverse demand functions depend on the marginal utility of income negatively. The marginal utility of income λ acts as a demand aggregator where a higher value indicates a lower demand for goods in every sector.

The producers aim to maximize their profits given the demand, the tax rates and the system of tax collection. Firms are assumed to have market power in their respective markets. However, they do not have direct influence on aggregate economic factors, as a continuum of sectors exists. Each firm is large in its own sector but small in the economy as a whole.

We assume that in both countries $n = n^* = 1$ firms exist in each sector z and that there are neither fixed costs of production nor transport costs. The firms play a static one-stage game where they compete in Cournot competition over output in the Home and Foreign market. They take the consumers' demand as given and perceive the inverse Frisch demand functions as linear – irrespective of the functional form of λ – as the companies do not have an individual influence outside their own sector.

Labour L is the only factor of production. It moves freely across sectors within a country but not across national borders. The wage rate w is determined at the country level such that the inelastically supplied labour L equals the demand for labour resulting from production of the companies.

Production occurs with constant returns to scale and common technology in each sector z, such that marginal costs in sector z are constant. To keep the model as simple as possible, we consider the case of identical technology across sectors as well as countries. The sector-specific common unit-labour requirements are $\gamma(z) = \gamma^*(z) = 1 \ \forall z$ so we can drop z throughout as the costs per unit is the wage rate w and independent from the sector. The model does not capture a Ricardian-style technological comparative advantage anymore. However, we retain the assumption of a multitude of sectors even though they will be symmetrical. As companies remain small in the large, they do not take their effect on wages into account when maximizing their profits. In this setting, international trade emerges due to strategic interactions among firms as in Quint and Rudsinske (2023) and profit shifting.

As all sectors are equal, there is no price heterogeneity that would affect the representative consumer's consumption decision and utility as in Neary (2016). Because of the strictly increasing marginal utility of consumption, we have a strictly monotonic relationship between consumption (or real income) and welfare defined as the representative consumer's utility. Accordingly, these terms can be used interchangeably when considering the direction of effects.

In their profit maximization, the firms have to take the tax system into account. One company comprises two distinct legal entities. On the one hand, the parent company produces the good in one country and sells the good in the same country. On the other hand, the subsidiary sells the good in the other country, where it is incorporated, but does not produce itself. Instead, the subsidiary imports the good only from its parent company. Therefore, a transaction between the two entities emerges that is not mediated over a market and does not have any consequences on the profits of the multinational company in the absence of taxation. In our model, however, the two entities fall under different tax jurisdictions. The parent is subject to taxation in one country, whereas the subsidiary is taxed by the other. To attribute the profits to the two entities, the company sets a transfer price $\Phi(z)$ per quantity of the good for the intrafirm transactions. If tax rates differ between countries, manipulations of this transfer price can reduce the company's overall tax bill as the transfer price affects the allocation of tax bases. The management of the multinational company sets the transfer price to maximize the company's aggregate after-tax profits subject to the constraints of the tax system.

We assume that the governments of Home and Foreign agreed to tax the multinational companies according to the source principle in conjunction with the territoriality principle. Profit streams resulting in a country will be taxed there, and not in the country where the parent company is located. Hence, profits realized in the subsidiary's residence country – and already taxed – are exempt from taxation in the parent companies' country.

The Home country taxes all companies active in Home. On the one hand, the multinational companies, which produce in Home, are subject to the Home tax with income generated by sales at Home less the cost of production for these sales. Additionally, exports to their affiliates in Foreign are taxed according to the difference between transfer price and unit costs. On the other hand, the Home government applies its tax on the subsidiaries, which only sell in Home, but import the goods from their parent companies in Foreign. For these subsidiaries, the tax base in Home results from the generated turnover, where the transfer price payment to the Foreign parent is deducted. There are no withholding taxes on dividend payments from subsidiaries to parents.

To ensure that the source principle holds, the governments commit to a common transfer price guideline. Companies are requested to set their transfer price Φ equal to the marginal costs of producing the good as is common in transfer pricing models (see e.g. Kind et al. (2005)). A company's net profit with the parent in Home and a subsidiary in Foreign is

$$\begin{aligned} \pi &= (1-\tau)\left[(p-w)y_h + (\Phi - w)y_f\right] + (1-\tau^*)\left[p^* - \Phi\right]y_f \\ &= (1-\tau)\left[p-w\right]y_h + (1-\tau^*)\left[p^*-w\right]y_f + \ (\tau^*-\tau)\left[\Phi - w\right]y_f, \end{aligned} \tag{2.3}$$

where the Home tax rate is τ and $0 \leq \tau < 1$.[5] Here, y_i indicates the amount of the good sold by the company in country $i \in h, f$. Equation (2.3) shows that if Φ is set equal to marginal costs, profit shifting will not occur and the source principle strictly holds. A company can increase its net profit – given differentiated tax rates – by adequately manipulating the transfer price if there is some scope for

[5]For Foreign, it holds that $0 \leq \tau^* < 1$.

deviations. If the Foreign tax rate is lower ($\tau > \tau^*$), the transfer price will be set as low as possible by the Home companies, such that a part of its profits is effectively shifted abroad. Additionally, we can see that the positive transfer price effect on a company's profit is tied to its exports. The more a company exports the more possibilities it has to shift profits to the low-tax jurisdiction. This means that a real activity is needed to shift profits towards the low-tax country.

To achieve some scope in the firms' transfer price decision, we assume that governments do not have complete information on the firms. Firms can deviate by g units from the marginal cost benchmark in either direction when setting the transfer price. To improve tractability, we do not assume concealment costs attached to the deviation from the benchmark, which is analogous to other models on transfer pricing (see e.g. Auerbach and Devereux (2018)). The deviation parameter is assumed to be equal across countries. The range of possible transfer prices for a firm producing in Home is given by

$$\Phi \quad \in \quad [w - g; w + g] \,. \tag{2.4}$$

For Foreign firms w^* is the benchmark. We assume that companies will not set transfer prices outside these ranges as this would result in harsh penalties. Here, g is a parameter and will not be deliberately set by governments. It may be interpreted as a general ineffectiveness or legal in-expertise by administrations.

2.2.2 Cournot Equilibrium

The firms maximize their after-tax profits given the marginal utilities of income λ and λ^* as well as the wage rates w and w^* by setting their supplied quantities and their transfer price. All companies active in that sector in the country compete in Cournot competition to satisfy this demand simultaneously. We consider Home firms, but profit maximization for Foreign firms is analogous. Given the demand and the other companies' supply, firms maximize their profits by choosing their supplied quantities in both countries.

$$\begin{aligned}\max_{y_h, y_f, \Phi} \pi &= (1-\tau)\,[p-w]\,y_h + (1-\tau^*)\,[p^*-w]\,y_f + (\tau^*-\tau)\,[\Phi-w]\,y_f \\ &\quad \text{with } \Phi \in [w-g; w+g]\,,\ p = {}^1\!/\!_\lambda[a-by] \text{ and } p^* = {}^1\!/\!_{\lambda^*}[a-by^*],\end{aligned}$$

where y describes the total supply of the good in Home and y^* in Foreign.

The first order conditions for the firms' profit maximization over their quantities sold are

$$\frac{\partial \pi}{\partial y_h} = (1-\tau)\left[\frac{1}{\lambda}\,(a-2\;b\;y_h-b\;y_h^*)-w\right] = 0 \tag{2.5}$$

$$\frac{\partial \pi}{\partial y_f} = (1-\tau^*)\left[\frac{1}{\lambda^*}\left(a-2\;b\;y_f-b\;y_f^*\right)-w\right] + (\tau^*-\tau)(\Phi-w) = 0. \tag{2.6}$$

These first order conditions can be transformed into reaction functions depending on the supply of Foreign companies in the respective markets. The supply to Foreign shows an effect of the transfer price on exports.

$$y_h = \frac{a-\lambda w-b\;y_h^*}{2b} \tag{2.7}$$

$$y_f = \frac{a-\lambda^* w-b\;y_f^*+\frac{\tau^*-\tau}{1-\tau^*}\lambda^*(\Phi-w)}{2b} \tag{2.8}$$

As mentioned above, the transfer price will be set taking the difference in tax rates into account. This follows from ${}^{\partial\pi}\!/\!_{\partial\Phi} = (\tau^*-\tau)y_f$, which commands the company to either set the transfer price as high as possible or as low as possible. If $\tau^* > \tau$, profits can be magnified with marginal increases of the transfer price. Hence, the rule is to set the transfer price as high as possible. This is reversed for $\tau^* < \tau$. Bearing in mind the admissible transfer price range in equation (2.4), it follows that the optimal transfer price is[6]

$$\Phi = \begin{cases} w + g & \text{if } \tau^* > \tau \\ w & \text{if } \tau^* = \tau \\ w - g & \text{if } \tau^* < \tau. \end{cases} \tag{2.9}$$

Combining the reaction function (2.8) and the optimal transfer price rule in equation (2.9) leads to

$$y_f = \frac{a - \lambda^* w - b\, y_f^* + \frac{|\tau^* - \tau|}{1-\tau^*}\lambda^* g}{2b}. \tag{2.10}$$

This formulation contains all cases for the optimal transfer price as the sign in equation (2.9) assures that the absolute value of the tax rate difference determines the transfer price effect on exports. The reaction function is continuous even though the optimal transfer price in equation (2.9) exhibits a discontinuity for equalized tax rates. Multiplying with the tax rate differential assures that this does not feature in the reaction function. At the point of the discontinuity in the optimal transfer price it is unaffected by the transfer price for equal tax rates.

By combining the reaction functions from Home's and Foreign's companies in the respective markets we obtain the Cournot-Nash-equilibrium supply for Home (Foreign) companies y_i (y_i^*) in both markets. The equilibrium supply of each

[6] We assume that if tax rates are equal and profit shifting via transfer price manipulation is not possible, companies will set the transfer price equal to marginal costs.

individual company to the Home market is[7]

$$y_h = \frac{\lambda}{3b}\left\{\frac{a}{\lambda} - w + (w^* - w) - \frac{|\tau^* - \tau|}{1-\tau}\, g\right\} \tag{2.11}$$

$$y_h^* = \frac{\lambda}{3b}\left\{\frac{a}{\lambda} - w^* + (w - w^*) + 2\,\frac{|\tau^* - \tau|}{1-\tau}\, g\right\}. \tag{2.12}$$

The supplied quantities in Foreign are analogous:

$$y_f = \frac{\lambda^*}{3b}\left\{\frac{a}{\lambda^*} - w + (w^* - w) + 2\,\frac{|\tau^* - \tau|}{1-\tau^*}\, g\right\}. \tag{2.13}$$

$$y_f^* = \frac{\lambda^*}{3b}\left\{\frac{a}{\lambda^*} - w^* + (w - w^*) - \frac{|\tau^* - \tau|}{1-\tau^*}\, g\right\} \tag{2.14}$$

The profit tax only affects the supplied quantities in both markets respectively via the transfer price, as the tax rate differential determines the gain from profit shifting via transfer pricing. However, if transfer prices are set according to marginal costs, taxes do not affect the companies' supply decision but only the after-tax profits. If profit shifting is possible and the high tax country further raises its tax rate, its firms increase their exports. Exporting becomes more profitable as even more tax payments can be avoided. The same is true for the low-tax country firms that increase their exports as well. This results in higher competitive pressure such that all firms are inclined to reduce their domestic supply.

An increase in the range for possible transfer prices g will reduce the supply in the production country while increasing exports. As mentioned above, the profit shifting via transfer pricing is tied to the exports, thereby making exporting more

[7] y_i indicates the supply of one company producing in Home (no asterisk) and selling in country i. y_i^* signals the supply of one company producing in Foreign (*) and selling this amount in country i.

profitable.[8] At the same time, increased exports lead to higher competitive pressure and a reduction in firms' domestic supply.

2.2.3 General Equilibrium

Labour Market

With the Cournot-Nash-equilibrium supply derived above we can turn to the clearing of the labour market. As described, the representative consumer inelastically supplies L and L^* units of labour in the respective countries. For simplicity we assume that countries are symmetric in their labour endowment and set $L = L^* = {}^1\!/_2$. The labour demand depends on the equilibrium supply of goods produced in the respective country. Each company in one sector in Home will produce $y_h + y_f$. The total labour demand is given by $L^D = \int_0^1 y_h + y_f \; dz$. In equilibrium demand has to equal supply. With $\bar{\lambda} \equiv \lambda + \lambda^*$ this yields in Home

$$
\begin{aligned}
L = \frac{1}{2} \quad &= \quad \int_0^1 y_h + y_f \; dz = y_h + y_f &(2.15)\\
&= \quad \frac{1}{3b}\left\{2a - \bar{\lambda}\; w + \bar{\lambda}(w^* - w) + |\tau^* - \tau| \; g \; \left(\lambda^* \frac{2}{1-\tau^*} - \lambda \frac{1}{1-\tau}\right)\right\}.
\end{aligned}
$$

In combination with the analogously defined equilibrium on the labour market in Foreign, wages in both countries are

$$
w \quad = \quad \frac{1}{\bar{\lambda}}\left\{2a - \frac{3}{2}b + \lambda^* \; g \; \frac{|\tau^* - \tau|}{1-\tau^*}\right\}; \tag{2.16}
$$

$$
w^* \quad = \quad \frac{1}{\bar{\lambda}}\left\{2a - \frac{3}{2}b + \lambda \; g \; \frac{|\tau^* - \tau|}{1-\tau}\right\}. \tag{2.17}
$$

[8] Transfer price possibilities work similar but opposite to tariffs in our model. See Rudsinske (2023) for an application of the model to the case of asymmetric import tariffs.

Both equilibrium wages depend positively on the transfer pricing scope g and the tax differential $|\tau^* - \tau|$, weighted by the marginal utility of income in the export destination country. Wages are equal across countries if $g = 0$.

General Oligopolistic Equilibrium

In equilibrium the model is characterized by nine equations in nine endogenous variables. The Cournot equilibrium quantities determine the supply of each multinational company to each country given the wages and the marginal utilities of income. The labour market clearing in each country determines the wage given the produced quantities in the respective country. Additionally, the prices are given by the representative consumers' inverse Frisch demand functions.

The last equation implicitly determines the marginal utility of income. We use the budget constraint of the representative consumer to attain this implicit definition. The budget constraint is given by $p(y_h + y_h^*) = w\ L + \pi + T$. This can be rearranged to obtain a straightforward relationship that has to hold in equilibrium:

$$-\left(p^*\ y_f - p\ y_h^*\right) = \tau\ y_h^*\ \left(p - w^* - g\right) - \tau^*\ y_f\left(p^* - w + g\right) \tag{2.18}$$

On the left hand side we have the (negative) balance of trade of the Home country which has to equal the balance of capital on the right hand side. The balance of payments has to be even in equilibrium. We allow for trade imbalances if these are offset by capital transfers, which are possible due to differing tax payments of the companies across countries.

To attain the general equilibrium values of the endogenous variables and to solve the system of equations, we further need to address the determination of the marginal utilities of income. Following Quint and Rudsinske (2023), we normalize the aggregate marginal utility of income to unity, i.e. $\bar{\lambda} = 1$. Hence, the aggregate marginal utility of income is used as numéraire. This translates into the relationship between λ and λ^* that $\lambda^* = 1 - \lambda$, which allows us to substitute all λ^*. Both marginal utilities of income will lie between zero and one which follows from the economic reasoning that a marginal utility has to be positive.

We can now further simplify the system of equations by expressing all endogenous variables, such that they only depend on exogenous parameters and λ.[9] These can then be used in the balance of payments condition such that we only have one equation in one variable left. To improve tractability, we introduce some further assumptions. Without loss of generality we assume that Home is the high tax country, i.e. $0 \leq \tau^* \leq \tau < 1$. To ensure that the condition of a positive marginal utility of consumption holds in any case, we set $b < a$.[10] Additionally, interior solutions to each firm's supply decision at $g = 0$ are given by setting $2a < 3b$. We show that there exists an equilibrium, which is unique if the transfer price scope is not too large, i.e. $g < \bar{g}$.

Lemma 2.1 (Existence and Uniqueness of $\hat{\lambda}$)**.** *There exists a solution to the condition of an even balance of payments in* $\lambda \in (0, 1)$*, which is unique if* $g \leq \bar{g}$*.*

Proof. See appendix B.3.1. □

Unfortunately, we cannot determine the equilibrium marginal utility of income $\hat{\lambda}$ in closed form as in our equilibrium condition (2.18) it is derived from a quintic polynomial. According to Abel's impossibility theorem, there is no solution to

[9] See appendix B.1 for these equations.

[10] The most extreme case would be $y = L + L^* = 1$. More generally, we need $b < {}^{a}\!/_{(L+L^*)}$. Under our assumptions this also assures positive wages at $g = 0$, which require $b < 2a\left(({}^{n+1}\!/_{n})L + L^*\right)$.

this polynomial in radicals. However, it is possible to determine derivatives of $\hat{\lambda}$ with respect to exogenous parameters by implicitly differentiating the equilibrium condition.

To give the reader some intuition about the mechanisms that determine the general equilibrium supplies, we can break the system down into two fundamental conditions. The consumption indifference condition (CI) states that for utility maximization the origin of the product is inconsequential. The representative consumer is indifferent between products in the same sector that are produced in Home and in Foreign. The market indifference condition (MI) states that in equilibrium firms have to be indifferent between selling the marginal unit in Home or in Foreign. We can plot these two conditions in a box diagram with the Home origin (0) in the lower left corner and the Foreign origin (0^*) in the upper right corner.

The CI is easily derived from the budget constraint.

$$CI: \qquad y_h = \frac{I}{p} - y_h^*$$

It gives us a function with perfect substitutability between Home and Foreign goods from the consumer's perspective, for whom real income I/p is exogenous. In equilibrium without tax rate differences, the intercept is exactly in the upper left corner of the graph. In this case both countries are symmetric and consume the same quantities such that $I/p = L = 1/2$. An increase in real income in Home, for example through taxation, shifts the CI line towards the upper-right corner.

The MI can be derived from the fact that the marginal revenues of a firm – including possible taxation effects – need to be equal in both markets in equilibrium.

For Home, this follows straightforwardly from the profit maximization and can be rearranged[11] to

$$MI: \qquad y_h = -\lambda(1-\lambda)\frac{g}{b}\frac{(\tau-\tau^*)(2-\tau-\tau^*)}{(1-\tau)(1-\tau^*)} + y_h^*. \tag{2.19}$$

This results in a linear relationship, which in our diagram again is the same from Foreign's perspective. The first term on the right-hand side is exogenous from the firm's perspective. This intercept can be interpreted as the aggregate export incentive across countries. When g or the tax rate difference increases, the MI shifts downwards. This implies higher export shares for all companies.

If countries have the same tax rates and there is no transfer pricing leeway, Figure 2.1 shows how the equilibrium is determined at the intersection of MI and CI. We will use this depiction later on to graphically illustrate the effects of asymmetric taxation and transfer pricing possibilities on the consumers and producers.

2.3 Effects of Asymmetric Taxation

To facilitate explaining the effects of transfer price manipulation, we first consider unilateral tax policy in our model, if the transfer prices are set equal to marginal costs.[12] If both countries set the same tax rate, countries are symmetric in all exogenous parameters. Therefore, the equilibrium values of the marginal utilities of income in both countries need to be equal, i.e. $\hat{\lambda} = 1/2$. Accordingly, supplied quantities, prices and wages are the same in both countries.

[11] For the derivation see appendix B.2.

[12] We do not consider equilibrium tax rates that result from tax competition between the two countries. Here, we are only interested in comparative statics as a way to make the effects of tax-motivated transfer pricing clearer.

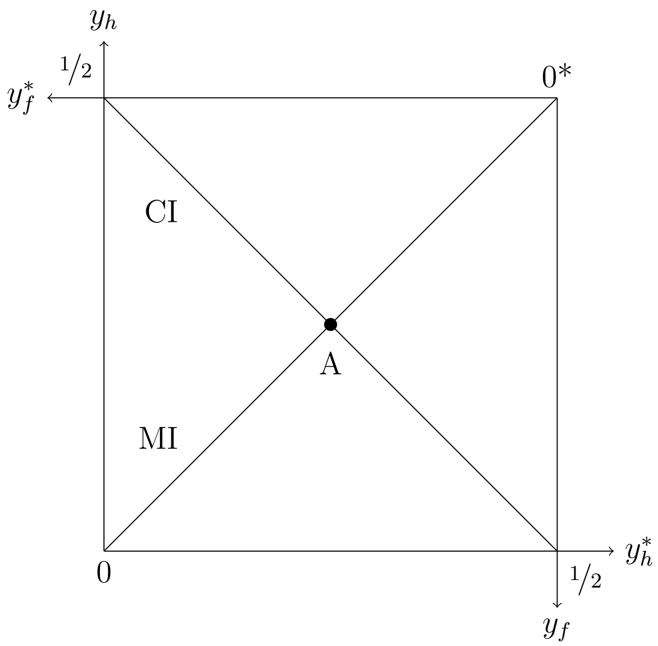

Figure 2.1: Symmetric Equilibrium with $\tau = \tau^*$

If the Home country increases its tax rate, this does not influence the multinational companies' supply decision directly, when $g = 0$. The same holds for the wages and the prices. However, the marginal utility of income in Home will fall in equilibrium and also affect the other variables in general equilibrium.[13]

Lemma 2.2 (Effect of Unilateral Tax Policy on $\hat{\lambda}$)**.** *A unilateral increase in the tax rate decreases the marginal utility of income domestically and increases it in the other country for any tax rate τ and τ^*.*

Proof. See appendix B.3.2. □

The reaction of the marginal utility of income stems from an income effect. All firms have to pay more taxes in the high-tax country Home, which are disbursed

[13] In the initial situation of equalized tax rates, both countries are identical which also implies identical marginal utilities of income. If then τ is increased, $\hat{\lambda}$ will decrease such that $\hat{\lambda} < 1/2$ for all $\tau > \tau^*$.

to the representative consumer in this country. While the tax payments of Home firms remain with the same consumer, Foreign firms' increased tax payments are a redistribution of income from Foreign to Home. The after-tax profits in Foreign and thus its income declines while the increased taxes end up with the Home representative consumer. In the high-tax country (Home) the income increases, while it decreases in the low-tax country (Foreign).

Following this surge in income, demand will rise in Home while it decreases in Foreign. The oligopolistic firms observe these changes in demands in the two markets. As prices increase in the high-tax country, firms sell more units of their good in this market, whereas reduced prices in Foreign lead to lower supplies. This is in line with the balance of payments equilibrium. As Home's balance of capital increases due to the increased inflow of tax payments by Foreign firms, the (negative) balance of trade has to increase as well through higher imports.

The changes in demand and the subsequent supply adjustments further reinforce the original income effect. All firms reduce their supply in Foreign where the smaller prices lead to falling mark-ups. This diminishes the tax base in Foreign and thus also the overall tax payments, especially by Home firms leading to an even smaller income. The opposite is true in the high-tax country Home, where income additionally increases.

Even though the companies react to the demand changes, they do not have an incentive to increase their overall production. As income shifts from Foreign to Home, firms follow this change with their supply, but do not want to expand production. Therefore, nominal wages in both countries remain unchanged after a unilateral tax increase if $g = 0$.

Proposition 2.1 (Effects of Unilateral Tax Policy)**.** *A unilateral tax increase in the high-tax country raises consumed quantities in the high-tax and decreases them in the low-tax country. Real wages rise in the low- and fall in the high-tax country, while real profits decrease in both countries. Labour income gains relative to profit income in both countries.*

Proof. See appendix B.3.3. □

A unilateral increase in the high-tax country's tax rate favours the high-tax country. Most importantly, total consumption increases there, even though prices increase as well. In our setting, the welfare strictly increases in the consumed quantity. A higher quantity directly leads to a higher utility for the representative consumer as given by the positive marginal utility of consumption. The incidence of the tax increase falls on the Foreign representative consumer, which we can characterize as tax exporting.

However, not only the distribution of income between countries, but also within countries is altered by unilateral tax policy. With constant nominal wage rates the price changes determine the effect of an increased tax rate on real wages. In Home prices increase leading to lower real wages, whereas prices decrease in Foreign, which raises real wages. Nevertheless, nominal after-tax profits fall in both countries. Therefore, the labour-to-profit ratio increases in Home and Foreign.

Figure 2.2 illustrates the situation of an increasing tax rate in Home. An increase in the Home tax rate shifts the CI line upwards, because Home gains tax revenue at the cost of Foreign profits and Foreign tax revenues, which increases the available real income of the Home representative consumer. The MI line is unaffected by unilateral taxation, because the tax does not directly influence firms'

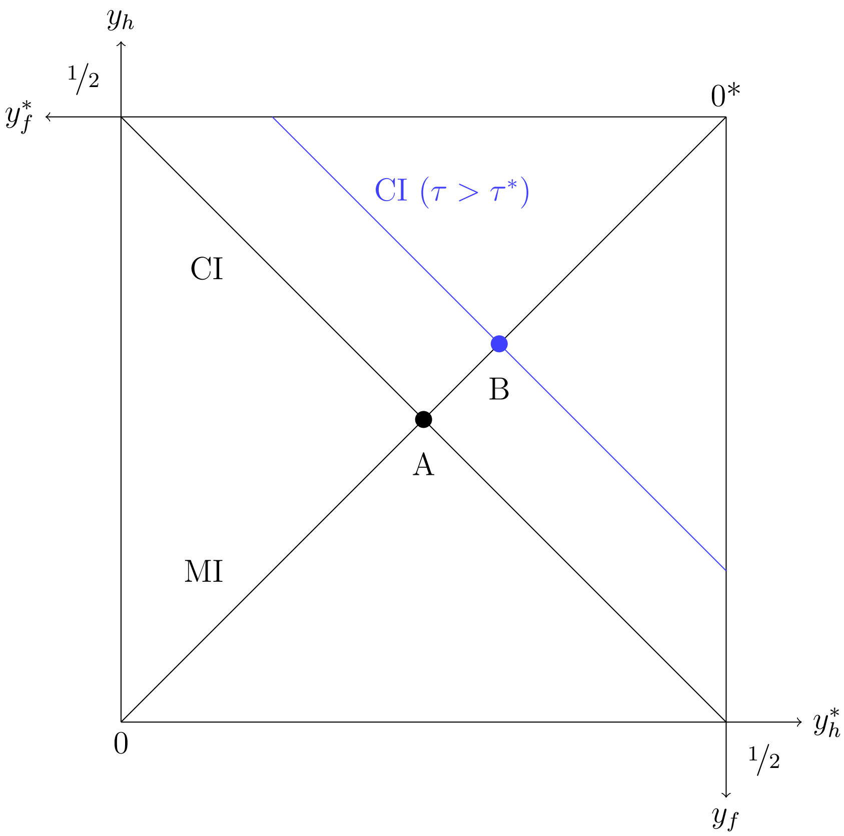

Figure 2.2: Unilateral Taxation with $\tau > \tau^*$

profit optimization with $g = 0$.[14] Firms only react to demand changes and shift their supply accordingly. The new equilibrium point B is at the intersection of the new CI and the unchanged MI line, where all firms have a higher supply in Home and a lower supply in Foreign.

[14] As seen in equation (2.19), the intercept is zero for $g = 0$.

2.4 Effects of Transfer Pricing

Up to this point, multinational companies were assumed to adhere to the arm's-length benchmark in their transfer pricing decisions. Effectively, unilateral tax policy led to a redistribution of profits towards tax revenue in the high-tax country. This stimulates demand in Home and reduces it in Foreign which causes an increase of supply and prices in the high-tax country and a reduction of these in the low-tax country.

Now we introduce some scope into the companies' transfer pricing decision. They can deviate from the transfer price benchmark of marginal costs in order to reduce their tax payments. This has two initial impacts on the economy. On the one hand, all companies have an additional incentive to export, as with each unit of exports more taxes can be avoided. This effect gives rise to a supply channel, where all firms want to increase their exports and indeed their overall production.

On the other hand, tax-motivated transfer pricing leads to changes in the two countries' incomes and thus demands. All firms reduce their tax payments by shifting tax base to the low-tax country. Thus, the high-tax country's tax revenues fall while those of the low-tax country rise. In the high-tax country, this loss of tax revenues is not offset by higher after-tax profits such that the income of the representative consumer decreases. In the low-tax country tax revenues increase, which leads to a higher income of the representative consumer. Following these income shifts, the demands in the two countries change. These demand effects constitute the demand channel of tax-motivated transfer pricing.

We analyse how the equilibrium is affected by the initial increase in the transfer pricing scope parameter g at $g = 0$. First, we show the reaction of the demand aggregator $\hat{\lambda}$ in Home.

Lemma 2.3 (Effect of g on $\hat{\lambda}$)**.** *The equilibrium marginal utility of income in the high-tax country $\hat{\lambda}$ increases in the transfer pricing scope g.*

Proof. See appendix B.3.4. □

Using this lemma, we can show the effect of g on the other variables. In equilibrium, all variables are affected by a marginal change of g. Firstly – and in line with partial equilibrium results – aggregate exports increase due to changing exporting incentives.

Proposition 2.2 (Trade Creation Effect)**.** *The number of exported units increases in the transfer pricing scope g.*

Proof. See appendix B.3.5. □

This trade creation effect reflects the supply channel through which firms want to increase their exports. Firms do not only want to shift their supply, but actually want to produce more. In general equilibrium, however, total production is fixed by the labour supply. Therefore, only the labour demand increases and nominal wages rise in both countries if g is marginally increased.

However, this supply channel is not the only effect of tax-motivated transfer prices in general equilibrium. Through the demand channel imbalances between countries' incomes from asymmetric taxation get reduced. We exemplify this demand channel by looking at the prices. Income is transferred across countries from the high- to the low-tax country. The change in incomes across countries affects demand, which leads to increasing prices in Foreign and decreasing prices in

Home. This in turn affects the firms' marginal revenues in the two countries such that they want to shift supply from the high-tax country to the low-tax country.

For Home country firms, both channels – demand and supply – operate in the same direction. These firms want to export more as they can avoid more taxes with increased exports. Additionally, the increased demand in Foreign stimulates their exports further. Firms producing in the low-tax country Foreign face conflicting incentives. On the one hand, they want to exploit tax saving possibilities by exporting more towards Home. On the other hand, prices in Home shrink making sales less profitable there.

Welfare is affected by the possibility of tax motivated transfer pricing. Again, welfare is measured by the consumed quantities in either country. However, we cannot clearly show where consumption increases if tax-motivated transfer pricing becomes possible for the general case. While Home firms' exports increase unambiguously, this is unclear for Foreign firms. The welfare effect hinges on the export activity of Foreign firms. If the demand channel they experience is weaker than the supply channel, their exports increase – possibly more than the exports from Home firms to Foreign. If, however, the demand channel outweighs the supply channel, Foreign firms supply more to their own country Foreign and reduce exports. In the latter case the reaction of consumption is straightforward, in the former case it is not possible to determine the sign of the changes in consumption in general.

Turning to welfare effects, we therefore set $\tau^* = 0$ to abstract from complicating tax revenue effects in Foreign.[15] In this setting, Foreign firms increase their exports but by smaller amounts than Home firms. This translates into increased

[15] Due to mathematical complexity, it is computationally difficult to proof the proposition for the general case of $\tau^* > 0$. However, for a specific case such as $a = 1$ and $b = 3/4$ we show in the supplement S2 that the welfare effect holds for all $0 < \tau^* < \tau < 1$.

consumption in Foreign. As total production is fixed, consumption in Home decreases.

Proposition 2.3 (Cross-Country Welfare Effect)**.** *Welfare decreases in the high-tax country and increases in the low-tax country in the transfer pricing scope g for $\tau^* = 0$.*

Proof. See appendix B.3.6. □

Graphically, the demand effect is captured by a shift in the CI line. The decreasing consumption corresponds to a downward-shift of the CI line as real income in Home (the intercept) decreases, while the opposite is true for Foreign. This effect persists for $g > 0$ as long as the transfer pricing scope does not become too large.[16] The supply effect corresponds to a downward-shift of the MI line as the intercept is no longer zero, but decreases. Figure 2.3 illustrates the effects of transfer pricing, when Home is the high-tax country. The intersection point C gives the new equilibrium with more exporting and less welfare-shifting between the countries as compared to point B without transfer pricing but with tax differences.

We now turn to within-country distributive effects of transfer pricing in our framework. We focus on real wages and real profits and show that the firms do not necessarily gain from the profit shifting possibility.

Proposition 2.4 (Within-Country Distribution Effect)**.** *If the transfer pricing scope g rises, real wages increase in both countries. Real profits increase in g in the high-tax country and decrease in the low-tax country for $\tau^* = 0$. Thus, in the low-tax country the labour share grows.*

[16] See the supplement S2 for this derivation.

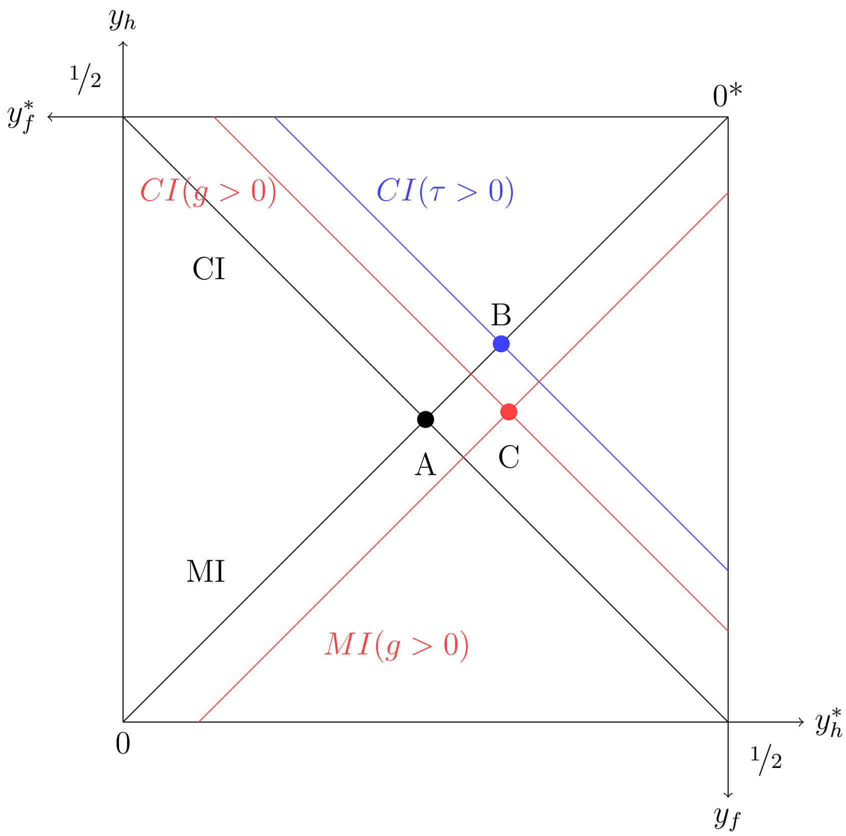

Figure 2.3: Transfer Pricing at $\tau^* = 0$

Proof. See appendix B.3.7. □

As firms want to increase their production, nominal wages increase. This effect also holds in real terms, even in the low-tax country where prices increase. Therefore, labour income increases in our general equilibrium model if firms are able to reduce their tax payments.

Nominal profits increase for Home firms and decrease for Foreign firms. All companies face higher wages, reducing their profits. However, wages in Foreign increase more strongly. Additionally, demand and supply channel work in the same direction for Home firms so they increase exports to Foreign, where prices increase. This affects their profits positively. Foreign firms, however, are faced with contradictory incentives and they export to the shrinking market with decreasing prices. The benefit is therefore reduced and the negative wage effect prevails in Foreign. These nominal profit effects are reinforced by the price changes. In Foreign prices increase and profits decrease such that real profits will decrease as well. In Home, nominal profits increase and prices decrease resulting in increasing real profits.

2.5 Conclusion

The taxation of multinational companies has to be viewed in its international context. Those firms have incentives to exploit the differences in tax systems and tax rates across countries to boost their profits. However, instruments to avoid tax payments like transfer pricing have real effects on firms' supply decisions and on the economy as a whole. To assess the welfare and distributional effects of those activities it is, therefore, essential to incorporate these instruments into general equilibrium frameworks.

We analyse the effects of corporate taxation and tax-motivated transfer pricing in a general oligopolistic equilibrium trade model with segmented markets. Without profit shifting, an increasing profit tax rate shifts welfare towards the tax-increasing country, where it also decreases real wages, whereas real wages rise in the other country. Labour income increases relative to profit income in both countries. Transfer pricing generates an additional benefit from exporting, such that companies want to expand production. Caused by this supply channel, nominal and real wages will rise in both countries. Due to shifting tax incomes, a cross-country demand channel relocates consumption from the high- to the low-tax country, thereby increasing welfare in the latter on cost of the former. In the low-tax country, real profits decrease such that the labour share of income rises.

The general oligopolistic equilibrium framework allows to analyse the interplay between firms with market power and government policies. It could be insightful to examine further types of taxation in this framework. For instance, it would be worthwhile to introduce some form of mobile capital, which then allows to consider tax competition between countries. Additionally, it would be interesting to develop a multi-country extension that might facilitate to bring some of the model's predictions to the data.

Chapter 3

Correlation, Communication and Career Concerns*

3.1 Introduction

One important aspect of elections is to elect competent politicians into office. In their seminal contribution, Persson and Tabellini (2002) show this selection property of elections in a career concern framework, where voters observe previous supply of public goods. However, the voters' perspective and their interactions are underdeveloped in this strand of the theoretical literature. Either voters are assumed to be commonly informed (e.g. Ashworth et al. (2017), Bruns and Himmler (2016)) or each voter has their individual signal as introduced by Aytimur and Bruns (2019). Contradicting these limiting cases, voters' information on previous actions may be correlated but not fully equal. For instance, voters living in the same village have similar observations of infrastructure and public transport, which

*I thank Emre Aytimur, Jonas F. Rudsinske and Robert Schwager for valuable comments.

may differ from the assessment by voters who live in another city. Additionally, recent technological innovations like social networks or instant messengers lead to increased communication between voters, facilitating the exchange of information. Incorporating such correlations as well as communication between voters can enhance our understanding of the accountability and selection properties of elections in career concern models.

If voters have similar information that determines their election decision, these decisions converge. Therefore, it is more likely that the electorate makes a worse decision, as all voters' information is steered into the same and possibly wrong direction. Communication between voters increases their respectively available information, and more informed decisions lead to better outcomes. The probability that an individual voter supports the more competent candidate becomes larger. However, if voters exchange information, their assessment becomes more equal as information is duplicated. The question is, whether correlated information and simple forms of communication can affect electoral outcomes and whether this is beneficial to voters.

I develop a model of voting where an incumbent seeks re-election and balances her re-election probability and future rents with the costs of current effort in providing public goods. Voters seek to elect the most competent politician as he or she supplies the largest amount of public goods after the election. Each voter observes some noisy signal about the incumbent's performance before the election, which is affected by her effort and determines the voter's decision whom to support. I consider the case in which voters' noises are correlated, either exogenously or endogenously through communication, while the literature only considers independent or equal signals. If the noises in the private signals are correlated, voting decisions tend to be synchronous. This reduces the incentives for the incumbent to exert effort and the probability to elect the more competent politician decreases. If the noise terms are correlated, all signals are bunched together, which means that the deciding signal of the median voter is pulled away from the correct value, increasing the chance of an imperfect election outcome.

When voters can exchange information and do not only observe their own, but also other signals, the election outcome becomes more favourable for them. Communication leads to the duplication of information, as voters observe the same signals, and thus signals become more correlated leading to a negative effect on accountability and selection. However, the overall level of individual knowledge within the electorate increases as each voter has a more precise updated signal than before communication, improving the electoral outcome. This information effect outweighs the duplication effect in such a way that the probability to elect the more competent politician increases if each voter observes more signals. At the same time, the incumbent has a larger incentive to provide public goods before the election, as the signal of the median voter becomes more precise and an increase in effort increases the re-election chances more strongly.

I extend the communication patterns in two ways. Firstly, voters may overemphasise their own signal compared to other observed signals in their updating process. This reduces the correlation and its negative effect, while voters still communicate, and their knowledge increases. At the same time, the information flow within the electorate is reduced, as they rely more heavily on their own signal, which leads to less informed voters. Only if their own signal is weighted just slightly more than the other signals, the election outcome is improved. Secondly, the formation of communicating groups of voters within the electorate is beneficial.

Even though information becomes more homogeneous within each group and each of these may decide en bloc, on average correlations decrease if more groups emerge. The more groups are formed the higher the probability that the deciding median voter has a signal that is close to the correct value, leading to a correct election outcome.

The present model is based on the career concern election model as described in Persson and Tabellini (2002). This framework is used by Aytimur and Bruns (2019) to analyse the properties of election outcomes with large electorates but without communication and thus uncorrelated noise terms. They most importantly outline the *sampling effect* of an increasing electorate which leads to higher effort levels and higher expected supply of public goods after the election. This supports the Condorcet Jury Theorem which states that the probability to choose the right alternative in an election becomes larger with larger electorates. The sampling effect is also present in the model below. However, if correlations between the noise terms are exogenously given, the outcome of a large election with the number of voters tending to infinity does not coincide with the decision by a social planner with a perfect signal as in Aytimur and Bruns (2019), but yields worse results.

While in Aytimur and Bruns (2019) all voters receive individual signals, Bruns and Himmler (2016) assess a model where the signal that determines the election outcome results from a newspaper investigation and is equal to all voters. They explain why voters pay for information provided by media even if they do not expect to be individually pivotal in the case of a small election. My model allows the information structure to be between these two examples and includes these two limiting cases. The noises in voters' signals are neither independent as in the former paper, nor equal as in the latter case, but lie between these two extremes with some positive correlation between noise terms.

In a similar framework Ashworth and De Mesquita (2014) focus on the interaction between politician's strategies and changes in voters' information. They additionally show that election outcomes can be improved if voters behave less rationally. Ashworth et al. (2017) show that there may be a trade-off between accountability and selection in a career concern type model. If accountability becomes stronger, this can reduce the information voters can get about the politician's competence. Kotsogiannis and Schwager (2008) also utilize the career concern framework to implement yardstick competition in multiple jurisdictions, focusing on the effects of fiscal equalisation schemes on the accountability of politicians. However, these models focus on a representative voter rather than an electorate of individual voters with individual signals. Ashworth and Fowler (2020) reconcile career concern type models with representative voters and with an electorate of individual voters. They show that a representative voter may be implied by a an electorate with voters that may differ in their utility, party affinity and their belief over the incumbent's competence.

Another relevant strand of literature considers the effects of information aggregation through elections as well as before elections through communication. It focuses on the conditions under which the Condorcet Jury Theorem holds. Most importantly, some models[1] criticise the original and implicit assumptions[2] resulting from Austen-Smith and Banks (1996) who argue that voting may be strategic rather than sincere. Information sharing in a committee setting is considered in Austen-Smith and T. J. Feddersen (2006). They discuss the extent of strategic

[1] See for example T. Feddersen and Pesendorfer (1998), Coughlan (2000).
[2] See for instance Grofman and Feld (1988), Young (1988).

information sharing and show that uncertainty over others' preferences can support full sharing under majority rule. By contrast, in my model communication is not deliberate to convey information (or false claims) but is unintended. Additionally, voters do not communicate with all other voters but have a fixed number of communication partners. Experimentally, Mercier and Claidière (2022) argue that discussions within large groups improve individual responses and do not impede on aggregate decisions if there is a correct answer. If there is an opinion leader, Grofman et al. (1983) show that the probability to choose the correct alternative drops compared to the case without such a leader. Dietrich and List (2004) argue that decisions in juries depend on the evidence provided to the jurors. Thus, the probability of a correct majority vote converges to the probability that the evidence is correct. Regarding group structures, Kao and Couzin (2019) consider their effects on collective decision-making in the context of animal group behaviour. They assume that a majority decision is reached within each subgroup before these are combined to determine the overall decision. They find that a moderate number of subgroups actually minimises the accuracy. However, when information is correlated, they also find that grouped structures improve the outcome. In contrast to my model their decision-making set-up involves two steps, whereas I assume the grouped structure only for the purpose of information sharing in the second extension.

Ladha (1992) describes under which conditions the majority of voters makes a better decision than the average voter if the votes are correlated. This is only the case if the votes are not highly correlated. However, he only considers exogenously given correlations between voting decisions. For negatively correlated votes following distinct distributions Ladha (1995) shows that a majority of voters does better than an individual voter. He also shows that for large electorates the election outcome matches the full information outcome, confirming Condorcet's Jury Theorem. The same result is given in Berg (1993), who also shows that for positively correlated votes an election improves on an individual vote but does not become infallible with an increasing electorate. My result on correlations follows these insights. However, I introduce correlation between voting decisions not only exogenously, but also endogenously by introducing communication. Additionally, I do not consider an average voter as a benchmark but a perfectly informed social planner without noise in his signal. In this case I can show that for exogenously correlated noise an election with a large electorate is worse than the decision by the social planner. Furthermore, previous work only focuses on the selection properties of elections. I also consider electoral accountability, which incentivises the incumbent to increase her effort before the election.

The article is structured as follows. I first describe the baseline model, where individuals' noises are exogenously correlated in Section 3.2. Section 3.3 considers communication between voters and its effect on accountability and selection. The communication patterns are extended in Section 3.4, where I describe the effects of different information weights as well as a grouped electorate. The last section concludes.

3.2 Voting with correlated signals

I introduce correlation and communication among the electorate into an agency model of elections. It follows the career concerns idea as outlined in Persson and Tabellini (2002). Voters aim to elect the most competent of two candidates to increase the supply of public goods after the election. The incumbent politician wants to maximise her expected utility by exerting some effort in the first period which increases her chance of re-election. Voters base their election decision on a signal they receive about the incumbent's performance before the election. On the one hand, the signal consists of the supply of public goods in the first period that results from a combination of the incumbent's effort and her competence. On the other hand, there is a noise term in each private signal. At the time of the election it is difficult for individuals to properly evaluate the public good supply by the incumbent. Voters may not be able to fully capture the long-term effects of public goods or may be skewed to view the supply of public goods (un)favourably. Therefore, it is not possible for them to observe the actual supply of public goods and voters need to rely on their noisy signal to evaluate the incumbent's competence.

The noise terms of each voters' signal can be correlated.[3] For instance, this could be the case when there is some event like a disaster that does not impact the supply of public goods the incumbent herself is responsible for. Voters in such situations can exhibit "blind retrospection" as described in Heersink et al. (2017). They have a tendency not to re-elect the incumbent. Other interpretations may be that some news emerge about the public goods in another jurisdiction which improves or worsens the electorate's perception of the supply of public goods in their own region. It may also be interpreted as a new report about the incumbent herself, which casts a shadow on her achievements or improves her image. For instance there could be a scandal around a candidate which becomes public before the election and impacts the noise of all voters' signals in the same way. All these

[3] The signals themselves are correlated in any case as they all depend on the supply of public goods in the first period. Thus, correlation between the noise terms leads to a higher level of correlation between the signals.

instances do not alter the amount of public goods and hence a voter's utility but the perception of the public good through the signals.

In each of the two periods, the politician in office provides public goods. In the first period, the incumbent exerts some effort e and together with her competence θ^I this determines the public good supply v_1. In the second period, the elected politician – the incumbent I or the challenger C – supplies v_2.[4]

$$
\begin{aligned}
v_1 &= \theta^I + e \\
v_2 &= \theta^\ell + e_2^\ell \qquad \text{with } \ell \in \{I, C\}
\end{aligned}
$$

Each politician's competence θ^ℓ does not change between the two periods and is unknown to the electorate and the politicians themselves. Both are independently drawn from a normal distribution with expectation $E[\theta^\ell] = \bar{\theta}$ and variance $Var[\theta^\ell] = {}^1/_{\tau_\theta}$ before the first period. This distribution is known by the politicians and the electorate.

A politician's effort is not costless to them. I assume a strictly convex effort cost function $c(e^\ell)$ with $c(0) = 0$, $c' > 0$, $c'' > 0$ and $\lim_{e \to 0} c' = 0$. As voters take the amount of public goods in the first period into consideration via their signals when deciding on their vote, the incumbent can influence the voting decision through her effort in the first period. If she increases her effort more public good is provided increasing the re-election probability $p(e)$. Politicians obtain some rent R in case of election, which incentivises effort by the incumbent in the first period. She

[4] The results qualitatively also hold for the case of production functions, where effort and competence are related both additively and multiplicatively. I use this reduced version for ease of exposition here and in all following model variations, whereas the formal proofs in the appendices use the more elaborate type of production function.

maximises her expected utility

$$p(e)\left(R - c(e_2^I)\right) - c(e).$$

There are n voters, where n is large and odd. All voters only care for the amount of public goods in the two periods such that their utility is $u_t = v_t$. They vote sincerely and take the signal they receive into account. Each voter i observes a private signal s_i, which consists of the amount of public good provided in the first period and some noise x_i.

$$s_i = v_1 + x_i$$

However, the noises of all n voters in the electorate are not independent. The noise terms are distributed according to an equicorrelated multivariate normal distribution. The expected value of each individual noise term is $E[x_i] = 0$ and its variance is $Var[x_i] = {}^1\!/_{\tau_x}$. All noise terms are pairwise correlated with the correlation coefficient $\rho \in [0, 1)$. Hence, the covariance between the noise terms of two voters is $Cov[x_i, x_j] = {}^\rho\!/_{\tau_x} \ \forall\ i \neq j$.

The timing is as follows. There are three stages in which agents take actions. In the first period, the incumbent determines her effort in providing the public goods. After this period an election is held in which the voters can vote in favour of the incumbent or the challenger. In the second period, the winner of the election decides on their effort in this period. Before the first stage the politicians' competences are determined but not observed by themselves or the voters. Before the voters take to the polls and after the public good supply is set in the first period they receive their signal.

The model is solved by backwards-induction. In the second period, the elected politician decides on their effort. As there is no further election and effort is costly, the politician exerts the smallest effort possible, which is zero in this case. Hence the supply of public goods in the second period is fully determined by the elected politician's competence.[5]

$$v_2 = \begin{cases} \theta & \text{if } I \text{ is re-elected} \\ \theta^C & \text{if } C \text{ is elected} \end{cases}$$

Therefore, voters want to elect the politician who they think is more competent. Each individual voter supports the incumbent only if they expect that her competence is at least as large as the challenger's competence. The voters determine the expected value of competence for the incumbent by taking their individual signal as well as their equilibrium belief over the incumbent's efforts e^* into account. Voter i's decision is determined by

$$d_i = \begin{cases} I & \text{for } E[\theta|s_i, e^*] \geq E[\theta^C] = \bar{\theta} \\ C & \text{for } E[\theta|s_i, e^*] < E[\theta^C] = \bar{\theta} \end{cases} \tag{3.1}$$

The incumbent's competence θ and an individual's signal s_i are bivariate normal distributed with

$$\begin{aligned} \theta &\sim N\left(\bar{\theta}, {}^1\!/\!_{\tau_\theta}\right) \\ s_i &\sim N\left(\bar{\theta} + e^*, {}^1\!/\!_{\tau_\theta} + {}^1\!/\!_{\tau_x}\right) \end{aligned}$$

[5] For the remainder of the paper I drop the I in θ^I for convenience.

$$Cov[\theta, s_i] \;=\; {}^1\!/\!{}_{\tau_\theta}.$$

Each voter updates the incumbent's expected competence using Bayes' rule. They know that their signal contains some information about this competence, but have to bear in mind that the signal includes some noise, which reduces its accuracy.

$$\begin{aligned} E[\theta|s_i, e^*] &= E[\theta] + \frac{Cov[\theta, s_i]}{(Var[\theta]Var[s_i])^{1/2}} \left(\frac{Var[\theta]}{Var[s_i]}\right)^{1/2} (s_i - E[s_i|e^*]) \\ &= \bar{\theta} + \frac{\tau_x}{\tau_x + \tau_\theta} \left(s_i - \bar{\theta} - e^*\right) \end{aligned}$$

With this result, each individual's voting decision can be determined. There is a cut-off signal value $\hat{s}$, under which a voter supports the challenger. If the voter's signal is large enough, they vote for the incumbent. With the voting rule in equation (3.1), voter i gives their vote to the incumbent if

$$\begin{aligned} E[\theta|s_i, e^*] &\geq \bar{\theta} \\ s_i &\geq \bar{\theta} + e^* \equiv \hat{s} \end{aligned} \tag{3.2}$$

Once the signal is larger than or equal to the expected value of public goods given the belief over effort e^*, an individual voter decides to support the incumbent. However, the incumbent is only re-elected if she can obtain at least half of the votes. This implies that she is re-elected only if the median voter $i = m$ votes for her. The voter who observes the median of the realised signals becomes the median voter. He supports the incumbent if his signal s_m is large enough. The median voter's signal needs to be characterised in order to calculate the incumbent's re-election probability $p(e, e^*)$ that depends on actual and expected effort.

All n signals are correlated, which also impacts the median's signal. For the realised effort e in the first period, all signals s_i are multivariate normal distributed with a common mean, a common variance and a common correlation coefficient due to the noises' correlation structure.

$$\begin{aligned} E[s_i|e] &= \bar{\theta} + e \\ Var[s_i|e] &= \frac{1}{\tau_\theta} + \frac{1}{\tau_x} \\ Cov[s_i, s_j|e] &= \frac{1}{\tau_\theta} + \frac{\rho}{\tau_x} \end{aligned}$$

Every linear combination of the signals is univariate normal distributed, which implies that the signals are multivariate normal distributed (Hamedani, 1992).

The fact that the signals are multivariate normal distributed with equal moments makes the asymptotic determination of the median signal's distribution possible. Firstly, the result of Arnold et al. (2008) assures that the median signal is asymptotically normal distributed. Secondly, Tong (1990) generalises the result of Owen and Steck (1962), which allows to determine approximations to the first and second moment of order statistics $X_{(i)}$ for random variables $\mathbf{X}$. This approximation holds for $\mathbf{X} = (X_1, ..., X_n)'$ that is equicorrelated multivariate normal distributed with common mean, variance, correlation coefficient ϱ and covariance $Cov[X_i, X_j]$. The moments of the i^{th} order statistic for $\mathbf{X}$ are approximately

$$E[X_{(i)}] = Var[X_i]^{1/2}\sqrt{1-\varrho}E[Z_{(i)}] + E[X_i] \tag{3.3}$$

$$Var[X_{(i)}] = Var[Z_{(i)}] \cdot Var[X_i] + (1 - Var[Z_{(i)}])\ Cov[X_i, X_j], \tag{3.4}$$

where $Z_{(i)}$ is the i^{th} order statistic of n i.i.d. standard normal variables. Lastly, the result in Cramér (1946) can be used to asymptotically determine the moments of the median of the i.i.d. normal variables. Accordingly, the expected value of the median of n standard normal variables is $E[Z_m] = 0$ and its variance is $Var[Z_m] = \pi/(2n)$.

Applying these three elements, the median signal s_m that determines the outcome of the election is asymptotically normal distributed with

$$\begin{aligned}
E[s_m] &= \bar{\theta} + e \\
Var[s_m] &= \frac{\pi}{2n} Var[s_i] + \left(1 - \frac{\pi}{2n}\right) Cov[s_i, s_j] \\
&= \frac{\pi}{2n}\left(\frac{1}{\tau_\theta} + \frac{1}{\tau_x}\right) + \left(1 - \frac{\pi}{2n}\right)\left(\frac{1}{\tau_\theta} + \frac{\rho}{\tau_x}\right) \\
&= \underbrace{\frac{1}{\tau_\theta}}_{\substack{\text{Variance due} \\ \text{to distribution} \\ \text{of competence}}} + \underbrace{\frac{\pi}{2n}\frac{1}{\tau_x}}_{\substack{\text{Median noise's} \\ \text{variance without} \\ \text{correlated noise terms}}} + \underbrace{\left(1 - \frac{\pi}{2n}\right)\frac{\rho}{\tau_x}}_{\substack{\text{Variance due to} \\ \text{the \textit{bunching effect}} \\ \text{of correlation}}} \qquad (3.5) \\
&\equiv \frac{1}{\tau_\theta} + \frac{1}{\tau_m}.
\end{aligned}$$

For uncorrelated noises, i.e. $\rho = 0$, the model reduces to the case of Aytimur and Bruns (2019), where the variance of the median noise is $1/\tau_m = \pi/(2\tau_x n)$. In equation (3.5), the second summand represents the variance of the median's noise without correlation. However, correlation between the noise terms increases the variance of the median voter's signal, which is described by the third summand in equation (3.5).

The positive correlation between noises and hence signals indicates that if some signal s_i is small, another signal s_j tends to be small as well. The larger the correlation, the stronger this tendency. If the correlation coefficient between the noises becomes closer to one, all signals would be the same. This relationship leads to the *bunching effect* of correlation in this model. An increase in correlation has the effect that individual signals tend to be closer together stochastically. Every realisation of the signals is bunched around some value that does not necessarily need to be the common expected value. This value may be above or below the expected value of the signals $E[s_i]$ such that the expected value of the median's signal $E[s_m]$ is unaffected by the inclusion of the correlation. However, the bunching makes it more difficult to predict the median value as it is pulled towards the bunched value. This increasing variance is captured in the third summand of equation (3.5). Therefore, the bunching effect increases the variance of the median's signal.[6]

$$\frac{\partial Var[s_m]}{\partial \rho} = \frac{\partial}{\partial \rho}\left(\frac{1}{\tau_m}\right) = \frac{1}{\tau_x}\left(1 - \frac{\pi}{2n}\right) > 0$$

With the median signal's asymptotic distribution it is possible to determine the re-election probability for the incumbent. This probability is described by the probability that the median signal is larger than or equal to the cut-off signal $\hat{s}$ given actual and expected effort.

$$\begin{aligned} p(e, e^*) &= Pr[s_m \geq \hat{s}] = 1 - Pr[s_m < \hat{s}] \\ &= 1 - \Phi\left[(e^* - e)\left(\frac{1}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-1/2}\right], \end{aligned}$$

where Φ is the cumulative distribution function of the standard normal distribution. The incumbent's effort affects the re-election probability positively.

$$\frac{\partial p(e, e^*)}{\partial e} = \varphi\left[(e^* - e)\left(\frac{1}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-1/2}\right] \cdot \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-1/2} > 0$$

[6] As n needs to be large the condition of $n \geq 2$ is fulfilled which assures that the derivative is positive.

Here, φ denotes the probability density function of the standard normal distribution.

Given the re-election probability the decision by the incumbent over her effort can now be analysed. In the first period she sets her effort e in order to maximise her expected pay-off. On the one hand a higher effort in the first period positively affects her re-election probability. With higher effort, more of the public good is supplied, which increases all of the individual signals and the median signal as well. At constant cut-off signal $\hat{s}$ this increases the incumbent's re-election probability, and, therefore, also the expected rent from the second period. On the other hand, however, effort is costly such that an increase results in an immediate increase in costs in the first period. In the optimal situation the incumbent balances these two marginal effects to determine her effort in the first period.

$$\frac{\partial p(e, e^*)}{\partial e} R = c'(e) \tag{3.6}$$

In equilibrium the belief of the electorate regarding the incumbent's effort in the first period is accurate. Hence $e^* = e$ has to hold. With this relation the equilibrium effects of correlated signals on the incumbent's effort can be determined. Her optimisation in equation (3.6) implicitly defines the effort in the first period such that with $e^* = e$

$$\begin{aligned} 0 &= \frac{\partial p(e, e^*)}{\partial e} R - c'(e) \\ 0 &= (2\pi)^{-1/2} \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_m} \right)^{-1/2} R - c'(e) \equiv \lambda. \end{aligned}$$

The marginal benefit of effort in equilibrium is determined by the change in the re-election probability. This change is given by the density of the median signal at its expected value. With a higher variance this density is reduced as deviations from the expected value become more likely. Once such errors are more prevalent, the incumbent cannot count on the desired effect of increased effort with the same probability such that she decides to reduce her effort.

Proposition 3.1. *Equilibrium effort decreases in the correlation coefficient ρ of the signals' noise terms.*

Proof. See appendix C.1.1. □

In a second step, the selection properties of the modelled election are considered. It is not only important to incentivise effort in the first period but also to have a high probability of electing the more competent politician in order to enjoy a high amount of public goods in the second period. The outcome of the election is determined by the median's signal, the voting rule in equation (3.1) and the cut-off signal in equation (3.2).

$$d^* = \begin{cases} I & \text{for } s_m \geq \hat{s} \\ C & \text{for } s_m < \hat{s} \end{cases}$$

The expected public good supply in the second period given competence θ results from the election probability of the respective politician in equilibrium and the ensuing supply by the respective politician. As the effort is zero in either case with the additive production function, the competences fully determine the public good supply in the two cases. The expected public good supply in the second period aggregates these values over all possible values of θ weighing them according to the probability that they occur.

$$E[v_2] = \int_{-\infty}^{\infty} \left\{ Pr[d^* = I|\theta]\ \theta + Pr[d^* = C|\theta]\ \bar{\theta} \right\} f(\theta)\ d\theta$$

$$= \int_{-\infty}^{\infty} \left\{ \theta + \left(\bar{\theta} - \theta\right) \Phi \left[\left(\bar{\theta} - \theta\right) \left({}^{1}\!/_{\tau_m}\right)^{-1/2} \right] \right\} f(\theta) \; d\theta,$$

where $f(\theta)$ is the density function of the incumbent's competence. If the correlation between the voters' noise terms increases, the expected utility decreases in equilibrium. Overall, the aggregate informational value of the signals decreases if they are related to each other. The probability to elect the less competent politician increases with reduced information.

Proposition 3.2. *If the correlation ρ between the noise terms of the voter's signals increases, the expected utility in the second period $E[v_2]$ decreases.*

Proof. See appendix C.1.2. □

The introduction of correlated noise terms to the voters' signals leads to the *bunching effect*, which reduces electoral accountability and worsens electoral selection in the model for a fixed number of voters. The Condorcet Jury Theorem, however, states that the efficiency of elections to choose the correct or better alternative increases if the electorate increases. Aytimur and Bruns (2019) found that this is indeed the case for accountability and selection in a career concern model if the noise terms are not correlated.

The *sampling effect* that the median's signal becomes more precise with a larger electorate remains influential when introducing correlated noise terms. The variance of the median signal decreases with an increased electorate.

$$\frac{\partial Var[s_m]}{\partial n} = \frac{\partial}{\partial n}\left(\frac{1}{\tau_m}\right) = -(1-\rho)\frac{1}{\tau_x}\frac{\pi}{2n^2} < 0$$

However, correlation between the signals weakens the sampling effect. The *bunching* of the noise terms leads to a persistent increase in the median signal's variance even for large electorates.

$$\lim_{n \to \infty} Var[s_m] = \frac{1}{\tau_\theta} + \frac{\rho}{\tau_x} \tag{3.7}$$

Even if the electorate becomes large there is some additional variance to the median's signal compared to the variance of a social planner's signal that does not include a noise term. This affects the accountability properties of the election for a large electorate. The incumbent's effort decision hinges on the value of this variance. A larger variance of the median signal implies a lower return of effort in terms of the re-election probability. Thus, the incumbent reduces her effort if she faces a large electorate in comparison to such a social planner.

Electoral selection is negatively affected by correlated noise terms in a large electorate as well. In contrast to a social planner with a perfect signal the bunching of noise terms leads to the possibility to elect the less competent candidate at every competence level θ. If $\theta > \bar{\theta}$ the median signal is expected to indicate that the incumbent is more competent. However, for a large electorate this signal's variance does not converge to zero but is positive at $\lim_{n \to \infty} Var[s_m|\theta] = {}^{\rho}/_{\tau_x}$. Only, if there is no correlation between noise terms, the signal's distribution becomes degenerate.

Proposition 3.3. *For correlated signals with $\rho \in (0, 1)$ equilibrium effort in the first period and the expected utility in the second period are smaller if a large electorate $n \to \infty$ decides compared to a social planner with a perfect signal.*

Proof. See appendix C.1.3. □

If voters' perceptions of the public good supply in the first period exhibit some commonalities it impacts the results of an election as well as the incumbent's incentives to exert effort. Even if the electorate becomes large, these negative effects persist and reduce accountability and selection.

3.3 Communication between voters

A different way to introduce correlation of voters' signals without assuming correlation between the noise terms is communication. I introduce a communication stage into the framework described above after the voters observe their own signal but before the election. Voters interact with other voters and observe their signals additionally to their own. These conversations may be between colleagues, neighbours or friends that do not primarily aim at a transfer of signals, but incidentally include this information. Voters do not actively search for new information and acquire the additional knowledge about the public goods in passing. Voters passively learn about their counterpart's signal, which can be used in their election decision.[7] With recent changes in communication technology such exchanges became more prevalent. Instant messengers like WhatsApp make timely information exchanges over long distances possible. This way, voters can obtain information from different parts of a country thereby increasing the accuracy of the information they base their voting decision on.

The timing as well as politicians' and voters' decisions are equally structured as in Section 2. Each voter receives their own private signal $s_i = v_1 + x_i$ about public good provision. For ease of exposition I do not include exogenous correlation

[7]See the literature on passive or incidental learning that focuses on media but also online social networks for example H. E. Krugman and Hartley (1970), Tewksbury et al. (2001), Bode (2016), Matthes et al. (2020) and Nanz and Matthes (2020).

between voters' noise terms to focus on the effects of communication. In the proofs I include such correlation, which does not affect the results qualitatively.

Communication occurs along a directed graph with the $n \times n$ adjacency matrix T. Neither the politicians nor the voters themselves know the (full) communication network that results from the adjacency matrix. If voter i observes j's signal, then $T_{ij} = 1$ and zero otherwise. The adjacency matrix does not need to be symmetric such that it is not necessary that two voters exchange their signal. Every voter observes their own signal $T_{ii} = 1$. After communication each voter has observed $k < n$ signals including their own signal, so $\sum_{j=1}^{n} T_{ij} = k \geq 1$. There is no exchange of information if $k = 1$. The directed links between voters are randomly generated. The probability to observe another voter's signal is equal for all voters such that there are no opinion leaders or influential voters. The probability that i observes j's signal is $Pr[T_{ij} = 1] = {}^{(k-1)}/_{(n-1)}$ and known by politicians and voters. Voters observe signals only once so there are no multiple rounds of updating.

After they gathered other persons' signals, voters update their signal which then is the basis for their voting decision. The updating of signals follows a DeGroot-style updating rule, where the updated signal is the average of all observed signals. The voters weight all signals with ${}^{1}/_{k}$ and aggregate them. After the updating round, the updated signal S_i of voter i is

$$\begin{aligned} S_i &= \sum_{j=1}^{n} \left(T_{ij} \frac{1}{k} s_j \right) \\ &= v_1 + \frac{1}{k} \sum_{j=1}^{n} (T_{ij} x_j) \\ &= \theta + e + X_i. \end{aligned} \tag{3.8}$$

The updated signal S_i is comparable to i's initial signal s_i insofar that it consists of the amount of public good v_1 and an (updated) noise term X_i. This term now consists not only of the voter's own initial noise but of the average of all noises they can observe. DeGroot-updating leads to the same result as Bayesian updating in this case as all signals have the same variance and should be weighted equally in Bayesian updating (Jackson, 2011).

The updated signal is still normal distributed as it is the sum of the normally distributed initial signals. Its moments are

$$E[S_i] = \bar{\theta} + e \tag{3.9}$$

$$\begin{aligned} Var[S_i] &= \frac{1}{\tau_\theta} + \frac{1}{k^2}\left[\sum_{j=1}^{n} T_{ij} Var[x_j]\right] \\ &= \frac{1}{\tau_\theta} + \frac{1}{k\tau_x} \end{aligned} \tag{3.10}$$

Communication between voters leads to more informed individuals in the sense that the variance of their updated signal is smaller than of their original signals. Individually, each voter is better informed when making their voting decision. This is the *information effect* of communication in this model.

However, communication and the exchange of signals duplicates information within the electorate. As one voter shares their signal with another voter, these two individuals have some common information and their updated signals are correlated. The updated signals are multivariate normal distributed. The expected values and variances are common for all updated signals and the covariance between two randomly chosen updated signals is

$$
\begin{aligned}
Cov[S_i, S_j] &= E[S_i \cdot S_j] - E[S_i] \cdot E[S_j] = \frac{1}{\tau_\theta} + E[X_i \cdot X_j] \\
&= \frac{1}{\tau_\theta} + E\left[\frac{1}{k}\left(x_i + \sum_{\epsilon \neq i} T_{i\epsilon} x_\epsilon\right) \cdot \frac{1}{k}\left(x_j + \sum_{\zeta \neq j} T_{j\zeta} x_\zeta\right)\right] \\
&= \frac{1}{\tau_\theta} + \frac{1}{k^2}\left\{E[T_{ji}]E[{x_i}^2] + E[T_{ij}]E[{x_j}^2] + \sum_{\epsilon \neq i}\sum_{\zeta \neq j} E\left[T_{i\epsilon} T_{j\zeta}\right] E[x_\epsilon x_\zeta]\right\} \\
&= \frac{1}{\tau_\theta} + \frac{1}{k^2 \tau_x}\frac{k-1}{n-1}\left(2 + (n-2)\frac{k-1}{n-1}\right).
\end{aligned}
$$

The covariance increases for increased communication. This results in the *duplication effect* of communication. With more common sources signals become more correlated. Overall, voters have better signals but also more similar ones, if they communicate with others.

Solving the equilibrium is analogous to the previous section. Voters support the incumbent if they expect her to have a higher competence given their signal and their belief e^* over the incumbent's effort. Otherwise they vote for the challenger. Competence θ and the signal S_i are bivariate normal distributed given e^* with

$$
\begin{aligned}
\theta &\sim N\left(\bar{\theta}, \frac{1}{\tau_\theta}\right) \\
S_i &\sim N\left(\bar{\theta} + e^*, \frac{1}{\tau_\theta} + \frac{1}{k\tau_x}\right) \\
Cov(\theta, S_i) &= \frac{1}{\tau_\theta}.
\end{aligned}
$$

The expected value of the competence given the updated signal is thus

$$
E[\theta | S_i, e^*] = \bar{\theta} + \frac{Cov[\theta, S_i]}{Var[S_i]}\left(S_i - (\bar{\theta} + e^*)\right)
$$

and a voter supports the incumbent if $S_i \geq \bar{\theta} + e^* \equiv \hat{s}$. A voter supports the incumbent if their information indicates that the supplied public good is larger than anticipated.

The outcome of the election is determined by the median voter's decision and thus the median updated signal S_M. The updated signals are multivariate normal distributed with common mean $E[S_i]$, common variance $Var[S_i]$ and common covariance $Cov[S_i, S_j]$. The same procedure as above can be used in order to approximate the moments of the – asymptotically normal distributed – median updated signal.

$$
\begin{aligned}
E[S_M] &= \bar{\theta} + e \\
Var[S_M] &= \frac{\pi}{2n} Var[S_i] + \left(1 - \frac{\pi}{2n}\right) Cov[S_i, S_j] \\
&= \underbrace{\frac{1}{\tau_\theta}}_{\substack{\text{Variance due} \\ \text{to distribution} \\ \text{of competence}}} + \underbrace{\frac{\pi}{2n}\frac{1}{k\tau_x}}_{\substack{\text{Median noise's} \\ \text{variance including} \\ \textit{information effect}}} + \underbrace{\left(1 - \frac{\pi}{2n}\right) \frac{1}{k^2\tau_x} \frac{k-1}{n-1} \left(2 + (n-2)\frac{k-1}{n-1}\right)}_{\substack{\text{Variance due to} \\ \text{the } \textit{duplication effect} \\ \text{of communication}}} \\
&\equiv \frac{1}{\tau_\theta} + \frac{1}{\tau_M}
\end{aligned}
$$

The variance of the median updated signal depends on the number of communication partners and changes when communication between voters is more frequent. On the one hand, the information effect reduces the variance and makes the median's signal more precise. On the other hand, the duplication effect leads to correlated updated signals, which in turn activates the bunching effect outlined above, thus increasing the median signal's variance. Overall, however, the information effect of communication outweighs the duplication effect as an increase of communication partners k decreases the variance.[8]

$$\begin{aligned}
\frac{\partial Var[S_M]}{\partial k} &= \frac{\partial}{\partial k}\left(\frac{1}{\tau_M}\right) \\
&= \underbrace{\frac{\pi}{2n}\frac{\partial Var[S_i]}{\partial k}}_{\substack{<0 \\ \textit{Information effect}}} + \underbrace{\left(1-\frac{\pi}{2n}\right)\frac{\partial Cov[S_i,S_j]}{\partial k}}_{\substack{>0 \\ \textit{Duplication effect}}} \\
&= \frac{1}{k^3\tau_x}\left(\left(1-\frac{\pi}{2n}\right)\frac{2}{(n-1)^2}(n-k)-\frac{\pi}{2n}k\right) < 0
\end{aligned}$$

In equilibrium, communication has the desired effect from the electorate's point of view. The incumbent exerts higher effort in the first period. As the median signal's variance is reduced with communication, deviations from the expected value of the signal are less likely such that the marginal effect of effort on the re-election probability is stronger. This incentivises the incumbent to increase her effort. At the same time, communication improves electoral selection.

Proposition 3.4. *An increase in the number of sources k per voter increases the incumbent's effort in equilibrium and increases the expected utility of the electorate in the second period.*

Proof. See appendix C.2.1. □

Communication between voters therefore is beneficial to voters as the electorate becomes more informed, even though it also becomes more homogeneously informed with increased communication. The former effect outweighs the latter even if the electorate is not large. If the electorate becomes larger $n \to \infty$, the Condorcet Jury Theorem holds for accountability and selection as the median's signal becomes more

[8] The derivative is not negative at $k = 1$. However, I show in the appendix that $Var[S_M|k = 1] > Var[S_M|k = 2]$.

precise and matches the precision of the signal of a social planner who observes a perfect signal in contrast to the case above.[9]

$$\lim_{n\to\infty} \frac{1}{\tau_\theta} + \frac{1}{k^2\tau_x}\left(\frac{\pi}{2n}k + \left(1-\frac{\pi}{2n}\right)\frac{k-1}{n-1}\left(2+(n-2)\frac{k-1}{n-1}\right)\right) = \frac{1}{\tau_\theta}$$

This convergence is due to low communication probabilities. If the number of voters n increases at constant communication partners k it is less likely for two randomly chosen voters to have common information and thus correlated signals. While the information effect prevails – each voter still has k sources –, the duplication effect decreases with a large electorate. For large electorates the sampling effect identified by Aytimur and Bruns (2019) assures higher accountability and better selection properties as the noises of two randomly selected updated signals are not correlated in the limit and the variance of the median noise converges to zero.[10]

3.4 Extensions

In the preceding section the base case of communication between individual voters was described drawing on the effects of correlated signals. It became clear that in this reduced form of communication it is beneficial for the voters to exchange

[9] In the presented case, there is no exogenous correlation between noise terms. With such correlation the same pattern as in the second section emerges due to exogenous correlation of noise terms.

[10] The inclusion of a public signal by Aytimur and Bruns (2019) is somewhat similar to communication but with different properties for large electorates. The driving difference is that their public signal affects all voters equally. It does not affect the noise terms of the signals which determine who becomes the median voter. Therefore, the sampling effect does not apply to the variance of the public signal and for large electorates it is detrimental. In my case, however, introducing communication does not affect all (updated) signals equally and changes the (updated) noise terms of each voter. Thereby, the median voter may change and the sampling effect assures the election's accountability and selection properties of a large electorate.

information. However, this exchange might not be as simple as assumed. Therefore I present two extensions to the previous communication structure. Firstly, the updating process is altered by allowing voters to give their own signal a higher weight when updating their signal. Secondly, I discuss the effects of communicating groups within the electorate, where there is no information exchange with voters outside one's group.

3.4.1 Over-weighting of own signal

In many cases, it is not reasonable to assume that an individual assigns equal weight to all the information they obtain. They might be more convinced by information they receive from close friends compared to the information conveyed by some acquaintance. Additionally, an individual may overemphasize their own signal when deciding on whom to give a vote. I translate this notion into the given model by assuming that each voter gives their own signal a larger weight than other signals they receive.

The one round of DeGroot-style updating of signals in the case above without differentiated weights leads to updated signals that correspond to Bayesian updating, i.e. a rational form of updating. This can be attributed to the fact that all signals have equal precision and are equally weighted. Hence, introducing unequal weights while signals are still equally precise leads to a deviation from Bayesian updating. Voters do not treat the available information properly what may lead to inefficient voting decisions.

As above, each voter updates their signal based on the information they get from all their sources that are defined by the communication network and the adjacency matrix T. The sum of all weights is normalized to one, where one's own signal is weighted by

$$w = \frac{\omega}{\omega + k - 1}$$

and all $k-1$ other signals a voter observes are each weighted by

$$\bar{w} = \frac{1}{\omega + k - 1}.$$

For $\omega = 1$, all signals have equal wights, whereas $\omega > 1$ implies that a voter overweights their own signal compared to any other signal. An increase in ω leads to a higher weight of the voter's own signal and reduces the weight a voter places on all the other signals they can observe.

With this weighting structure voter i's updated signal is

$$\begin{aligned} S_i &= w s_i + \sum_{j \neq i} T_{ij} \bar{w} s_j \\ &= \theta + e + \frac{1}{\omega + k - 1} \left(\omega x_i + \sum_{j \neq i} T_{ij} x_j \right). \end{aligned}$$

For $\omega = 1$ this is the same as above in equation (3.8), where all signals have equal weight in the updated signal. If $\omega \to \infty$ in the limit, only the voter's own signal is considered in their decision-making process. Each updated signal is again normal distributed and expected to be accurate. However, its variance now differs and increases with ω.[11]

$$Var[S_i] = \frac{1}{\tau_\theta} + \frac{\omega^2 + k - 1}{(\omega + k - 1)^2} \frac{1}{\tau_x}$$

[11] Again, I do not consider exogenous correlation between signals in the text for convenience, but include them in the proofs in the appendix.

$$\frac{\partial}{\partial \omega} Var[S_i] \quad = \quad 2\frac{(k-1)(\omega-1)}{(\omega+k-1)^3}\frac{1}{\tau_x} > 0 \;\; \forall\, \omega > 1$$

If a voter considers their own signal more strongly than the signals of their neighbours, the additional information is not optimally incorporated in the updated signal. They feature their own signal more prominently such that the precision of the updated signal converges to the precision of their original signal. Consequently, overweighting a signal rolls back some of the positive information effect of communication for the voters. Each voter is individually less precisely informed about the amount of public goods supplied.

However, for the initial marginal change of ω, the variance is unchanged and for all $\omega < 2$ the variance's second derivative is positive. This indicates that initially the effect of overweighting on the variance – and thus the information effect – is small, but increasing. As long as their own signal only has a slightly higher weight than the other signals, the $k-1$ observed signals remain important. Only after increased reliance on one's own signal, these other signals become irrelevant – individually as well as aggregated.

Additionally, the correlation between two randomly chosen voters' updated signals diminishes if voters rely more heavily on their own signal. As both voters focus on their own signal, the possibly duplicated information is weighted less overall such that the covariance between two voters' signals decreases if voters increase their focus on their own signal in the updating process.

$$Cov[S_i, S_j] \quad = \quad \frac{1}{\tau_\theta} + \frac{1}{(\omega+k-1)^2}\left(\underbrace{2\omega\frac{k-1}{n-1}\frac{1}{\tau_x}}_{\text{A}} + \underbrace{(n-2)\frac{(k-1)^2}{(n-1)^2}\frac{1}{\tau_x}}_{\text{B}}\right)$$

Part A describes the covariance between the two signals that comes from the possibility that these two voters observe each other's signal. The probability that i observes j's signal is ${}^{(k-1)}/_{(n-1)}$ and j weights this signal with w, whereas i gives it the weight $\bar{w}$. The same occurs for j receiving i's signal. The more influential part of this covariance are the $n-2$ other signals which both voters may observe simultaneously, depicted in part B in the above equation. In such cases, both voters i and j weight them equally with the lower value $\bar{w}$. An increase in the weighting parameter decreases the covariance especially because both voters give a lower weight to all other signals in part B.

The derivative of the covariance is negative for all $\omega \geq 1$.

$$\frac{\partial}{\partial \omega} Cov[S_i, S_j] = 2 \frac{k-1}{(n-1)^2} \frac{k-1-\omega(n-1)}{(\omega+k-1)^3} \frac{1}{\tau_x} < 0.$$

Already the first marginal increase in ω reduces the covariance and thus dampens the duplication effect. If two voters have a common (third) source, this signal's influence is reduced by both voters such that the correlation is reduced through both sides.

Changes in the weighting structure away from Bayesian updating have these two immediate effects that may counteract each other. On the one hand, the information effect of communication is weakened, as each voter focuses on their own signal. On the other hand, the duplication effect is reduced as well. The signals do not correlate that much anymore.

The cut-off value for a voter's decision to re-elect the incumbent is not affected by communication. Voters support the incumbent if their updated signal is larger than or equal to the expected amount of public goods $\hat{s} = \bar{\theta} + e^*$. The outcome of the election is determined by the value of the median signal. This median signal is a normally distributed random variable, which is expected to have the actual value of the public good. Its variance can be determined with the formula by Owen and Steck (1962), as the signals are multivariate normal distributed and have equal expected values, variances and pairwise correlations.

$$\begin{aligned} Var[S_{\tilde{M}}] &= \frac{\pi}{2n} Var[S_i] + \left(1 - \frac{\pi}{2n}\right) Cov[S_i, S_j] \\ &= \frac{1}{\tau_\theta} + \frac{1}{(\omega + k - 1)^2} \frac{1}{\tau_x} \left[\frac{\pi}{2n} (\omega^2 + k - 1) \right. \\ &\qquad \left. + \left(1 - \frac{\pi}{2n}\right) \frac{k-1}{n-1} \left(2\omega + (n-2)\frac{k-1}{n-1}\right) \right] \\ &\equiv \frac{1}{\tau_\theta} + \frac{1}{\tau_{\tilde{M}}} \end{aligned}$$

Given $1/\tau_{\tilde{M}}$, the procedure is the same as above and the change in the median signal's variance accordingly affects electoral accountability and selection. An increase in the variance decreases effort in the first period and the expected supply of the public good in the second period. For small values of ω an increase of ω reduces the median signal's variance. Accordingly, focusing on one's own signal does not need to have detrimental effects on this variance and the effort in the first period and the expected supply in the second period.

Proposition 3.5. *An increase in the relative weight of voters' own signals in the updating process increases the effort of the incumbent in the first period as well as the expected amount of public goods in the second period as long as*

$$\omega < \frac{1}{(\pi - 2)(n - 1)} \left[\pi \left(n + \frac{k}{n} \right) - 2(k + \pi - 1) \right].$$

Proof. See appendix C.2.2. □

This is not a straightforward result as it implies that deviating from an effectively rational updating procedure leads to better outcomes for the voters. For small values of ω the reduced information effect is outweighed by the reduced duplication effect. The duplication effect depends more strongly on the weighting procedure compared to the information effect. As the weight on others' signals decreases the duplication of these other signals is reduced as both randomly chosen voters reduce their weight on these signals. The higher ω is, the less it is possible that these signals' influence is reduced as they are already almost negligible in voters' updated signals.

3.4.2 Groups within the electorate

In many cases, communication networks are not as uniform as assumed above, where the probability to observe someone else's signal is equal for the whole of the electorate. Communication often occurs in groups of people, for instance in families or between friends and colleagues. Additionally, communication networks are shaped by geographical distance. It is more likely to talk to one's direct neighbour than to communicate with someone from another city (Jackson and Rogers, 2007). Online, phenomenons like filter bubbles or echo chambers can influence voters' information (Flaxman et al., 2016). I do not consider the formation of such groups but their effect on accountability and selection. Therefore, I introduce groups into the structure of the communication network and discuss the effects of group sizes and the number of groups in an electorate.

There are g groups that consist of q voters each. With this structure the number of voters is $n = g \cdot q$. Voters may only communicate with voters from their own group. They do not observe the signals of individuals outside their group. Each voter has k sources within their group and the probability to observe the signal of another voter is $P[T_{ij} = 1] = {}^{(k-1)}/_{(q-1)}$ if both are from the same group. I assume $k \leq {}^{q}/_{4}$, which allows to determine the median signal.[12] Voters weight all observed signals equally when they update their signal in order to inform their voting decision.

The introduction of groups within the electorate is implemented in the adjacency matrix $\mathbf{T}$. There are blocs of group-specific $q \times q$ adjacency matrices T_f for $f = 1, ..., g$ such that the $n \times n$ matrix $\mathbf{T}$ is

$$\mathbf{T} = \begin{pmatrix} \mathbf{T}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{T}_g \end{pmatrix},$$

where $\mathbf{0}$ are $q \times q$ zero-matrices. This allows to use the updating function in equation (3.8) with

$$S_i = \theta + e + \frac{1}{k} \sum_{j=1}^{n} T_{ij} x_j = v_1 + X_i.$$

Accordingly, the updated signals are again normally distributed with expected value and variance as in equations (3.9) and (3.10). In this section I focus on the

[12] For ease of presentation, I do not consider (intra-group) correlation of the noise terms. The results are qualitatively unchanged if they are included, as shown in the proof in appendix C.3. However, an additional assumption restricting the intra-group correlation is needed. This assures that the distribution of the median noise term can be approximated, which is only given for correlations smaller ${}^{1}/_{4}$. For the same reason the number of sources is restricted.

noise terms X_i of the signals. As all signals are affected equally by the public good supply v_1, only the noise terms determine the order statistic of the updated signals. The updated noise terms are expected to be zero, but have a variance of $Var[X_i] = {}^1\!/\!_{(k\tau_x)}$. The variance decreases with more sources and thus each signal becomes more precise. However, it is unaffected by the groups in the electorate. The groups do not affect the precision of the signals, only the number of sources does. Accordingly, the information effect of communication is not affected by the introduction of groups. Each voter has an equally precise signal to base their decision on for any group size or number of groups.

The duplication effect is connected to the correlation of the noise terms X_i of the updated signals. If two voters are not part of the same group the noise terms have a covariance of zero. If two randomly chosen voters of the same group are considered, their noise terms are correlated and have the covariance and correlation

$$\begin{aligned} Cov[X_i, X_j] &= \frac{1}{k^2\tau_x}\frac{k-1}{q-1}\left(k+1-\frac{k-1}{q-1}\right) \\ \rho_S &= \frac{k-1}{k(q-1)}\left(k+1-\frac{k-1}{q-1}\right). \end{aligned}$$

An increase in the number of voters per group decreases the correlation between two voters' updated noise terms. The more members there are in a group, the less likely it is that two voters share common sources. Less common information leads to less correlated signals.

To determine changes in the duplication effect, not only the within group correlation has to be considered but also the correlation of zero with voters outside one's own group. Taking the average correlation a voter has to all other voters in the electorate, the group effects become apparent. If the number of groups is fixed and the voters per group increase, average correlation decreases. On the one hand, correlation with all voters within the group decreases. On the other hand, there are now even more voters who are not part of the own group further decreasing the average. If, however, the number voters is fixed and an increase of q is reached by reducing the number of groups, average correlation increases. While correlation within groups decreases, there are now less outsiders whose signals are uncorrelated. Thus, the effect of group formation on average information duplication depends on the changes in n, q and g.

The noise terms of the updated signals are multivariate normal distributed with common expected values and the $n \times n$ variance-covariance-matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_g & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_g & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Sigma}_g \end{pmatrix}, \text{ with } \boldsymbol{\Sigma}_g = \frac{1}{k\tau_x} \begin{pmatrix} 1 & \rho_S & \cdots & \rho_S \\ \rho_S & 1 & \cdots & \rho_S \\ \vdots & \vdots & \ddots & \vdots \\ \rho_S & \rho_S & \cdots & 1 \end{pmatrix},$$

where $\boldsymbol{\Sigma}_g$ is a $q \times q$ matrix.

The electorate aims to elect the more competent candidate. Thus, a voter supports the incumbent only if his updated signal is large enough. The cut-off value does not change such that individual voters give the incumbent their vote if their signal is larger than or equal to $\hat{s}$. The outcome of the election depends on the voter with the median signal $S_{\mathcal{M}}$, which is determined by the median of the the noise terms of the updated signals. As the noise terms are multivariate normal distributed with common expected value and variance and this specific form of unequal correlations, the results in Rawlings (1975) and Rawlings (1976) can be used to determine the moments of the distribution of the noise terms' median. He expands the approximation for moments of order statistics in Owen and Steck (1962) to the case, where correlations are unequal, but there are groups of equally correlated variables. He proposes to use the average correlation

$$\varrho_a = \frac{q-1}{n-1} \frac{Cov[X_i, X_j]}{(Var[X_i]Var[X_j])^{1/2}}$$

in the approximation equations (3.3) and (3.4), if the intra-group correlations are smaller than a quarter. However, he does not elaborate on the accuracy of the approximation of the variance. Using a Monte-Carlo-simulation I can show that the approximation is valid as well to determine the median noise's variance for $\rho_S \leq 1/4$.[13]

The expected value of the median noise is zero, whereas the variance is

$$Var[X_{\mathcal{M}}] = \frac{1}{\tau_{\mathcal{M}}} = \frac{1}{k\tau_x} \left[\frac{\pi}{2n} + \left(1 - \frac{\pi}{2n}\right) \frac{k-1}{k(n-1)} \left(k + 1 - \frac{k-1}{q-1}\right) \right].$$

From this follows that the median signal $S_{\mathcal{M}} = \theta + e + X_{\mathcal{M}}$ is normally distributed with

$$\begin{aligned} E[S_{\mathcal{M}}] &= \bar{\theta} + e \\ Var[S_{\mathcal{M}}] &= \frac{1}{\tau_\theta} + \frac{1}{\tau_{\mathcal{M}}}. \end{aligned}$$

It allows to determine the accountability and selection effects of the grouped electorate. These effects are wholly determined by changes in the variance of the median noise.

[13] See appendix C.4 and the supplement S3.

As $n = g \cdot q$, changes in one of the three variables also affects at least one of the other two as well to satisfy the relation. I consider the extreme cases where if q increases either the overall number of voters n increases and the number of groups is fixed ($g = \bar{g}$) or the number of groups g decreases and the number of voters is fixed ($n = \bar{n}$). Symmetrically, I consider changes in the number of groups. These changes are given respectively by

$$\begin{aligned} \frac{\partial n}{\partial q} &= g & \qquad \frac{\partial n}{\partial g} &= q \\ \frac{\partial g}{\partial q} &= -\frac{n}{q^2} & \qquad \frac{\partial q}{\partial g} &= -\frac{n}{g^2} \end{aligned}$$

In these cases the change in the variance of the median noise is

$$\begin{aligned} \frac{\partial}{\partial q}\left(\frac{1}{\tau_{\mathcal{M}}}\right)\bigg|_{g=\bar{g}} &< 0 & \qquad \frac{\partial}{\partial g}\left(\frac{1}{\tau_{\mathcal{M}}}\right)\bigg|_{q=\bar{q}} &< 0 \\ \frac{\partial}{\partial q}\left(\frac{1}{\tau_{\mathcal{M}}}\right)\bigg|_{n=\bar{n}} &> 0 & \qquad \frac{\partial}{\partial g}\left(\frac{1}{\tau_{\mathcal{M}}}\right)\bigg|_{n=\bar{n}} &< 0 \end{aligned}$$

The variance of the median's noise term decreases, if the increase in the number of voters per group q or the number of groups g results in an increase of the electorate overall. In this case, the duplication effect and the sampling effect work in the same direction and make the median's signal more precise.

If, however, the increase of the voters per group results from a reduction of the number of groups, the median signal becomes less precise. On the one hand, the correlation of the noise terms within each group decreases. On the other hand, the average correlation which is used for approximating the variance increases, as now each voter's noise term is correlated with more noise terms of other voters. If the voters per group are reduced as g increases, the median's signal becomes more precise reversing the argument above.

The same procedure as outlined in previous sections allows to determine the effort of the incumbent in the first period as well as the expected utility in the second period. If the median signal is more precise it is more beneficial for the incumbent to increase her efforts in the first period, as a marginal increase of effort yields higher expected returns due to larger increases in the re-election probability. Additionally, the probability to elect the more competent politician and getting a higher supply of public goods increases, if the signal that determines the election is more precise.

Proposition 3.6. *An increase in the number of voters per group q or the number of groups g that is accompanied by an increase of the size of the electorate n increases effort in the first period and the expected supply of public goods in the second period.*

If q increases (decreases) while g decreases (increases) and the number of voters is fixed, effort in the first period as well as the expected supply of public goods in the second period decrease (increase).

Proof. See appendix C.3. □

If the number of communication partners k is fixed, it is beneficial for voters to have as much groups as possible. While the signals are highly correlated within each of the groups and these voters most likely vote for the same candidate, the large number of groups again leads to a relatively precise median signal. Suppose two groups at the outset, where voters in one group have a positive view of the incumbent and voters in the other have a negative view. In this case, the median voter is from one of these groups and exhibits the respective bias at least partially, which yields an imprecise signal. If now these two groups are reduced and a third group emerges filled with voters from the other groups, these voters' initial biases would counteract each other. Their signals would lie between the two original groups more closely to the correct estimate, such that the new median signal would be more precise.

3.5 Conclusion

In previous work on elections – especially with career concern frameworks – it is assumed that the information a voter bases his decision on is independent from other voters' information if it is considered at all. This as well as equal information does not capture actual informational situations before elections. Therefore, I introduced individual information that can be correlated between voters. I show that common information reduces the accountability and selection properties of elections.

Connected information of voters can be explained by exchange of information between individual voters. However, if voters can transmit additional pieces of information to their communication partners it does not only lead to more equal signals but also to more precise ones. Therefore, exchange of signals is beneficial to voters as all of them and especially the median voter are more precisely informed. This leads to higher accountability of incumbent politicians and higher expected public good supply after the election. Extensions in the communication structure show the effects of differential treatment of received signals in the individual decision process as well as the effect of information bubbles within the electorate. Differential weighting of signals can improve the election outcome only if one signal is not too heavily relied on. Groups within the electorate in which communication occurs are beneficial to voters as the average correlation to all other members of the electorate is reduced even if each block has a relatively homogeneous voting decision.

For models of elections that focus on informed voting decisions it is crucial to consider how each individual's information relate to others' information. Most importantly, voters do communicate before elections and discuss their appreciation of politicians and their ideas. This does not need to be deliberate but can also be incidental. Election campaigns rest upon these societal conversations. In order to fully grasp disinformation campaigns or the effect of influential persons within political economic voting models, there needs to be some understanding of simpler and incidental forms of communication within those. Therefore I argue that such informational linkages need to be incorporated for example in the way I propose in this paper, which lays the ground for further analyses of communication patterns before elections.

Chapter 4

Accountability in Elections with Regional Structures*

4.1 Introduction

Most democratic countries have some form of regional structure – states, counties, cities – that organise the administration within these regions. Such geographic units often constitute the basis of the electoral systems. The people living in a city form the electorate that determines who governs them as mayor. At the same time, regions are also important in nationwide elections. Parliamentarians often represent one specific constituency and the people living there. Naturally, the question arises how such regional structures influence politicians who want to be re-elected and how to determine the optimal size of these regions.

If the voters directly decide on the executive like mayors or governors, other

*I thank Jonas F. Rudsinske, Lukas Schulze-Eschenbach and Robert Schwager for valuable comments.

regions provide a benchmark against which the electorate compares the incumbent in my model. This classical yardstick competition argues in favour of regions as they increase the information people have before the election. However, this argument cannot be made for ever more regions as the electorate becomes ever smaller which reduces the elections' effectiveness. It is also possible that each region only elects a representative, and all of these from all over the country determine policy in parliament. In this case, voters in a constituency need to infer information about their parliamentarian from this common outcome, which can reduce their available information. Thus, increasing the number of regions only improves elections as long as each incumbent still has enough influence on the policy outcomes.

Institutionally, there are two ways how the addition of a new region can be incorporated. Firstly, such a region could enter a federation from outside with additional voters and thus increasing the number of voters overall. An example is the entry of new member states into the EU, which may provide additional information or increase the number of parliamentarians in the EU parliament. Secondly, adding a new region can be thought of as redistricting. Out of the already existing regions – with their voters – a new region is formed in such a way that a federation is split into more but smaller regions. In this case the number of voters overall does not change, but they are allocated to more regions. This second variant has additional effects to the first because not only is a new region created, but all already existing regions change and become smaller.

I introduce regional structure into the career concern framework developed by Persson and Tabellini (2002) and further developed by Aytimur and Bruns (2019) to include electorates with many voters. In the first part I consider local elections that are embedded in regional structures so voters do not only observe a signal about their own region but also about other regions. If there is a common exogenous shock to all regions, signals from other regions include useful information for voters akin to yardstick competition. By getting an idea about the extent of the shock voters predict their incumbent's competence more precisely such that effort has more impact on the re-election probability. Therefore, adding new regions is beneficial as it increases effort before the election. However, this is not always the case when increasing the number of regions means a reduction in the number of voters in each region. If the size of the electorate in each region becomes small, the decisive signal is less precise due to the sampling effect identified by Aytimur and Bruns (2019). This can outweigh the former positive effect of yardstick competition and decreases effort. It follows that there is an optimal number of regions in a federation with a set number of voters which maximises the effort of the politicians.

Secondly, I assume that each parliamentarian represents a constituency in the legislature, which determines the policies. However, each incumbent has a strong influence on the supply of public goods in their own constituency as they especially lobby for this region's interests. Increasing the number of constituencies in this setting has two opposing effects. Voters observe more precise signals about the supply of public goods which averages all parliamentarians' contributions. This allows for incumbents to have more influence on their re-election chances by increasing effort. However, politicians have less influence on the supply of public goods as they are part of an ever larger parliament. Therefore, voters get less information about the incumbent, and politicians' influence on the re-election probability decreases. Only if the second effect is not too large is increasing the size of parliament beneficial to voters.

The career concerns idea was introduced into political economy by Persson and

Tabellini (2002), where incumbent politicians want to get re-elected and therefore exert effort before the election in order to appear competent to voters. Large parts of the literature on career concerns such as Bruns and Himmler (2016), Ashworth and De Mesquita (2014) or Ashworth et al. (2017) focus on settings with representative voters in the career concern framework. In my context, however, there exists a trade-off between the number of jurisdictions and the number of voters in each jurisdiction. Therefore, this approach with a representative voter is not productive in a setting with multiple regions. Aytimur and Bruns (2019) introduce a large electorate into this framework and show that increasing the number of voters increases the accountability, as the decisive voter – with the median signal – becomes more informed. Using the model with a large electorate, Quint (2023) shows that exchanging information among the electorate improves the election outcome. In this paper voters can gather additional information not by communicating with others, but by observing other regions.

The idea of yardstick competition as introduced by Besley and Case (1995) has been utilised in the theoretical literature extensively using different contexts. Adopting a career concern framework with a representative voter, Kotsogiannis and Schwager (2008) analyse the effects of an equalisation scheme on the accountability properties of elections. They find two counteracting effects of equalisation on accountability, one due to a higher weight on remaining differences between countries and the other following from the complexity of the scheme which reduces the received information. Allers (2012) considers asymmetric jurisdictions in a similar framework and shows that such differences affect yardstick competition, which can be ameliorated with an equalisation scheme. However, this might incentivise cooperation of incumbent politicians (Di Liddo and Giuranno, 2016). Konishi

(2010) also employs a career concern framework with a representative voter to study yardstick competition but focuses on multiple tasks that can be assigned to different governmental levels following the tradition of Seabright (1996). This features a trade-off between increased yardstick competition and increased noise. He also shows that an increased number of jurisdictions increases effort. However, he disregards the size of each jurisdiction and the federal state overall, which I account for. Sand-Zantman (2004) also shows that if countries experience similar economic shocks, yardstick competition improves incumbents' efforts. He additionally considers a context that is somewhat similar to how I model the determination of public good supply in parliament, where the action of an incumbent in another country has an influence on public goods. He finds that effort decreases in this setting as it has less influence on welfare, which is similar to my model, but disregards the more precise information voters can get with more regions. Generally, Sand-Zantman (2004) does not consider multiple countries and the relation between the overall number of voters and the number of regions, as he focuses on the comparison of two countries in the context of (economic) integration.

To the best of my knowledge, there is no literature on parliamentarians' incentives within a career concern framework. In elections for parliament it is natural to assume that not only the individual parliamentarian's competence is decisive for voting decisions. Still, it is shown empirically that the effort of parliamentarians affects their re-election chances. Box-Steffensmeier et al. (2003) find that legislative activity – especially sponsoring locally important bills – increased name recognition and re-election chances of incumbent US Representatives in their constituency. Similar effects are found for the French National Assembly (François and Navarro, 2019). Considering Belgium, where voters can rank politicians within the same party, Däubler et al. (2016) show that legislative activity leads to a higher rank for politicians. In Italy, misbehaviour of incumbents, such as being absent in parliament, is punished by voters (Nannicini et al., 2013). Additionally, Soontjens (2022) shows that members of parliament are aware of voters considering their activities in parliament, especially in candidate-centred electoral systems. Even though the evidence on the effect of constituency work is mixed (e.g. Kaslovsky (2022)), there are arguments that it improves electoral chances of incumbents (e.g. Tucker (2019), King (1991)). Thus, such a personal vote can be an important element in voting decisions additional to party-centric motives. Therefore, in this paper I abstract from additional voting motives such as ideology or party membership in order to focus on parliamentarians' incentives within a representative framework.

The paper is structured as follows. Section 4.2 deals with yardstick competition more generally, where it is shown that having more regions is only beneficial to voters if there are enough voters in each of the regions. In Section 4.3 I discuss the effect of the number of parliamentarians on each incumbent's incentive to exert effort in parliament and find that these incentives only increase as long as the politicians have a meaningful influence on the supply of public goods. The last Section concludes.

4.2 Yardstick Competition

A classical argument in favour of decentralised elections is the idea of yardstick competition. Voters compare their own politicians with those in other jurisdictions which provides them with a possibility to better assess their politician's competence. To this end, the supply of public goods in the other regions has to be connected to the supply in the region of the voter. I introduce a common shock to public good supply in all regions. Thus, public goods in region j depend on the contribution of the incumbent in their region – her competence and effort – and the exogenous shock. Voters receive a signal about the supply of public goods not only in their own but in all k regions. They use these signals to obtain more information about their incumbent's competence. Voters can improve their assessment of their politician, which may lead to higher accountability.

I introduce this yardstick idea into the career concern framework that was developed by Persson and Tabellini (2002), where incumbent politicians exert effort before elections as long as the positive effect of a higher re-election chance is larger than the costs of effort. I incorporate the assumption of a large electorate introduced by Aytimur and Bruns (2019) into a system of k regions with n voters each.

A politician's political work is characterised by two elements. Firstly, each politician ℓ in region j has some level of competence $\theta^{\ell,j}$, which captures their inherent capabilities in office and does not change across time. Competences – of incumbent and challenger in each region ($\ell = I, C$) – are independently drawn from the same normal distribution with expected value $E[\theta^{\ell,j}] = \bar{\theta}$ and variance $Var[\theta^{\ell,j}] = {}^{1}\!/\!{}_{\tau_\theta}$. Neither politicians nor voters know the competences of the various politicians. Secondly, politicians who are in office in region j can exert costly effort e_t^j in the period before and after the election ($t \in \{1, 2\}$). With more effort, more public goods are provided. However, the cost of effort $c(e_t^j)$ incurred by the incumbent increases with higher levels, $c'(e_t^j) > 0$, $c''(e_t^j) > 0$ and $\lim_{e_t^j \to 0} c'(e_t^j) = 0$. More supply of the public good increases the incumbent's re-election probability $p^j(e_1^j)$. In case of their election to office, politicians obtain some rent R, which incentivises effort. The incumbent in constituency j thus maximises her pay-off

$$p^j(e_1^j)\left(R - c(e_2^j)\right) - c(e_1^j).$$

The supply of public goods in region j consists of two elements in period t. On the one hand, the incumbent contributes $\theta^j + e_t^j$, i.e. via her competence θ^j and her effort e_t^j.[1] In the second period, the contribution of the elected politician is decisive. On the other hand, there is a common exogenous shock η_t to all regions in the federation in period t that affects the supply of public goods positively or negatively. It is assumed to be normal distributed with $E[\eta_t] = 0$ and $Var[\eta_t] = {}^1/_{\tau_\eta}$. Thus, the supply of public goods is

$$\begin{aligned} v_1^j &= \theta^j + e^j + \eta_1 \\ v_2^j &= \theta^{\ell_j,j} + e_2^{\ell_j,j} + \eta_2 \end{aligned}$$

in the two periods, where $\ell_j \in \{I, C\}$ signifies which politician is elected.[2]

In each region there are n voters, where n is large and odd. All voters in j only care for the supply of public goods there, i.e. their utility is $u_t^j = v_t^j$. They vote sincerely between the incumbent and one challenger where voter i's decision is based on their signals. Each voter in a region does not only observe a signal on their own region's public good supply but also of the $k-1$ other regions. The signal a voter receives about region j consists of the public good supply there and

[1] For ease of exposition, I drop I in $\theta^{I,j}$ for the remainder of the paper.
[2] For convenience, I drop the time index for effort in the first period.

a – region-specific – noise term x_i^j, which is normal distributed.

$$\begin{aligned} s_i^j &= v_1^j + x_i^j = \theta^j + e^j + \eta_1 + x_i^j \\ \text{with} \qquad & x_i^j \sim N\left(0, \frac{1}{\tau_x}\right) \end{aligned}$$

Thus, the individual signals are also normal distributed with an expected value of $E[s_i^j] = \bar{\theta} + e^j$ and a variance of $Var[s_i^j] = {}^1/_{\tau_\theta} + {}^1/_{\tau_\eta} + {}^1/_{\tau_x}$. The signals from the k regions are correlated due to the common shock with $Cov[s_i^j, s_i^{-j}] = {}^1/_{\tau_\eta}$. However, they only contain direct information about the incumbent's competence from the specific region

$$\begin{aligned} Cov[\theta^j, s_i^j] &= \frac{1}{\tau_\theta} \\ Cov[\theta^j, s_i^{-j}] &= 0. \end{aligned}$$

The voters use the signals to inform their voting decision. They need to form an expectation about their incumbent's competence to compare it to the challenger's expected competence.

There are three stages in the model. In the first period before the election, all incumbents from the k regions decide on their effort and the local supplies of public goods are determined as all θ^j and η_1 are drawn. Secondly, voters observe their signals and the election between incumbent and challenger is held. Lastly, in the second period the elected politicians decide on their effort which leads to the supply of public goods in this period.

The model is solved by backwards-induction. The elected politicians decide on their effort in the second period. Effort is costly and it does not have any reward in or after the second period to the elected politicians. Therefore, they do not exert any effort and the supply of public goods in j only depends on the competence of the elected politician in j and the exogenous shock

$$v_2^j = \theta^{\ell_j,j} + \eta_2.$$

Voters thus want to elect the politician who they think is more competent. For the challenger they expect him to have a competence of $E[\theta^{C,j}] = \bar{\theta}$. For the incumbent they can take their signals as well as their belief over the effort of all incumbents before the election into account to determine her expected competence.

The signals from the $k-1$ other regions do not directly provide insights about the incumbent's competence. However, they do contain information about the (common) exogenous shock η_1. For instance, if in all other regions the supply of public goods – and thus also the received signals – is high and the signal from their own region is small, voters can deduce that most likely the shock was positive but their own incumbent is incompetent.

All signals and the competence of the incumbent in j are multivariate normal distributed. Their expected values are given above. The variance-covariance matrix of all signals voter i from region j receives and the competence of the incumbent in this region j, i.e. of $(\theta^j, s_i^1, ..., s_i^j, ..., s_i^k)'$, is

$$\Sigma = \begin{pmatrix} \frac{1}{\tau_\theta} & 0 & 0 & \cdots & \frac{1}{\tau_\theta} & \cdots & 0 \\ 0 & \frac{1}{\tau_\theta}+\frac{1}{\tau_\eta}+\frac{1}{\tau_x} & \frac{1}{\tau_\eta} & \cdots & \frac{1}{\tau_\eta} & \cdots & \frac{1}{\tau_\eta} \\ 0 & \frac{1}{\tau_\eta} & \frac{1}{\tau_\theta}+\frac{1}{\tau_\eta}+\frac{1}{\tau_x} & \cdots & \frac{1}{\tau_\eta} & \cdots & \frac{1}{\tau_\eta} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ \frac{1}{\tau_\theta} & \frac{1}{\tau_\eta} & \frac{1}{\tau_\eta} & \cdots & \frac{1}{\tau_\theta}+\frac{1}{\tau_\eta}+\frac{1}{\tau_x} & \cdots & \frac{1}{\tau_\eta} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & \frac{1}{\tau_\eta} & \frac{1}{\tau_\eta} & \cdots & \frac{1}{\tau_\eta} & \cdots & \frac{1}{\tau_\theta}+\frac{1}{\tau_\eta}+\frac{1}{\tau_x} \end{pmatrix}$$

$$\equiv \begin{pmatrix} \frac{1}{\tau_\theta} & \Sigma_\theta \\ \Sigma_\theta{}' & \Sigma_s \end{pmatrix},$$

where Σ_θ is a $1 \times k$ vector and Σ_s a $k \times k$ matrix. The variance-covariance matrix has a positive determinant and the rank of $k+1$. To predict θ^j given all the signals an individual receives they have to be aggregated to obtain the conditional distribution of θ^j with the lowest possible variance (Tong (1990)). To this end, all k signals are aggregated according to

$$S_i^j = \Sigma_\theta \cdot \Sigma_s{}^{-1} \cdot (s_i^1, ..., s_i^k)'$$

$$\text{with} \quad \Sigma_s{}^{-1} = \left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-1}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-1}\begin{pmatrix} \alpha & \beta & \cdots & \beta \\ \beta & \alpha & \cdots & \beta \\ \vdots & \vdots & \ddots & \vdots \\ \beta & \beta & \cdots & \alpha \end{pmatrix},$$

$$\text{where} \quad \alpha = \frac{1}{\tau_\theta}+(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}$$

$$\beta = -\frac{1}{\tau_\eta}.$$

The aggregated signal of an individual in region j is

$$S_i^j = \frac{1}{\tau_\theta}\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-1}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-1}\left[\left(\frac{1}{\tau_\theta}+(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)s_i^j-\frac{1}{\tau_\eta}\sum_{f\neq j}s_i^f\right].$$

While the signal from region j features positively, all other signals reduce the value of this aggregated signal. This follows from the argument above that the only connection between the different regions lies in the common shock. If this shock is positive – and thus all other signals tend to be large – this information is used to reduce the value of the aggregated signal and thus reducing the effect of the shock on the aggregated signal. Comparing the relative importance of a voter's signal about their own region to a signal about another region, their own signal is more important and influences the aggregated signal more strongly. It contains the more relevant information on the competence of the incumbent. If the exogenous shock is less precise (smaller τ_η), other regions' signals become less important relative to the signal from region j. If it becomes more difficult to predict the value of the exogenous shock, other signals are less informative because it becomes harder to distinguish the shock from other elements of the signal.

Considering the signals s_i^j for the regions, the aggregated signal can be rearranged to

$$\begin{aligned} S_i^j \;=\; & \frac{1}{\tau_\theta}\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-1}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-1} \\ & \cdot\left[\left(\frac{1}{\tau_\theta}+(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)(\theta^j+e^j)-\frac{1}{\tau_\eta}\sum_{f\neq j}(\theta^f+e^f)+\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right)\eta_1\right] \\ & +X_i^j \end{aligned} \tag{4.1}$$

with

$$\begin{aligned} X_i^j &= \frac{1}{\tau_\theta}\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-1}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-1} \\ &\quad \left[\left(\frac{1}{\tau_\theta}+(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)x_i^j-\frac{1}{\tau_\eta}\sum_{f\neq j}x_i^f\right]. \end{aligned}$$

The former part is equal for all voters in region j, whereas X_i^j is specific to each voter. It is apparent that the exogenous shock vanishes if more regions can be observed by the voters. Also the contributions of incumbents in other regions $\theta^f + e^f$ become less important if more signals are considered by the voter. In the limit,

$$\lim_{k\to\infty} S_i^j = \frac{1}{\tau_\theta}\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-1}(\theta^j+e^j+x_i^j),$$

such that the aggregated signal depends neither on the contribution of politicians from other regions nor on the exogenous shock. The information of all other signals is used to deduce the exogenous shock which is then eliminated from the signal as well. This process is partially done for smaller values of k.

Each voter forms an expectation of their incumbent's competence conditional on their signal S_j^j and their expectation on all incumbents' efforts $\mathbf{e}^* = (e_1^*, ..., s_k^*)$. Following Tong (1990), the competence θ^j and the aggregated signal S_i^j are bivariate normal with

$$\begin{aligned} \rho_{\theta^j,S_i^j} &= \left(\frac{1}{\tau_\theta}\right)^{-1/2}\left(\Sigma_\theta\cdot{\Sigma_s}^{-1}\cdot{\Sigma_\theta}'\right)^{1/2} \\ &= \left[\frac{1}{\tau_\theta}\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-1}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-1}\left(\frac{1}{\tau_\theta}+(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)\right]^{1/2}. \end{aligned}$$

Thus, the expected competence of the incumbent in j given the signal S_i^j and the expected effort of the incumbents $\mathbf{e}^*$ is

$$\begin{aligned} E[\theta^j|S_i^j,\mathbf{e}^*] &= E[\theta^j] + \rho_{\theta^j,S_i^j}\left(\frac{Var[\theta^j]}{Var[S_i^j]}\right)^{1/2}(S_i^j - E[S_i^j|\mathbf{e}^*]) \\ &= \bar{\theta} + S_i^j - E[S_i^j|\mathbf{e}^*]. \end{aligned} \tag{4.2}$$

A voter supports the politician who they expect to be more competent.

$$d_i = \begin{cases} I & \text{for } E[\theta^j|S_i^j,\mathbf{e}^*] \geq E[\theta^{C,j}] \\ C & \text{for } E[\theta^j|S_i^j,\mathbf{e}^*] < E[\theta^{C,j}] \end{cases}$$

There is a cut-off value $\hat{S}$ of an individual's signal given the voting rule and the expected value of the incumbent's competence in equation (4.2).

$$\begin{aligned} E[\theta^j|S_i^j,\mathbf{e}^*] &\geq E[\theta^C] \\ \bar{\theta} + S_i^j - E[S_i^j|\mathbf{e}^*] &\geq \bar{\theta} \\ S_i^j &\geq E[S_i^j|\mathbf{e}^*] \equiv \hat{S} \end{aligned}$$

Once the signal is larger than this cut-off $\hat{S}$, a voter supports the incumbent for re-election. If the signal is smaller than $\hat{S}$, the individual votes for the challenger.

A politician needs half of the electorate to vote for them in order to be elected to serve in the second period. Therefore, the voter with the median signal is decisive. He decides who gets elected as at least half of the electorate votes the same way as him. Only the individual noise terms determine the order of the signals described in (4.1) as the supply of public goods equally affects all voters. Using the result of Cramér (1946), the median value of the error terms X_m^j is asymptotically normal distributed with an expected value of zero and a variance of

$$\begin{aligned} Var[X_m^j] &= \left(\frac{1}{\tau_\theta}\right)^2 \frac{1}{\tau_x} \left\{\frac{1}{\tau_\theta} + \frac{1}{\tau_x}\right\}^{-2} \left\{\frac{1}{\tau_\theta} + k\frac{1}{\tau_\eta} + \frac{1}{\tau_x}\right\}^{-2} \\ &\cdot \left[\left(\frac{1}{\tau_\theta} + (k-1)\frac{1}{\tau_\eta} + \frac{1}{\tau_x}\right)^2 + (k-1)\left(\frac{1}{\tau_\eta}\right)^2\right] \frac{\pi}{2n}. \end{aligned}$$

The median signal is thus normal distributed with

$$\begin{aligned} E[S_m^j] &= E[S_i^j] \\ Var[S_m^j] &= \left(\frac{1}{\tau_\theta}\right)^2 \left\{\frac{1}{\tau_\theta} + \frac{1}{\tau_x}\right\}^{-2} \left\{\frac{1}{\tau_\theta} + k\frac{1}{\tau_\eta} + \frac{1}{\tau_x}\right\}^{-2} \\ &\cdot \Bigg[\left(\left(\frac{1}{\tau_\theta} + (k-1)\frac{1}{\tau_\eta} + \frac{1}{\tau_x}\right)^2 + (k-1)\left(\frac{1}{\tau_\eta}\right)^2\right) \\ &\left(\frac{1}{\tau_\theta} + \frac{\pi}{2n}\frac{1}{\tau_x}\right) + \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_x}\right)^2 \frac{1}{\tau_\eta}\Bigg]. \end{aligned}$$

The incumbent is re-elected, if the median voter (with the median signal) supports her, i.e. $d_m = I$. With the distribution of the median signal, the re-election probability of the incumbent in j given the voters' expectation $\mathbf{e}^*$ can be determined. To this end, the probability for S_m^j to be larger than the cut-off $\hat{S}$ has to be computed.

$$p^j(e^j) = Pr[S_m^j \geq \hat{S}] = 1 - Pr[S_m^j < \hat{S}] = 1 - \Phi\left[(\hat{S} - E[S_m^j])Var[S_m^j]^{-\frac{1}{2}}\right],$$

where Φ represents the standard normal distribution function. This re-election probability can be positively affected by the incumbent in region j. Given the expectations of voters about effort $\mathbf{e}^*$ by politicians, increasing actual effort also increases the re-election probability.

$$\begin{aligned}\frac{\partial p^j(e^j)}{\partial e^j} \quad = \quad & \frac{1}{\tau_\theta}\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-1}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-1}\left(\frac{1}{\tau_\theta}+(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right) \\ & \cdot Var[S_m^j]^{-1/2}\varphi\left[(\hat{S}-E[S_m^j])Var[S_m^j]^{-1/2}\right],\end{aligned}$$

where φ is the standard normal density function.

The incumbent in region j can exert effort before the election to increase the chance of re-election and obtaining the rent of being in office after the election. However, this benefit of increased effort has to be balanced with the costs of increased effort before the election. The incumbent's rationale is

$$\frac{\partial p^j(e^j)}{\partial e^j}R - c'(e^j) = 0.$$

In equilibrium, the voters' expectation about incumbents' effort is correct ($\mathbf{e}^* = \mathbf{e}$). Using this equilibrium condition, the incumbent's rationale implicitly determines the effort exerted by the incumbent.

$$\begin{aligned}0 = \lambda \quad \equiv \quad & \frac{1}{\tau_\theta}\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-1}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-1}\left(\frac{1}{\tau_\theta}+(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right) \\ & \cdot Var[S_m^j]^{-1/2}(2\pi)^{-1/2}R \\ & -c'(e^j) \qquad (4.3)\end{aligned}$$

This implicit definition of effort can be used to determine the effect of the number of regions voters can observe on the effort of the incumbent.

Proposition 4.1. *Increasing the number of regions k, where voters receive a signal from, increases the effort of the incumbent before the election.*

Proof. The effort of the incumbent is implicitly defined in equation (4.3). Thus, the change in effort with respect to the number of observed signals from different regions k is

$$\frac{\partial e^j}{\partial k} = -\frac{\partial\lambda/\partial k}{\partial\lambda/\partial e^j}$$

with

$$\begin{aligned}
\frac{\partial\lambda}{\partial k} &= \frac{1}{2}(2\pi)^{-1/2}R\left(\frac{1}{\tau_\theta}\right)^3\left(\frac{1}{\tau_\eta}\right)^2\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-3}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-3}Var[S_m]^{-3/2} \\
&\quad\cdot\left[\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right)^2+\left(1-\frac{\pi}{2n}\right)\frac{1}{\tau_x}\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right)+(k-1)\frac{1}{\tau_\eta}\left(\frac{1}{\tau_\theta}+\frac{\pi}{2n}\frac{1}{\tau_x}\right)\right]>0 \\
\frac{\partial\lambda}{\partial e^j} &= -c''(e^j)<0
\end{aligned}$$

Given the signs of the two derivatives of λ, effort e^j increases if the number of regions k increases. □

The number of regions which voters observe has a positive effect on the accountability property of elections. Voters are enabled to enhance their information about the incumbent's competence because they can remove the common shock from their signal. As voters – and especially the median voter – are better informed, effort by the incumbent has a stronger effect on the re-election probability. Therefore, she increases her effort before the election and the supply of public goods increases.

Such additions of new regions may not occur often. An example would be the entry of new countries into the European Union. These countries feature more heavily in the media and therefore provide new insights to the voters about common trends in the EU. However, it is more reasonable to think about additional regions in the context of redistricting within a country. The differentiating factor between these two examples is that in the former case new voters are added to the overall number of voters. In the latter case, the number of voters is fixed but they are allocated to a larger number of regions. In the model this results in $n = {}^{N}\!/_{k}$, i.e. the number of voters n in a region depends on the overall number of voters in the federation N and the number of regions k.

If the overall number of voters is fixed, increasing the number of regions has an additional effect to the yardstick argument above. The number of voters in each region declines which affects the sampling effect identified by Aytimur and Bruns (2019). The median voter has a less precise signal, which reduces the effect of effort on the re-election probability thus decreasing the effort before the election. Therefore, two counteracting mechanisms influence the effort in equilibrium. On the one hand, increasing the number of regions increases the incumbent's effort before the election as voters are better informed. On the other hand, the number of voters in each region becomes smaller with an additional region, which reduces effort. Thus, in order to increase effort, the overall number of voters N needs to be large enough when a new region is added. Each region's electorate ${}^{N}\!/_{k}$ is still large enough that the negative effect of the reduced sampling effect does not outweigh the positive effect of the increased information.

Proposition 4.2. *Assume that the number of voters in a federation is N and equally distributed to all regions, i.e. $n = {}^{N}\!/_{k}$. An increase in the number of regions only increases the effort e^j before the election, if N is large enough,*

$$\begin{aligned} N > \tilde{N} \quad \equiv \quad & \frac{\pi}{2} \left\{ \frac{1}{\tau_\eta} \right\}^{-2} \frac{1}{\tau_x} \left(\frac{1}{\tau_\theta} + k \frac{1}{\tau_\eta} + \frac{1}{\tau_x} \right) \\ & \cdot \left(\left(\frac{1}{\tau_\theta} + (k-1) \frac{1}{\tau_\eta} + \frac{1}{\tau_x} \right) \left(\frac{1}{\tau_\theta} + k \frac{1}{\tau_\eta} + \frac{1}{\tau_x} \right) \right. \end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{\tau_\eta}\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right)^2+\left(\frac{1}{\tau_\eta}\right)^2\left(\frac{1}{\tau_\theta}-k(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)\Bigg)\\
\cdot\Bigg\{&\left(\frac{1}{\tau_\theta}\right)^2\left(\frac{1}{\tau_\theta}+(2k-1)\frac{1}{\tau_\eta}+4\frac{1}{\tau_x}\right)\\
&+\left(\frac{1}{\tau_x}\right)^2\left(5\frac{1}{\tau_\theta}+2k\frac{1}{\tau_\eta}+2\frac{1}{\tau_x}\right)\\
&+\frac{1}{\tau_\theta}\frac{1}{\tau_\eta}\left(k(k-1)\frac{1}{\tau_\eta}+(4k-1)\frac{1}{\tau_x}\right)\Bigg\}^{-1}.
\end{aligned}
$$

Proof. The effort in equilibrium is still implicitly defined in equation (4.3) with $n = {}^N\!/_k$. The change in effort e^j for an increase in k is again defined by

$$
\frac{\partial e^j}{\partial k} = -\frac{{}^{\partial\lambda}/_{\partial k}}{{}^{\partial\lambda}/_{\partial e^j}}
$$

with

$$
\begin{aligned}
\frac{\partial\lambda}{\partial e^j} &= -c''(e^j) < 0\\
\frac{\partial\lambda}{\partial k} &= \frac{1}{2}(2\pi)^{-1/2}R\left(\frac{1}{\tau_\theta}\right)^3\left\{\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right\}^{-3}\left\{\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right\}^{-4}Var[S_m]^{-3/2}\\
&\Bigg[\left(\frac{1}{\tau_\eta}\right)^2\Bigg(\left(\frac{1}{\tau_\theta}\right)^2\left(\frac{1}{\tau_\theta}+(2k-1)\frac{1}{\tau_\eta}+4\frac{1}{\tau_x}\right)\\
&\qquad+\left(\frac{1}{\tau_x}\right)^2\left(5\frac{1}{\tau_\theta}+2k\frac{1}{\tau_\eta}+2\frac{1}{\tau_x}\right)\\
&\qquad+\frac{1}{\tau_\theta}\frac{1}{\tau_\eta}\left(k(k-1)\frac{1}{\tau_\eta}+(4k-1)\frac{1}{\tau_x}\right)\Bigg)\\
&-\frac{\pi}{2N}\frac{1}{\tau_x}\left(\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)\\
&\quad\cdot\Bigg(\left(\frac{1}{\tau_\theta}+(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)\left(\frac{1}{\tau_\theta}+k\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)\\
&\qquad-\frac{1}{\tau_\eta}\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_x}\right)^2+\left(\frac{1}{\tau_\eta}\right)^2\left(\frac{1}{\tau_\theta}-k(k-1)\frac{1}{\tau_\eta}+\frac{1}{\tau_x}\right)\Bigg)\Bigg],
\end{aligned}
$$

where the expression in brackets is positive (negative) for $N > (<)\tilde{N}$. □

The effort only increases if the electorate is large enough. In this case the number of voters per region becomes smaller and the median signal less precise. However, this negative effect is counteracted by the positive effect of yardstick competition. The additional information each voter obtains is more beneficial than the slightly less precise signal of the median voter due to the reduced sampling effect.

The cut-off size $\tilde{N}$ depends on the number of regions positively ($\partial\tilde{N}/\partial k > 0$). The more regions there are the larger the electorate has to be in order to support an additional region. This result implies that for a fixed number of voters, introducing regions first increases effort before the election but eventually decreases it. Thus, there is an optimal number of regions which leads to the largest effort by the incumbents.

4.3 Parliamentarians in their Constituencies

Policy decisions are often made by multiple politicians. In representative democracies parliaments are the main legislative bodies that deliberate and decide on policies. Politicians in such parliaments are often directly elected from the people in their districts or constituencies such as in the USA or in the UK. Voters therefore need to asses the outcome of parliament when they want to get an idea about their own parliamentarian's competence in office. However, as the parliament collectively decides on policy, all parliamentarians impact the supply of public goods in each region. Thus, voters have to disentangle their parliamentarian's contribution to the public goods from all other parliamentarians' contributions. They want to elect the more competent of two candidates in their constituency to increase the supply of public goods after the election.

In comparison to the model above, voters now only observe a signal about the supply of public goods in their own constituency. More importantly, the determination of the supply of public goods in each constituency changes. All incumbent parliamentarians affect the public goods in all jurisdictions, but there is no exogenous shock to public good supply. The timing of the model is unchanged.

Political outcomes that voters observe are determined by all parliamentarians who are in office. However, they might differ between constituencies. I assume that the supply of public goods in a constituency is a weighted average of all politicians' competence and effort. While all other politicians have equal weights, the incumbent parliamentarian elected in constituency j may be more important for the supply of public goods in region j. There are two aspects that motivate this higher weight. Firstly, the local parliamentarian can offer advice to the people in her constituency on a wide range of topics in her surgery. Additionally, she can raise questions in parliament about topics related to her constituency.[3] Secondly, she especially cares for projects that are important to her constituency such as construction of new infrastructure or upgrades to the local hospital.

I assume that the supply of public goods in period $t = 1$ before the election in region j is given by

$$v_1^j = \alpha(k)\ (\theta^j + e^j) + \beta(k)\ \sum_{f \neq j} (\theta^f + e^f) \text{ with } 0 < \beta(k) \leq \alpha(k) < 1,$$

where $\alpha(k)$ and $\beta(k)$ are the weights on the contributions of the region's parliamen-

[3] Chiru (2018) shows that bringing up such local issues improves the vote share an incumbent parliamentarian obtains.

tarian and all other parliamentarians respectively. I assume $\alpha(k) + (k-1)\beta(k) = 1$ such that $\beta(k) = \frac{1-\alpha(k)}{k-1}$. This assures that the supply of public goods does not merely increase because there is an additional politician in parliament. Each politician's weight is assumed to decrease if a new constituency emerges. For this to hold for $\beta(k)$, I assume

$$-\frac{1-\alpha(k)}{k-1} \leq \frac{\partial \alpha(k)}{\partial k} \leq 0.$$

With these assumptions, the supply of public goods in constituency j before the election is[4]

$$v_1^j = \alpha(\theta^j + e^j) + \frac{1-\alpha}{k-1} \sum_{f \neq j} (\theta^f + e^f).$$

In the second period, the elected politicians' competence and effort are decisive. Thus,

$$v_2^j = \alpha(\theta^{\ell_j, j} + e_2^{\ell_j, j}) + \frac{1-\alpha}{k-1} \sum_{f \neq j} (\theta^{\ell_f, f} + e_2^{\ell_f, f}),$$

where $\ell_j \in \{I, C\}$ signifies whether the incumbent I or the challenger C is elected in the respective constituencies.

Each of the n voters in a constituency receives a signal about the supply of public goods in their constituency in the first period. All voters in j only care for the supply of public goods there, i.e. their utility is $u_t^j = v_t^j$. They vote sincerely and voter i's decision is based on their signal that consists of the actual supply of public goods before the election v_1^j and a voter specific noise term x_i^j. This noise is normal distributed with an expected value of zero and a variance of $Var[x_i^j] = {}^1/_{\tau_x}$.

[4]For convenience, I drop the k in $\alpha(k)$.

Thus, the signal is given by

$$s_i^j = v_1^j + x_i^j = \alpha(\theta^j + e^j) + \frac{1-\alpha}{k-1}\sum_{f \neq j}(\theta^f + e^f) + x_i^j. \tag{4.4}$$

Again, voters want to elect the politician who seems to be more competent, as the politician in office does not have any incentive to exert effort in the second period. Voters want to have the most competent politicians in parliament, even though they can only decide on their own representative. For the challenger they expect him to have a competence of $E[\theta^{C,j}] = \bar{\theta}$. For the incumbent they can take their signal s_i^j as well as their belief $\mathbf{e}^* = (e_1^*, ..., e_k^*)$ over the effort of all incumbents into account to determine her expected competence.

The signal as defined by equation (4.4) is normal distributed with

$$\begin{aligned} E[s_i^j] &= \alpha\left(\bar{\theta} + e^j\right) + \frac{1-\alpha}{k-1}\sum_{f \neq j}(\bar{\theta} + e^f) \\ Var[s_i^j] &= \left(\alpha^2 + \frac{(1-\alpha)^2}{k-1}\right)\frac{1}{\tau_\theta} + \frac{1}{\tau_x}. \end{aligned}$$

The number of constituencies affects the precision of the signal. With more parliamentarians the variance decreases for given effort of the incumbents. The driving factor is that each politician – and their competence – becomes less important in the determination of the public good supply, which underlies the individual signals. This *averaging effect* of more constituencies implies that for given efforts $\mathbf{e}$ signals show less variance if new constituencies are added.

Each voter's signal in j is positively correlated with the competence of the incumbent politician in their constituency j. With higher competence, more public goods are supplied and therefore the signal becomes larger. However, the incumbent's competence is only partly responsible for public goods such that it may become difficult to disentangle all other politicians' contributions from the incumbent's in j. The covariance between the signal and the competence of the incumbent decreases if more regions exist.

$$\begin{aligned} Cov[\theta^j, s_i^j] &= \alpha \frac{1}{\tau_\theta} \\ \frac{\partial Cov[\theta^j, s_i^j]}{\partial k} &= \frac{\partial \alpha}{\partial k} \frac{1}{\tau_\theta} < 0 \end{aligned}$$

Due to the lower weight α with more constituencies, the incumbent in j has less influence on the supply of public goods in their own constituency j – both through her competence and her effort. This is the *political influence effect* of increasing the number of constituencies. One result of the *political influence effect* is that the signal an individual in j receives becomes less informative about the parliamentarian's competence from j which reduces the informational value of the signals.

The signal s_i^j that individual i in region j receives and the competence θ^j of their parliamentarian are bivariate normal distributed. The expected value of the competence given the signal and some expectation $\mathbf{e}^*$ about all politicians' effort before the election is

$$E[\theta^j | s_i^j, \mathbf{e}^*] = \bar{\theta} + \frac{Cov[\theta^j, s_i^j]}{Var[s_i^j]} \left(s_i^j - E[s_i^j | \mathbf{e}^*] \right) \tag{4.5}$$

If now the number of constituencies increases, both the *averaging effect* and the *political influence effect* affect how strongly voters consider their signal when determining this expected value. On the one hand, the signal becomes more important through the *averaging effect* as with a lower variance of the signal, the fraction ${Cov[\theta^j, s_i^j]}/{Var[s_i^j]}$ increases. The signal is more precise and should therefore be taken into consideration more strongly. On the other hand, the covariance decreases due to the first element of the *political influence effect* which decreases the fraction ${Cov[\theta^j, s_i^j]}/{Var[s_i^j]}$ and thus the influence of the signal. As the signal is less informative about the incumbent in j, it should be taken into account to a lesser extent.

A voter gives the incumbent their vote only if they expect that she is more competent than the challenger. They do not receive any additional information on the challenger such that the original expectation of $\bar{\theta}$ holds. Thus an individual's voting decision is determined by

$$d_i^j = \begin{cases} I & \text{for } E[\theta | s_i, \mathbf{e}^*] \geq E[\theta^{C,j}] = \bar{\theta} \\ C & \text{for } E[\theta | s_i, \mathbf{e}^*] < E[\theta^{C,j}] = \bar{\theta}. \end{cases}$$

The cut-off value for an individual's signal to support the incumbent is

$$\hat{s} \equiv \alpha \left(\bar{\theta} + e^{j*} \right) + \frac{1-\alpha}{k-1} \sum_{f \neq j} (\bar{\theta} + e^{f*}).$$

If their signal is larger than the cut-off, a voter supports the incumbent.

Again, the election is decided by the voter with the median signal. If his signal is large enough to vote for the incumbent, half of the electorate has an even larger signal. Whoever he votes for is elected as at least half of the electorate votes just like him. The order of the error terms determines the order of the signals as above. Using the result of Cramér (1946), the median value of error terms x_m^j is asymptotically normal distributed with an expected value of zero and a variance of $\pi/(2n) \cdot 1/\tau_x$. Consequently, the median signal is normally distributed with

$$\begin{aligned} E[s^j_m] &= E[s^j_i] \\ Var[s^j_m] &= \left(\alpha^2 + \frac{(1-\alpha)^2}{k-1}\right)\frac{1}{\tau_\theta} + \frac{\pi}{2n}\frac{1}{\tau_x}. \end{aligned}$$

If the median voter observes a signal that is large enough, the incumbent is re-elected. This probability is

$$p^j(e^j, \mathbf{e}^*) = Pr[s^j_m \geq \hat{s}] = 1 - \Phi\left[\left(\hat{s} - E[s^j_m]\right) Var[s^j_m]^{-1/2}\right].$$

The incumbent can influence her re-election chances by exerting more effort before the election, which leads to a higher re-election probability.

$$\frac{\partial p^j(e^j, \mathbf{e}^*)}{\partial e^j} = \alpha\varphi\left[\left(\hat{s} - E[s^j_m]\right) Var[s^j_m]^{-1/2}\right] Var[s^j_m]^{-1/2} > 0.$$

The incumbent has an incentive to exert effort before the election, which improves the re-election chances and thus enjoying the ensuing rent for being in office in the second period as well. However, effort is costly and the incumbent has to balance these two effects in her decision.

$$\frac{\partial p^j(e^j, \mathbf{e}^*)}{\partial e^j} R = c'(e^j)$$

An equilibrium is reached, when the effort level voters expect is correct, i.e.

$\mathbf{e}^* = \mathbf{e}$. The effort of the incumbent is then implicitly determined by

$$0 = \alpha(2\pi)^{-1/2} Var[s_m^j]^{-1/2} R - c'(e^j) \equiv \lambda. \tag{4.6}$$

This in turn can be used to analyse the accountability properties of this election with several constituencies. If the number of regions increases, incumbent politicians may in- or decrease their effort before the election.

Proposition 4.3. *If the number of constituencies increases, effort by incumbent politicians only increases as long as the negative change in the weighting parameter $\alpha(k)$ for more regions is not too strong.*

Proof. Equation (4.6) implicitly determines the effort of the incumbent in equilibrium. Thus, the change in effort if the number of constituencies increases follows from implicit differentiation

$$\frac{\partial e^j}{\partial k} = -\frac{\partial\lambda/\partial k}{\partial\lambda/\partial e^j}$$

with

$$\begin{aligned} \frac{\partial \lambda}{\partial e^j} &= -c''(e^j) < 0 \\ \frac{\partial \lambda}{\partial k} &= \{2\pi\}^{-\frac{1}{2}} Var[s_m^j]^{-\frac{3}{2}} R \left(\frac{\partial \alpha}{\partial k} \left(\frac{1-\alpha}{k-1} \frac{1}{\tau_\theta} + \frac{\pi}{2n} \frac{1}{\tau_x} \right) + \frac{1}{2} \alpha \left(\frac{1-\alpha}{k-1} \right)^2 \frac{1}{\tau_\theta} \right). \end{aligned}$$

With $c''(e^j) > 0$, the former derivative is negative. Thus, the sign of the second derivative determines the change of effort. This in turn follows from the sum within the brackets. It is positive for

$$\frac{\partial \alpha}{\partial k} > -\{k-1\}^{-1} \left\{ 2n(1-\alpha)\frac{1}{\tau_\theta} + (k-1)\pi \frac{1}{\tau_x} \right\}^{-1} \alpha \; n(1-\alpha)^2 \frac{1}{\tau_\theta} \equiv \tilde{\alpha}$$

and negative else. Thus,

$$\frac{\partial e^j}{\partial k} \begin{cases} > 0 & \text{if } \frac{\partial \alpha}{\partial k} > \tilde{\alpha} \\ < 0 & \text{if } \frac{\partial \alpha}{\partial k} < \tilde{\alpha} \end{cases}.$$

□

The *averaging* and the *political influence effect* have an impact on the incumbent's influence on the re-election probability through her effort. Firstly, the *averaging effect* reduces the variance of the median signal thus increasing her effort's influence on re-election. The more precise the median signal the higher the possibility to affect the voting decision of the median voter through her effort. However, the *political influence effect* reduces this influence of the incumbent on the supply of public good and thus the signals. It becomes more difficult for the incumbent to meaningfully increase the signal through effort which would lead to a higher re-election probability.

If the *averaging effect* outweighs the *political influence effect* increasing the effort is optimal for the incumbent. She can increase her expected rent from being in office more with effort than the additional costs of effort she has to bear. This is the case if the change in her weight α is small as the *political influence effect* hinges on this parameter. With smaller changes in α in absolute terms the *political influence effect* is small and outweighed by the positive outcome of the *averaging effect* improving accountability.

I assume that the parliamentarians' influence on public goods decreases if parliament becomes larger, because they become more specialised on specific topics and more people have to work together to legislate. This suggests strong decreases in the weighting parameter α, if new constituencies are added, which would lead to decreases in effort according to Proposition 4.3. However, there is also an argument against such a strong decrease and thus positive effects of larger parliaments. Firstly, parliamentarians might specialise in policy fields that are important for their constituencies, for instance in rural constituencies where the parliamentarian might opt to focus on agricultural topics. Secondly, they can give advice to their constituents and lobby for their constituency in parliament to attract additional (infrastructure) spending. This aspect of local services should not decrease with a larger parliament giving an argument for only a moderate decrease in the weighting parameter α.

The effect on effort and thus the supply of public goods before the election does not only hold in any specific constituency j but in all of the k constituencies. Thus, the effect of an additional constituency on public goods is even more pronounced. For these symmetric regions, all parliamentarians act equally such that overall the supply of public goods increases or decreases everywhere as all incumbents adjust their effort.

Up to this point I assumed that new constituencies emerge out of thin air with an additional set of n voters. However, it is more convincing to think about redistricting where a new constituency is created by taking some voters from all other constituencies. The fixed number N of voters overall is now allocated to the k constituencies. The more constituencies there are, the less voters are in each constituency $n = {}^{N}\!/\!_{k}$. As above, the median voter has a less precise signal if the electorate becomes smaller, which reduces effort. In this case, the main result of Proposition 4.3 remains intact but with another cut-off of

$$\bar{\alpha} = -\{k{-}1\}^{-1}\left\{2n(1-\alpha)\frac{1}{\tau_\theta} + (k-1)\pi\frac{1}{\tau_x}\right\}^{-1}\alpha\frac{N}{k}\left((1-\alpha)^2\frac{1}{\tau_\theta} - \frac{\pi}{2N}(k-1)^2\frac{1}{\tau_x}\right).$$

Only if the weight of the incumbent does not decrease too strongly, increasing the number of constituencies improves effort. However, the smaller the overall number of voters N, the smaller the range of derivatives of α that allows for higher effort. Fixing the number of voters further restricts the cases of positive effects of the number of constituencies compared to Proposition 4.3.

4.4 Conclusion

Elections in many different countries have some relation to regional boundaries, even if they are federal elections. Regional elements can officially be part of the institutional setting of (federal) elections. Additionally, neighbouring regions provide a context to voters with additional information that can help to decide whom to vote for. These regional aspects should be considered when accountability properties of elections are analysed.

I propose to account for regional structures in two types of elections. Firstly, regional structures are effectively part of the election when voters can observe how political outcomes are in neighbouring districts. These other regions can be used as a benchmark against which their own politician is measured especially in regional elections for mayors or governors. Secondly, federal parliamentary elections are often decided in constituencies. Voters have to choose a parliamentarian who represents them. In this setting, regions are part of the institutional background that determines how public goods are supplied.

For both types of regional influences – effective and institutional – I show that they have an impact on accountability of incumbents. If the regional structure only effectively impacts the election – i.e. the supply of public goods is determined locally, but all regions face a common shock – the other regions provide additional information to voters. The classic argument of yardstick competition applies. Voters can observe other regions and learn about their own incumbent's competence as they use the information to determine the common shock. As the electorate is more precisely informed, effort is more effective to increase re-election chances. However, if the overall number of voters is fixed, increasing the number of regions reduces the size of the electorate in each region. This implies that the median signal is less precise which reduces the incentives for incumbents to exert effort. Thus, only for large number of voters it is beneficial to further increase the number of regions and there is an optimal number of regions to maximise effort before the election.

If the supply of public goods in a region is determined by parliamentary process, but depends especially on the regional parliamentarian, increasing the number of constituencies has two channels through which it affects the effort of incumbents before the election. Firstly, the *averaging effect* leads to more precise signals for the voters, as the supply of public goods does not rely on only few politicians' contributions. This increases effort as it becomes more effective for incumbents to increase their re-election probabilities. However, the *political influence effect* reduces the accountability as each parliamentarian has a smaller impact on the supply of public goods and thus the signals, if there are more politicians in parliament. Only if the second effect is not too large, increasing the number of constituencies improves accountability.

These regional characteristics of elections should be taken into account when discussing effects of elections. Incumbents face different incentives depending on regional structures. Here, I only consider the accountability effect within a career concern framework, but it would be interesting to further investigate the outcomes for instance in terms of the selection properties in the same framework.

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Appendix A

Appendix to Asymmetric General Oligopolistic Equilibrium

A.1 Proofs

A.1.1 Proof of Proposition 1.1

Proof. First, we reformulate the condition that the balance of payments is zero. In this paper's setting the balance of payments is equal to the balance of trade (BoT).

$$\begin{aligned} BoP \equiv BoT = 0 &= n \int_0^1 p_f(z)\ y_f(z)\ dz - n^* \int_0^1 p_h^*(z)\ y_h^*(z)\ dz \\ 0 &= x_1 + x_2\lambda + a^2 \left(\frac{n}{1-\lambda} - \frac{n^*}{\lambda} \right), \end{aligned}$$

where

$$x_1 = 2a\frac{\eta}{2-\eta}\alpha(n-n^*)(nw+n^*w^*) + a\frac{\eta}{2-\eta}\alpha\rho(n^*w^* - nw)$$

$$
\begin{aligned}
& -n\beta w^2\left(1+(n-1)\eta+\frac{1}{2-\eta}n^*(2+(n-2)\eta)\right) \\
& +\frac{\eta^2}{(2-\eta)^2}nn^{*2}\beta(w^{*2}-(1-\eta)w^2) \\
& +\frac{\eta^2}{(2-\eta)^2}\Delta nwn^*w^*(2n-\rho) \\
x_2 \;=\; & n\beta w^2\Bigg(1+(n-1)\eta+\frac{\eta}{2-\eta}n^*(2+(n-2)\eta) \\
& \qquad +\frac{\eta^2}{(2-\eta)^2}(n^{*2}(1-\eta)-nn^*)\Bigg) \\
& +n^*\beta w^{*2}\Bigg(1+(n^*-1)\eta+\frac{\eta}{2-\eta}n(2+(n^*-2)\eta) \\
& \qquad +\frac{\eta^2}{(2-\eta)^2}(n^2(1-\eta)-nn^*)\Bigg) \\
& +2\frac{\eta^2}{(2-\eta)^2}\Delta nwn^*w^*(\rho-n-n^*),
\end{aligned}
$$

and wages are defined by equation (1.10) with $\bar{\lambda}=1$ for Home and analogously for Foreign.

In the limiting cases of the admissible λ, the BoT is

$$
\lim_{\lambda\to 0^+} BoT = -\infty
$$
$$
\lim_{\lambda\to 1^-} BoT = \infty.
$$

Additionally, the BoT is differentiable with respect to λ, which implies continuity of the BoT. Therefore, there has to be at least one solution of the above equation for $\lambda \in (0,1)$.

At the same time the balance of trade has a strictly positive derivative with respect to λ in $\lambda \in (0,1)$.

$$\frac{\partial BoT}{\partial \lambda} = x_2 + a^2 \left(\frac{n}{(1-\lambda)^2} + \frac{n^*}{\lambda^2} \right) > 0.$$

For $\Delta \leq \beta$, it holds that $x_2 \geq 0$. As the second summand is positive, the derivative is strictly positive. Therefore, there exists a unique solution. □

A.1.2 Proof of Lemma 1.1

Proof. In the equilibrium condition in equation (1.11), $\hat{\lambda}$ is implicitly defined. The derivative of $\hat{\lambda}$ with respect to L is then given by

$$\frac{\partial \hat{\lambda}}{\partial L} = -\frac{\partial BoT/\partial L}{\partial BoT/\partial \lambda}.$$

The derivative of the balance of trade with regard to λ is positive as shown in the proof of uniqueness. Additionally, the derivative of the balance of trade with respect to L is given under our assumptions by

$$\frac{\partial BoT}{\partial L} = (2\lambda - 1) \underbrace{\left[b(L + L^*) - \frac{4}{3}a \right]}_{<0} + \underbrace{a - bL}_{>0}, \tag{A.1}$$

where the signs follow from the assumption that $b(L + L^*) < a$. As we show in appendix A.2, $\hat{\lambda} = 1/2$ for $L = L^*$. Therefore,

$$\left. \frac{\partial BoT}{\partial L} \right|_{L=L^*} = a - bL > 0.$$

This implies a decreasing $\hat{\lambda}$ for a marginally increasing L at $L = L^*$. If now L is increased further, we have $\hat{\lambda} < {}^1\!/_2$. In this case

$$\left.\frac{\partial BoT}{\partial L}\right|_{L>L^*} = \underbrace{(2\lambda - 1)}_{<0} \underbrace{\left[b(L+L^*) - \frac{4}{3}a\right]}_{<0} + \underbrace{a - bL}_{>0} > 0,$$

which assures a declining $\hat{\lambda}$ when L rises in equilibrium for $L > L^*$. It follows that $\hat{\lambda} < {}^1\!/_2$, if $L > L^*$.

We know that $\hat{\lambda} \in ({}^1\!/_4, {}^3\!/_4)$. In equation (A.1) the term in parentheses becomes largest if λ takes the largest possible value and therefore, the equation as a whole may become negative. However, for the largest value $\lambda = {}^3\!/_4$ the derivative is still positive for

$$\begin{aligned} 0 &< \frac{\partial BoT}{\partial L} = \frac{1}{2}\left[b(L+L^*) - \frac{4}{3}a\right] + a - bL \\ b(L - L^*) &< \frac{2}{3}a \end{aligned}$$

This condition is fulfilled for all $L^* > L$, as a is positive. Therefore, the derivative of the BoT with respect to L is positive for all $L^* > L$. It follows that the derivative ${}^{\partial\hat{\lambda}}\!/_{\partial L}$ is negative, as the derivative ${}^{\partial BoT}\!/_{\partial L} > 0$.

With this result $\hat{\lambda}$ is

$$\hat{\lambda} \begin{cases} \in \left(\frac{1}{4}, \frac{1}{2}\right) & \text{for } L > L^* \\ = \frac{1}{2} & \text{for } L = L^* \\ \in \left(\frac{1}{2}, \frac{3}{4}\right) & \text{for } L < L^* \end{cases}$$

□

A.1.3 Proof of Proposition 1.2

Proof. Changes in consumption in Home and Foreign are given by

$$\begin{aligned}
\frac{\partial}{\partial L}(y_h + y_h^*) &= \hat{\lambda} + \frac{\partial \hat{\lambda}}{\partial L}\left(L + L^* - \frac{4}{3}\frac{a}{b}\right) \\
\frac{\partial}{\partial L}(y_f + y_f^*) &= (1 - \hat{\lambda}) + \frac{\partial \hat{\lambda}}{\partial L}\left(\frac{4}{3}\frac{a}{b} - (L + L^*)\right)
\end{aligned}$$

The increase of consumption in Home is straightforward as $\frac{\partial \hat{\lambda}}{\partial L} < 0$ and $L + L^* < {}^{4a}\!/_{3b}$ under our assumption of $b(L + L^*) < a$. For consumption in Foreign, we show with mathematical software in the supplement S1 that the derivative is positive in equilibrium.

The sum of exports is affected by an increase in L according to

$$\frac{\partial}{\partial L}(y_f + y_h^*) = 1 - \hat{\lambda} + \frac{\partial \hat{\lambda}}{\partial L}(L^* - L).$$

For $L \geq L^*$ it follows immediately that the derivative is positive, but also for $L < L^*$ we show with mathematical software in the supplement S1 that the derivative remains positive.

Without loss of generality we now assume that Home is the more liberal country with $L > L^*$. Consumption in Home is smaller than its production if

$$y_h + y_h^* \quad < \quad L.$$

We show with mathematical software in the supplement S1 that this is given in equilibrium, such that Home's consumption is lower under free trade than under autarky, where $y_h = L$. □

A.1.4 Proof of Proposition 1.3

Proof. The changes in nominal wages and prices in Home, where L is increased, are given by

$$\begin{aligned}\frac{\partial w}{\partial L} &= -2b\\ \frac{\partial p}{\partial L} &= -\frac{1}{3}\frac{a}{\hat{\lambda}^2}\frac{\partial\hat{\lambda}}{\partial L} - b.\end{aligned}$$

The change in real wages is accordingly

$$\frac{\partial}{\partial L}\left(\frac{w}{p}\right) = \frac{1}{p^2}\left(\frac{\partial w}{\partial L}p - w\frac{\partial p}{\partial L}\right),$$

where the expression in parentheses determines the sign of the derivative. This expression is negative in general equilibrium as

$$\begin{aligned}\frac{\partial w}{\partial L}p - w\frac{\partial p}{\partial L} &= -2b\, p - w\left(-\frac{1}{3}\frac{a}{\hat{\lambda}^2}\frac{\partial\hat{\lambda}}{\partial L} - b\right)\\ &= \underbrace{\frac{1}{3}\frac{a}{\hat{\lambda}^2}\frac{\partial\hat{\lambda}}{\partial L}w}_{<0} - b\left(\underbrace{\frac{2}{3}a\frac{1-\hat{\lambda}}{\hat{\lambda}}}_{>0} + \underbrace{\frac{4}{3}a - bL^*}_{>0}\right) < 0,\end{aligned}$$

where the sign of the last two summands follows from $b(L+L^*) < a$.

If Foreign's labour endowment is smaller and L increases, wage payments in Foreign (w^*L^*) decrease, as

$$\frac{\partial w^*L^*}{\partial L} = -bL^*.$$

The profits in Foreign change according to

$$\frac{\partial \pi^*}{\partial L} = \frac{1}{9}\frac{a^2}{b}\frac{2\hat{\lambda}-1}{\hat{\lambda}^2(1-\hat{\lambda})^2}\frac{\partial \hat{\lambda}}{\partial L}.$$

As for $L > L^*$ we know that $\hat{\lambda} < 1/2$, this derivative is positive. Therefore the labour share decreases. The price in foreign changes according to

$$\frac{\partial p^*}{\partial L} = \frac{1}{3}\frac{a}{(1-\hat{\lambda})^2}\frac{\partial \hat{\lambda}}{\partial L} - b < 0$$

if L increases. As this expression is negative, real profits increase whenever nominal profits increase. $\square$

A.2 Marginal Utility of Income in General Equilibrium

Under the assumptions of Section 4 with $\gamma(z) = \gamma^*(z) = 1\ \forall z$, $n = n^* = 1$, $b(L+L^*) < a$ and $2/5 < L/L^* < 5/2$, the balance of trade equilibrium condition can be rearranged to

$$0 = \Xi \equiv \epsilon\lambda^3 + \zeta\lambda^2 + \kappa\lambda + \theta$$

with

$$\begin{aligned}
\epsilon &= -\left(b(L+L^*) - \frac{4}{3}a\right)^2 \\
\zeta &= \frac{8}{3}a^2 - 5abL - 3abL^* + 2b^2L^2 + 3b^2LL^* + b^2L^{*2} \\
\kappa &= \left(bL - \frac{1}{3}a\right)(2a - b(L+L^*))
\end{aligned}$$

$$\theta = -\frac{1}{9}a^2.$$

Accordingly, the discriminant is

$$\zeta^2\kappa^2 - 4\epsilon\kappa^3 - 4\zeta^3\theta - 27\epsilon^2\theta^2 + 18\epsilon\zeta\kappa\theta > 0.$$

As the coefficients are real numbers and the discriminant is positive, this cubic polynomial has three distinct real roots. However, two of these are complex. In the proof of Proposition 1.2 we use mathematical software to determine the applicable root. We can further restrict the values of the marginal utility of income in equilibrium using Newton's method to approximate roots. At $\lambda = {}^1\!/\!_4$ and $\lambda = {}^3\!/\!_4$ it holds respectively that

$$\frac{1}{4} - \frac{\Xi}{{}^{\partial\Xi}\!/\!_{\partial\lambda}}\bigg|_{\lambda={}^1\!/\!_4} > \frac{1}{4} \quad \text{and} \quad \frac{3}{4} - \frac{\Xi}{{}^{\partial\Xi}\!/\!_{\partial\lambda}}\bigg|_{\lambda={}^3\!/\!_4} < \frac{3}{4}.$$

This implies that ${}^1\!/\!_4 < \hat{\lambda} < {}^3\!/\!_4$. For the case of symmetric countries ($L = L^*$), the balance of trade condition can be rearranged to

$$\begin{aligned} 0 &= -\frac{1}{9}a^2 + \lambda\left\{-\frac{2}{3}a^2 + \frac{8}{3}abL - 2b^2L^2\right\} \\ &\quad +\lambda^2\left\{\frac{8}{3}a^2 - 8abL + 6b^2L^2\right\} - \lambda^3\left\{2bL - \frac{4}{3}a\right\}^2 \\ 0 &= (2\lambda - 1)\left\{\frac{1}{3}\frac{a^2}{b}(8\lambda(\lambda-1) - 1) + \lambda(\lambda-1)L\left(6bL - 8a\right)\right\} \end{aligned}$$

The condition is fulfilled for symmetric countries with $\lambda = {}^1\!/\!_2$.

A.3 Supplement S1

The supplement S1 (Mathematica Notebook) is available from the authors upon reasonable request.

Appendix B

Appendix to International Trade and Tax-Motivated Transfer Pricing

B.1 Endogenous Variables Depending on Exogenous Parameters and λ

We present these equations in a more general way without applying the assumptions on labour endowments $L = L^* = 1/2$ and number of firms $n = n^* = 1$. Still, we set $0 \leq \tau^* \leq \tau$. The supplied quantities in Cournot equilibrium are

$$
\begin{aligned}
y_h &= \frac{\lambda}{b}\left\{b\frac{L}{n} + \frac{1-2\lambda}{n+n^*+1}\frac{a}{\lambda} - (1-\lambda)g\frac{(\tau-\tau^*)}{n+n^*+1}\left(\frac{n^*}{1-\tau} + \frac{n^*+1}{1-\tau^*}\right)\right\} \\
y_f &= \frac{1-\lambda}{b}\left\{b\frac{L}{n} + \frac{2\lambda-1}{n+n^*+1}\frac{a}{1-\lambda} + \lambda g\frac{(\tau-\tau^*)}{n+n^*+1}\left(\frac{n^*}{1-\tau} + \frac{n^*+1}{1-\tau^*}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
y_h^* &= \frac{\lambda}{b}\left\{b\frac{L^*}{n^*} + \frac{1-2\lambda}{n+n^*+1}\frac{a}{\lambda} + (1-\lambda)g\frac{(\tau-\tau^*)}{n+n^*+1}\left(\frac{n}{1-\tau^*} + \frac{n+1}{1-\tau}\right)\right\} \\
y_f^* &= \frac{1-\lambda}{b}\left\{b\frac{L^*}{n^*} + \frac{2\lambda-1}{n+n^*+1}\frac{a}{1-\lambda} - \lambda g\frac{(\tau-\tau^*)}{n+n^*+1}\left(\frac{n}{1-\tau^*} + \frac{n+1}{1-\tau}\right)\right\}.
\end{aligned}
$$

The prices are given by

$$
\begin{aligned}
p &= a\left(1 + \frac{n+n^*+{}^{1-\lambda}/_{\lambda}}{n+n^*+1}\right) - b(L+L^*) - (1-\lambda)g\frac{(\tau-\tau^*)}{n+n^*+1}\left(\frac{n^*}{1-\tau} - \frac{n}{1-\tau^*}\right) \\
p^* &= a\left(1 + \frac{n+n^*+{}^{\lambda}/_{1-\lambda}}{n+n^*+1}\right) - b(L+L^*) - \lambda g\frac{(\tau-\tau^*)}{n+n^*+1}\left(\frac{n}{1-\tau^*} - \frac{n^*}{1-\tau}\right)
\end{aligned}
$$

and wages are

$$
\begin{aligned}
w &= 2a - b\left(\frac{n+1}{n}L + L^*\right) + (1-\lambda)g\frac{(\tau-\tau^*)}{1-\tau^*} \\
w^* &= 2a - b\left(\frac{n^*+1}{n^*}L^* + L\right) + \lambda g\frac{(\tau-\tau^*)}{1-\tau}.
\end{aligned}
$$

B.2 Derivation of the Market Indifference Curve

Here, we derive the Market Indifference (MI) curve resulting from profit maximization. Equations (2.5) and (2.6) can be rearranged to

$$
\frac{1}{\lambda}(a-by) = \frac{1}{\lambda}by_h + \frac{1}{\lambda^*}(a-by^*-by_f) + \frac{\tau-\tau^*}{1-\tau^*}g
$$

and

$$
\frac{1}{\lambda}(a-by) = \frac{1}{\lambda}by_h^* + \frac{1}{\lambda^*}(a-by^*-by_f^*) - \frac{\tau-\tau^*}{1-\tau}g.
$$

Because these equations have the same left-hand side, we can set the right-hand sides equal, rearrange, and get

$$\frac{g}{b}\frac{(\tau-\tau^*)(2-\tau-\tau^*)}{(1-\tau)(1-\tau^*)}=\frac{y_h^*-y_h}{\lambda}+\frac{y_f-y_f^*}{\lambda^*}.$$

Using the normalization $\lambda^*=1-\lambda$, as well as $y_f+y_h=1/2$ and $y_h^*+y_f^*=1/2$, this equation can be rearranged to get the function we use to illustrate the equilibrium:

$$y_h(y_h^*)=-\lambda(1-\lambda)\frac{g}{b}\frac{(\tau-\tau^*)(2-\tau-\tau^*)}{(1-\tau)(1-\tau^*)}+y_h^*.$$

B.3 Proofs

B.3.1 Lemma 2.1

Proof. The balance of payments can be rearranged to

$$BoP=\frac{1}{b}\left(x_1+x_2\ \lambda+x_3\ \lambda^2+x_4\ \lambda^3+\frac{1}{9}a^2\left(\frac{\tau-1}{\lambda}+\frac{\lambda-\tau^*}{(1-\lambda)}\right)\right)$$

with

$$\begin{aligned}
x_1 &= -\frac{1}{9}a^2(7+4\tau)-\frac{1}{2}b^2\left(1+\frac{1}{2}\ \tau^*\right)+\frac{1}{3}ab(4+\tau+\tau^*)+\frac{1}{3}ag(\tau^*-\tau)-\frac{1}{2}b\tau^*g\\
&\quad+\frac{1}{3}a\Delta\left(-1+\frac{5}{3}\tau-\frac{2}{3}\tau^*+\frac{2}{3}\tau\tau^*-\frac{2}{3}\tau^2\right)+\frac{1}{2}b\tau^*\Delta(1-\tau)\\
x_2 &= \left(\frac{2}{3}a-\frac{1}{2}b\right)^2(4+\tau+\tau^*)+\left(\frac{2}{3}a-\frac{1}{2}b\right)g(\tau-\tau^*)\\
&\quad+\Delta\Bigg[a\left(\frac{2}{3}-\frac{23}{9}\tau+\frac{2}{3}\tau^2+\frac{17}{9}\tau^*-\frac{2}{9}\tau^{*2}-\frac{4}{9}\tau\tau^*\right)\\
&\qquad+b\left(\frac{7}{6}\tau-\frac{1}{3}\tau^2-\frac{13}{6}\tau^*+\frac{1}{3}\tau^{*2}+\tau\tau^*\right)
\end{aligned}$$

$$
\begin{aligned}
& \quad +g\left(-\tau+\frac{1}{3}\tau^2-\tau^*+\frac{1}{3}\tau^{*2}+\frac{4}{3}\tau\tau^*\right)\Bigg] \\
& \Delta^2\left(\frac{1}{3}\tau-\frac{4}{9}\tau^2+\frac{1}{9}\tau^3+\frac{2}{3}\tau^*-\frac{1}{9}\tau^{*2}-\frac{13}{9}\tau\tau^*+\frac{7}{9}\tau^2\tau^*+\frac{1}{9}\tau\tau^{*2}\right) \\
x_3 \;&=\; \Delta\Bigg[a\left(\frac{20}{9}\tau-\frac{4}{9}\tau^2-\frac{20}{9}\tau^*+\frac{4}{9}\tau^{*2}\right)+b\left(-\frac{5}{3}\tau+\frac{1}{3}\tau^2+\frac{5}{3}\tau^*-\frac{1}{3}\tau^{*2}\right) \\
& \quad +g\left(\tau-\frac{1}{3}\tau^2+\tau^*-\frac{1}{3}\tau^{*2}-\frac{4}{3}\tau\tau^*\right) \\
& \quad +\Delta\left(-\frac{4}{3}\tau+\tau^2-\frac{2}{9}\tau^3-\frac{5}{3}\tau^*+\frac{2}{3}\tau^{*2}-\frac{1}{9}\tau^{*2}+\frac{13}{3}\tau\tau^*-\frac{5}{3}\tau^2\tau^*-\tau\tau^{*2}\right)\Bigg] \\
x_4 \;&=\; \Delta^2\left[\tau-\frac{5}{9}\tau^2+\frac{1}{9}\tau^3+\tau^*-\frac{5}{9}\tau^{*2}+\frac{1}{9}\tau^{*3}-\frac{26}{3}\tau\tau^*+\frac{8}{9}\tau^2\tau^*+\frac{8}{9}\tau\tau^{*2}\right] \\
\Delta \;&=\; \frac{(\tau-\tau^*)\,g}{(1-\tau)(1-\tau^*)}
\end{aligned}
$$

In the limiting cases of the admissible $\lambda \in (0,1)$, the BoP is determined by the last summand $1/9a^2\left((\tau-1)/\lambda+(\lambda-\tau^*)/(1-\lambda)\right)$. This yields

$$
\begin{aligned}
\lim_{\lambda\to 0^+} BoP &= -\infty \\
\lim_{\lambda\to 1^-} BoP &= \infty.
\end{aligned}
$$

Additionally, the BoP is differentiable with respect to λ, which implies continuity of the BoP. As in equilibrium $BoP=0$ has to hold, there has to be at least one solution for $\lambda \in (0,1)$.

In order to ensure uniqueness of our equilibrium, we derive a sufficient condition on the transfer price parameter g. Uniqueness is sufficiently ensured, if the derivative of the balance of payments is positive for all $\lambda \in (0,1)$. The derivative of the balance of payment with respect to λ is

$$
\frac{\partial BoP}{\partial \lambda}=\frac{1}{b}\left(x_2+2\;x_3\;\lambda+3\;x_4\;\lambda^2+\frac{1}{9}a^2\left(\frac{1-\tau}{\lambda^2}+\frac{1-\tau^*}{(1-\lambda)^2}\right)\right),
$$

where

$$\begin{aligned}
x_2 &\gtrless 0 \\
2\ x_3\ \lambda &\gtrless 0 \\
3\ x_4\ \lambda^2 &> 0 \\
\frac{1}{9}a^2\left(\frac{1-\tau}{\lambda^2}+\frac{1-\tau^*}{(1-\lambda)^2}\right) &> 0
\end{aligned}$$

for $0 \leq \tau^* \leq \tau < 1$ as well as $\lambda \in (0,1)$ as derived in the supplement S2. For $\tau = \tau^*$, $\frac{\partial BoP}{\partial \lambda}$ is strictly positive and uniqueness is given. This results from $\Delta = 0$ which implies $x_3 = x_4 = 0$, but $x_2 > 0$ as $a > b$. To ensure a positive derivative for $\tau > \tau^*$, we derive a sufficient upper bound on g based on the first three summands of the derivative. As we know that the last summand will be positive, it suffices to show that for a range of g, the first three summands are non-negative in aggregate to assure a strictly positive derivative. The remaining three summands of the derivative ($\overline{dBoP}(\lambda)$) are a quadratic polynomial.

$$\begin{aligned}
\overline{dBoP}(\lambda) &= \frac{\partial BoP}{\partial \lambda} - \frac{1}{b}\left(\frac{1}{9}a^2\left(\frac{1-\tau}{\lambda^2}+\frac{1-\tau^*}{(1-\lambda)^2}\right)\right) \\
&= \frac{1}{b}\left(x_2 + 2\ x_3\ \lambda + 3\ x_4\ \lambda^2\right)
\end{aligned}$$

This polynomial will have one global minimum at $\tilde{\lambda}$, where the derivative of $\overline{dBoP}(\lambda)$ with respect to λ is zero.

$$\begin{aligned}
0 &= \frac{1}{b}(2\ x_3 + 6\ x_4\ \lambda) \\
\tilde{\lambda} &= -\frac{1}{3}\frac{x_3}{x_4}
\end{aligned}$$

To ensure non-negativity of this global minimum, we need to show the conditions under which the minimum value $\overline{dBoP}(\tilde{\lambda})$ is non-negative.

$$\begin{aligned} \overline{dBoP}(\tilde{\lambda}) &\geq 0 \\ \frac{1}{b}\left(x_2 - \frac{1}{3}\frac{{x_3}^2}{x_4}\right) &\geq 0 \end{aligned}$$

This can be rearranged to determine a lower and an upper bound on g which assure non-negativity. As $g \geq 0$ by the model assumptions, we have an upper bound $\bar{g}$ as given in the supplement S2, which only depends on the exogenous parameters.

As the disregarded element of the derivative of the balance of payments is strictly positive, the complete derivative of the balance of payments will be strictly positive if g is below the value $\bar{g}$. This assures the uniqueness of the equilibrium $\hat{\lambda}$:

$$\frac{\partial BoP}{\partial \lambda} > \overline{dBoP} \geq \overline{dBoP}(\tilde{\lambda}) \geq 0 \qquad \forall\ 0 \leq g < \bar{g}.$$

□

B.3.2 Lemma 2.2

Proof. In the equilibrium condition (2.18), $\hat{\lambda}$ is implicitly defined for $0 \leq \tau^* \leq \tau$. Hence, the derivatives of $\hat{\lambda}$ with respect to τ and τ^* are given by

$$\begin{aligned} \frac{\partial \hat{\lambda}}{\partial \tau} &= -\frac{\partial BoP/\partial \tau}{\partial BoP/\partial \lambda}, \\ \frac{\partial \hat{\lambda}}{\partial \tau^*} &= -\frac{\partial BoP/\partial \tau^*}{\partial BoP/\partial \lambda}. \end{aligned}$$

At $g = 0$, the derivative of the balance of payments with regard to λ is positive resulting from the proof of uniqueness. Additionally,

$$\begin{aligned}
\frac{\partial BoP}{\partial \tau} &= \frac{1}{36\ b\ \lambda} \left(2\ a\ (1-2\lambda) + 3\ b\ \lambda\right)^2 > 0, \\
\frac{\partial BoP}{\partial \tau^*} &= -\frac{1}{36\ b\ (1-\lambda)} \left(2\ a\ (1-2\lambda) - 3\ b\ (1-\lambda)\right)^2 < 0.
\end{aligned}$$

Therefore, $\hat{\lambda}$ has a negative first derivative with respect to the Home tax rate and a positive derivative with respect to the Foreign tax rate. This shows that the high-tax (low-tax) country's marginal utility of income $\hat{\lambda}$ $(1-\hat{\lambda})$ decreases, if they increase their tax rate τ (τ^*) respectively for all $\tau^* \leq \tau$.

Additionally, we established that at equalized tax rates across countries $\hat{\lambda} = 1/2$ holds. For $g = 0$ Newton's method yields a lower bound on the equilibrium $\hat{\lambda}$ for any combination of tax rates $0 \leq \tau^* \leq \tau$ as shown in the supplement S2, such that $1/5 < \hat{\lambda} \leq 1/2$.

$$\frac{1}{5} - \left.\frac{BoP}{\partial BoP/\partial \lambda}\right|_{\lambda = 1/5} > \frac{1}{5}$$

Therefore, it suffices to proof the following propositions only for values $\lambda \in (1/5, 1/2]$.

□

B.3.3 Proposition 2.1

Proof. The proposition follows from inspection of the endogenous variables' derivatives with respect to the Home tax rate τ with $\frac{\partial \hat{\lambda}}{\partial \tau} < 0$ and $g = 0$. For the supplied quantities we have

$$\frac{\partial y_h}{\partial \tau} = \frac{\partial y_h^*}{\partial \tau} = \underbrace{\left(\frac{1}{2} - \frac{2}{3}\frac{a}{b}\right)}_{<0} \underbrace{\frac{\partial \hat{\lambda}}{\partial \tau}}_{<0} > 0 \tag{B.1}$$

$$\frac{\partial y_f}{\partial \tau} = \frac{\partial y_f^*}{\partial \tau} = \underbrace{\left(\frac{2}{3}\frac{a}{b} - \frac{1}{2}\right)}_{>0} \underbrace{\frac{\partial \hat{\lambda}}{\partial \tau}}_{<0} < 0. \tag{B.2}$$

All companies supply more to the high-tax country Home, such that consumption and welfare increases there. The nominal wages do not change in either country

$$\frac{\partial w}{\partial \tau} = \frac{\partial w^*}{\partial \tau} = 0.$$

The changes in the countries' prices after an increase of τ are

$$\begin{aligned} \frac{\partial p}{\partial \tau} &= -\frac{1}{3} a \frac{\partial \hat{\lambda}/\partial \tau}{\hat{\lambda}^2} > 0 \\ \frac{\partial p^*}{\partial \tau} &= \frac{1}{3} a \frac{\partial \hat{\lambda}/\partial \tau}{(1-\hat{\lambda})^2} < 0. \end{aligned}$$

Nominal wages remain constant while the prices increase in Home and decrease in Foreign. Therefore, the real wages decrease in high-tax country Home, when the tax rate is increased.

$$\begin{aligned} \frac{\partial \left(w/p\right)}{\partial \tau} &= \frac{\partial w/\partial \tau \; p - w \; \partial p/\partial \tau}{p^2} \\ &= -\frac{w}{p^2}\frac{\partial p}{\partial \tau} < 0. \end{aligned}$$

In low-tax country Foreign, real wages will increase.

$$\begin{aligned}\frac{\partial \left({}^{w^*}\!/_{p^*} \right)}{\partial \tau} &= \frac{{}^{\partial w^*}\!/_{\partial \tau}\ p^* - w^*\ {}^{\partial p^*}\!/_{\partial \tau}}{p^{*2}} \\ &= -\frac{w^*}{p^{*2}} \frac{\partial p^*}{\partial \tau} > 0.\end{aligned}$$

For $g = 0$, profits will be equal across countries as provided quantities are equal in the respective markets. Additionally, the wage rates are equal in both countries. Therefore, the nominal profit changes are symmetric in both countries. We can show with mathematical software in the supplement S2 that the nominal profits decrease in both countries. Straightforwardly, real profits will decrease in the high-tax country Home as well due to increasing prices. In the low-tax country Foreign, however, nominal profits as well as the price level decrease. Still, we can show in the supplement S2 that real profits decrease in Foreign as well.

The labour-to-profit ratio will decrease in both countries

$$\frac{\partial}{\partial \tau} \left(\frac{{}^{1}\!/_{2}\ w}{\pi} \right) = \frac{1}{2\pi^2} \left(\underbrace{\frac{\partial w}{\partial \tau}}_{=0} \pi - w \underbrace{\frac{\partial \pi}{\partial \tau}}_{<0} \right) > 0$$

The ratio in Foreign is defined analogously and increases as well. □

B.3.4 Lemma 2.3

Proof. We know that for equalized tax rates ($\tau = \tau^*$), the countries are identical and therefore $\hat{\lambda} = {}^{1}\!/_{2}$. Lemma 2.2 showed that the marginal utility of income in Home decreases with the tax rate τ. Therefore, at $g = 0$ the marginal utility of income in Home has to be smaller than a half for any tax rate with $\tau^* < \tau$.

The derivative $\frac{\partial \hat{\lambda}}{\partial g}$ can be determined by implicit differentiation of the balance of payments condition.

$$\frac{\partial \hat{\lambda}}{\partial g} = -\frac{\partial BoP/\partial g}{\partial BoP/\partial \lambda}$$

We know from showing the uniqueness of the equilibrium solution that $\frac{\partial BoP}{\partial \lambda} > 0$ for any $g < \bar{g}$. The derivative of the balance of payments with respect to g at $g = 0$ is given by

$$\begin{aligned}
\frac{\partial BoP}{\partial g} = \frac{1}{b}\Bigg\{ & \frac{1}{3}a(\tau^* - \tau) - \frac{1}{2}b\tau^* \\
& + \frac{\tau - \tau^*}{(1-\tau)(1-\tau^*)}\left(\frac{1}{3}a\left(-1 + \frac{5}{3}\tau - \frac{2}{3}\tau^* + \frac{2}{3}\tau\tau^* - \frac{2}{3}\tau^2\right)\right. \\
& \qquad \left. + \frac{1}{2}b\tau^*(1-\tau)\right) \\
& + \lambda\Bigg\{\frac{2}{3}a(\tau - \tau^*) + \frac{1}{2}b(\tau^* - \tau) \\
& \quad + \frac{\tau - \tau^*}{(1-\tau)(1-\tau^*)}\Bigg[a\left(\frac{2}{3} - \frac{23}{9}\tau + \frac{2}{3}\tau^2 + \frac{17}{9}\tau^* - \frac{2}{9}\tau^{*2} - \frac{4}{9}\tau\tau^*\right) \\
& \qquad + b\left(\frac{7}{6}\tau - \frac{1}{3}\tau^2 - \frac{13}{6}\tau^* + \frac{1}{3}\tau^{*2} + \tau\tau^*\right)\Bigg]\Bigg\} \\
& + \lambda^2 \frac{\tau - \tau^*}{(1-\tau)(1-\tau^*)}\Bigg[a\left(\frac{20}{9}\tau - \frac{4}{9}\tau^2 - \frac{20}{9}\tau^* + \frac{4}{9}\tau^{*2}\right) \\
& \qquad + b\left(-\frac{5}{3}\tau + \frac{1}{3}\tau^2 + \frac{5}{3}\tau^* - \frac{1}{3}\tau^{*2}\right)\Bigg]\Bigg\}
\end{aligned}$$

It follows that for all possible parameter values and $\lambda \leq 1/2$ this derivative is negative, as shown in the supplement S2. Hence, for all $\lambda \leq 1/2$ it follows that $\frac{\partial \hat{\lambda}}{\partial g} > 0$ at $g = 0$. As we have that $\hat{\lambda} < 1/2$ for $g = 0$, this is the case for the initial marginal increase in g. $\square$

B.3.5 Proposition 2.2

Proof. The sum of exports for $\tau > \tau^*$ is given by

$$\begin{aligned} y_f + y_h^* &= \frac{1-\hat{\lambda}}{b}\left\{\frac{1}{2}b + \frac{1}{3}a\frac{2\hat{\lambda}-1}{1-\hat{\lambda}} + \frac{1}{3}\hat{\lambda}\frac{\tau-\tau^*}{(1-\tau)(1-\tau^*)}(3-2\tau-\tau^*)g\right\} \\ &\quad + \frac{\hat{\lambda}}{b}\left\{\frac{1}{2}b + \frac{1}{3}a\frac{1-2\hat{\lambda}}{\hat{\lambda}} + \frac{1}{3}(1-\hat{\lambda})\frac{\tau-\tau^*}{(1-\tau)(1-\tau^*)}(3-\tau-2\tau^*)g\right\} \\ &= \frac{1}{2} + \frac{g}{b}\hat{\lambda}(1-\hat{\lambda})\frac{\tau-\tau^*}{(1-\tau)(1-\tau^*)}(2-\tau-\tau^*). \end{aligned}$$

Its derivative with respect to g is

$$\frac{\partial}{\partial g}(y_f + y_h^*) = \frac{1}{b}\frac{\tau-\tau^*}{(1-\tau)(1-\tau^*)}(2-\tau-\tau^*)\left[\hat{\lambda}(1-\hat{\lambda}) + g(1-2\hat{\lambda})\frac{\partial\hat{\lambda}}{\partial g}\right].$$

The derivative is positive for $g = 0$. □

B.3.6 Proposition 2.3

Proof. For $\tau^* = 0$ we can show in the supplement S2 that the exports of Foreign companies increase at $g = 0$. Additionally, we can show that the Home companies' increase in exports is larger than Foreign firms' increase in exports. This implies that consumption in Home will decrease, but increase in Foreign. □

B.3.7 Proposition 2.4

Proof. The changes in prices are given by

$$\frac{\partial p}{\partial g} = \frac{1}{3}\left[-\frac{a}{\hat{\lambda}^2}\frac{\partial\hat{\lambda}}{\partial g} - \frac{(\tau-\tau^*)^2}{(1-\tau)(1-\tau^*)}\left(1-\hat{\lambda}-g\frac{\partial\hat{\lambda}}{\partial g}\right)\right]$$

$$\frac{\partial p^*}{\partial g} = \frac{1}{3}\left[\frac{a}{(1-\hat{\lambda})^2}\frac{\partial \hat{\lambda}}{\partial g} + \frac{(\tau-\tau^*)^2}{(1-\tau)(1-\tau^*)}\left(g\frac{\partial \hat{\lambda}}{\partial g} + \hat{\lambda}\right)\right] > 0.$$

In Foreign, the sign of the price change is straightforward for $\frac{\partial \hat{\lambda}}{\partial g} > 0$. For Home, the derivative is negative if $g = 0$:

$$\frac{\partial p}{\partial g} = \frac{1}{3}\left[-\frac{a}{\hat{\lambda}^2}\frac{\partial \hat{\lambda}}{\partial g} - \frac{(\tau-\tau^*)^2}{(1-\tau)(1-\tau^*)}\left(1-\hat{\lambda}\right)\right] < 0.$$

The signs of the derivative of nominal profits for $g = 0$ and $\tau^* = 0$ are shown with mathematical software in the supplement S2 for Home and Foreign. The Home companies' nominal profits increase, while they decrease for Foreign firms. The derivative of real profits in Foreign is given by

$$\begin{aligned}\frac{\partial(\pi^*/p^*)}{\partial g} &= \frac{\partial\pi^*/\partial g\; p^* - \pi^*\; \partial p^*/\partial g}{p^{*2}} < 0 \\ \text{with} &\quad \partial\pi^*/\partial g < 0 \\ \text{and} &\quad \partial p^*/\partial g > 0\end{aligned}$$

Analogously, it holds for Home firms and their increasing real profits.

$$\begin{aligned}\frac{\partial(\pi/p)}{\partial g} &= \frac{\partial\pi/\partial g\; p - \pi\; \partial p/\partial g}{p^{2}} > 0 \\ \text{with} &\quad \partial\pi/\partial g > 0 \\ \text{and} &\quad \partial p/\partial g < 0\end{aligned}$$

In equilibrium the derivatives of the nominal wages are given by:

$$\frac{\partial w}{\partial g} = \frac{\tau-\tau^*}{1-\tau}\left(1-\hat{\lambda}-g\frac{\partial \hat{\lambda}}{\partial g}\right)$$

$$\frac{\partial w^*}{\partial g} = \frac{\tau - \tau^*}{1 - \tau^*} \left(\hat{\lambda} + g \frac{\partial \hat{\lambda}}{\partial g} \right) > 0$$

In Home, wages will increase, if $g = 0$. As prices decrease in Home, real wages increase in this country. In Foreign, prices as well as nominal wages increase. However, at $\tau^* = 0$, there is no tax income in Foreign and real profits decrease. At the same time, real income increases such that real wages have to increase in Foreign as well. The distribution effect in Foreign follows straightforwardly from the wage and the profit effects. $\square$

B.4 Supplement S2

The supplement S2 (Mathematica Notebook) is available from the authors upon reasonable request.

Appendix C

Appendix to Correlation, Communication and Career Concerns

C.1 Proofs for exogenously correlated noise terms

Throughout the proofs, I assume that the production of public goods follows

$$\begin{aligned} v_1 &= \theta(\alpha e + \beta) + \delta e \\ v_2 &= \theta^\ell(\alpha e_2^\ell + \beta) + \delta e_2^\ell \quad \text{for } \ell \in \{I, C\}, \end{aligned}$$

where ℓ is the winning politician and $\alpha, \beta, \delta \geq 0$ and at least one strictly positive.[1] Competences as well as noise terms are distributed as above

$$\begin{aligned} \theta^{\ell} &\sim N\left(\bar{\theta}, \frac{1}{\tau_{\theta}}\right) \\ \mathbf{x} &\sim MVN\left(0, \frac{1}{\tau_x}, \frac{\rho}{\tau_x}\right). \end{aligned}$$

As the noise terms $\mathbf{x} = (x_1, x_2, ..., x_n)'$ of the voters are multivariate normal distributed, so are the signals.

$$s_i = \theta(\alpha e + \beta) + \delta e + x_i$$

This follows from Hamedani (1992), as all linear combinations of the signals have univariate normal distributions. For realised effort e in the first period, the signals have an equicorrelated multivariate normal distribution with

$$\begin{aligned} E[s_i] &= \bar{\theta}(\alpha e + \beta) + \delta e \\ Var[s_i] &= \frac{(\alpha e + \beta)^2}{\tau_{\theta}} + \frac{1}{\tau_x} \\ Cov[s_i, s_j] &= \frac{(\alpha e + \beta)^2}{\tau_{\theta}} + \frac{\rho}{\tau_x} \\ \rho_s &= \frac{\rho \tau_{\theta} + (\alpha e + \beta)^2 \tau_x}{\tau_{\theta} + (\alpha e + \beta)^2 \tau_x} \end{aligned}$$

In the second period, the elected politician exerts the minimum possible effort level $\tilde{e} > 0$, as he or she only bears the costs of the effort, but has no benefits. Accordingly, voters want to have the more competent candidate in office from which results the voting rule in equation (3.1). An individual determines the

[1] The additive production function used above is given for $\alpha = 0$ and $\beta = \delta = 1$.

expected value of the incumbent's competence given their signal and belief e^* for the incumbent's effort. Competence and signal are bivariate normal distributed with the covariance

$$Cov[\theta, s_i] = \frac{\alpha e + \beta}{\tau_\theta}.$$

Therefore, the expected competence is

$$\begin{aligned} E[\theta|s_i, e^*] &= E[\theta] + \rho_s \left(\frac{Var[\theta]}{Var[s_i]}\right)^{1/2} (s_i - E[s_i|e^*]) \\ &= \bar{\theta} + \frac{Cov[\theta, s_i]}{Var[s_i]}(s_i - \bar{\theta}(\alpha e^* + \beta) - \delta e^*). \end{aligned}$$

An individual voter supports the incumbent if this expected value is larger than what he expects from the challenger's competence $E[\theta^C] = \bar{\theta}$. Therefore, the cut-off signal is

$$\hat{s} = \bar{\theta}(\alpha e^* + \beta) - \delta e^*.$$

The incumbent gets re-elected, if at least half of the electorate votes for her. Therefore, the median's signal gives the election result. If this signal is larger than the cut-off value, the incumbent wins a second term. The median's signal is asymptotically normal distributed with approximated moments according to

$$\begin{aligned} E[s_m] &= \bar{\theta}(\alpha e + \beta) + \delta e \\ Var[s_m] &= \frac{\pi}{2n} Var[s_i] + \left(1 - \frac{\pi}{2n}\right) Cov[s_i, s_j] \\ &= \frac{\pi}{2n}\left[\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_x}\right] + \left(1 - \frac{\pi}{2n}\right)\left[\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{\rho}{\tau_x}\right] \\ &= \frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_x}\left((1-\rho)\frac{\pi}{2n} + \rho\right) \\ &\equiv \frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_m}. \end{aligned}$$

The median signal's variance depends on the number of voters n negatively and on the correlation between noise terms ρ positively.

$$\begin{aligned}
\frac{\partial Var[s_m]}{\partial n} = \frac{\partial}{\partial n}\left(\frac{1}{\tau_m}\right) &= -(1-\rho)\frac{\pi}{2n^2\tau_x} < 0 \\
\frac{\partial Var[s_m]}{\partial \rho} = \frac{\partial}{\partial \rho}\left(\frac{1}{\tau_m}\right) &= \frac{1}{\tau_x}\left(1-\frac{\pi}{2n}\right) > 0
\end{aligned}$$

The re-election probability $p(e, e^*)$ is given by the probability that the median signal is larger or equal to the cut-off value $\hat{s}$. As s_m is asymptotically normal distributed,

$$\begin{aligned}
p(e,e^*) &= Pr[s_m \geq \hat{s}] = 1 - Pr[s_m < \hat{s}] \\
&= 1 - \Phi\left[(\hat{s} - E[s_m])Var[s_m]^{-1/2}\right] \\
&= 1 - \Phi\left[(e^* - e)(\bar{\theta}\alpha + \delta)\left(\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-1/2}\right],
\end{aligned}$$

where Φ is the cumulative distribution function of the standard normal distribution. If the incumbent increases her effort in the first period, she can influence the likelihood of re-election

$$\begin{aligned}
\frac{\partial p(e,e^*)}{\partial e} = {} & \varphi\left[(e^* - e)(\bar{\theta}\alpha + \delta)\left(\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-1/2}\right](\bar{\theta}\alpha + \delta) \\
& \cdot \left[\left(\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-1/2} + (e^* - e)\alpha\frac{\alpha e + \beta}{\tau_\theta}\left(\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-3/2}\right],
\end{aligned}$$

where φ is the probability density function of the standard normal distribution. She utilises it in order to maximise her pay-off $p(e, e^*)R - c(e)$, such that the marginal benefit of an increased re-election chance equals the marginal costs of increased effort

$$0 = \frac{\partial p(e, e^*)}{\partial e} R - c'(e)$$

As in equilibrium the electorate's belief on effort in the first period is correct ($e^* = e$), the incumbents' maximisation result implicitly defines her effort and becomes

$$\begin{aligned} 0 &= \varphi[0](\bar{\theta}\alpha + \delta)\left(\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-1/2} R - c'(e) \\ 0 &= (2\pi)^{-1/2}(\bar{\theta}\alpha + \delta)\left(\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_m}\right)^{-1/2} R - c'(e) \equiv \lambda \end{aligned} \tag{C.1}$$

The supply of public goods in the second period – and hence the expected utility in the second period – is determined by the election outcome and the elected politician's competence. The electoral outcome is

$$d^* = \begin{cases} I & \text{for } \; s_m \geq \bar{\theta}(\alpha e + \beta) + \delta e \\ C & \text{for } \; s_m < \bar{\theta}(\alpha e + \beta) + \delta e \end{cases}$$

The expected utility in the second period with minimum effort $\tilde{e} > 0$ is

$$\begin{aligned} E[v_2] &= \int_{-\infty}^{\infty} \Big[Pr[d^* = I|\theta](\theta(\alpha\tilde{e} + \beta) + \delta\tilde{e}) \\ &\qquad + Pr[d^* = C|\theta](\bar{\theta}(\alpha\tilde{e} + \beta) + \delta\tilde{e})\Big] f(\theta)\ d\theta \\ &= \int_{-\infty}^{\infty} \Big[\theta(\alpha\tilde{e} + \beta) + \delta\tilde{e} \\ &\qquad + (\bar{\theta} - \theta)(\alpha\tilde{e} + \beta) Pr[s_m < \bar{\theta}(\alpha e + \beta) + \delta e|\theta]\Big] f(\theta)\ d\theta \\ &= \int_{-\infty}^{\infty} \Bigg[\theta(\alpha\tilde{e} + \beta) + \delta\tilde{e} \end{aligned}$$

$$+(\bar{\theta}-\theta)(\alpha\tilde{e}+\beta)\Phi\left[(\bar{\theta}-\theta)(\alpha e+\beta)\left(\frac{1}{\tau_m}\right)^{-1/2}\right]\Bigg]f(\theta)\ d\theta$$

Here, it is important that the re-election probability given θ is used. This gives rise to different expected value and and variance of the median signal, which are

$$\begin{aligned} E[s_m|\theta] &= \theta(\alpha e+\beta)+\delta e \\ Var[s_m|\theta] &= \frac{1}{\tau_m} \end{aligned}$$

C.1.1 Proof of Proposition 3.1

The effort is implicitly defined by $\lambda = 0$. Therefore, the effect of ρ on effort is given by

$$\frac{\partial e}{\partial \rho} = -\frac{\partial\lambda/\partial\rho}{\partial\lambda/\partial e} < 0,$$

as

$$\begin{aligned} \frac{\partial\lambda}{\partial\rho} &= R(2\pi)^{-1/2}(\bar{\theta}\alpha+\delta)\left[-\frac{1}{2}\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_m}\right)^{-3/2}\underbrace{\frac{\partial\left(1/\tau_m\right)}{\partial\rho}}_{>0}\right] < 0 \\ \frac{\partial\lambda}{\partial e} &= R(2\pi)^{-1/2}(\bar{\theta}\alpha+\delta)\left[-\frac{1}{2}\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_m}\right)^{-3/2}\frac{2\alpha(\alpha e+\beta)}{\tau_\theta}\right] - c''(e) < 0. \end{aligned}$$

C.1.2 Proof of Proposition 3.2

To show the selection result, the expected utility $E[v_2]$ has to decrease for increased correlation.

$$\frac{\partial E[v_2]}{\partial\rho} = \int_{-\infty}^{\infty}\left[(\bar{\theta}-\theta)(\alpha\tilde{e}+\beta)\varphi\left[(\bar{\theta}-\theta)(\alpha e+\beta)\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_m}\right)^{-1/2}\right]\right.$$

$$
\begin{aligned}
&(\bar{\theta}-\theta)\left\{\alpha\frac{\partial e}{\partial \rho}\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_m}\right)^{-1/2}\right. \\
&\qquad -\frac{1}{2}(\alpha e+\beta)\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_m}\right)^{-3/2} \\
&\qquad\qquad \left.\cdot\left(\frac{2\alpha(\alpha e+\beta)}{\tau_\theta}+\frac{\partial\left(1/\tau_m\right)}{\partial\rho}\right)\right\}\Bigg] f(\theta)\ d\theta \\
= &\left[\alpha\underbrace{\frac{\partial e}{\partial \rho}}_{<0}\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_m}\right)^{-1/2}\right. \\
&\left.-\frac{1}{2}(\alpha e+\beta)\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_m}\right)^{-3/2}\left(\frac{2\alpha(\alpha e+\beta)}{\tau_\theta}+\underbrace{\frac{\partial\left(1/\tau_m\right)}{\partial\rho}}_{>0}\right)\right] \\
&\cdot\int_{-\infty}^{\infty}(\bar{\theta}-\theta)^2(\alpha\tilde{e}+\beta)\ \varphi\left[(\bar{\theta}-\theta)(\alpha e+\beta)\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_m}\right)^{-1/2}\right] f(\theta)\ d\theta<0
\end{aligned}
$$

C.1.3 Proof of Proposition 3.3

Analogously to Aytimur and Bruns (2019), a social planner observes a perfect signal $s=\theta(\alpha e+\beta)+\delta e$. However, they cannot observe e and θ individually and they re-elect the incumbent if $s \geq \bar{\theta}(\alpha e^*+\beta)+\delta e^*$. The signal s is normally distributed with

$$
\begin{aligned}
E[s] &= \bar{\theta}(\alpha e+\beta)+\delta e \\
Var[s] &= \frac{(\alpha e+\beta)^2}{\tau_\theta}
\end{aligned}
$$

Using the same procedure as above to determine the re-election probability and the incumbent's effort, yields the equilibrium effort determined by

$$0 = (2\pi)^{-1/2}(\bar{\theta}\alpha + \delta)\left(\frac{(\alpha e + \beta)^2}{\tau_\theta}\right)^{-1/2} R - c'(e)$$

Comparing this with the equilibrium effort condition for the election with a large electorate in equation (C.1), the variance element $1/\tau_m$ differentiates the two. If the number of voters increases and goes toward infinity, this element of the variance does not vanish.

$$\lim_{n\to\infty} \frac{1}{\tau_x}\left((1-\rho)\frac{\pi}{2n} + \rho\right) = \frac{\rho}{\tau_x}$$

Consequently, the equilibrium effort is smaller if there is an election where the noise terms are correlated.

To determine the effect of a large electorate $n \to \infty$ on electoral selection, I first determine the re-election probability of the incumbent in equilibrium for a given θ.

$$\begin{aligned}
Pr[d^* = I|\theta] &= \lim_{n\to\infty} 1 - Pr[s_m < \bar{\theta}(\alpha e + \beta) + \delta e|\theta] \\
&= \lim_{n\to\infty} 1 - \Phi\left[(\bar{\theta} - \theta)(\alpha e + \beta)(1/\tau_m)^{-1/2}\right] \\
&= 1 - \Phi\left[(\bar{\theta} - \theta)(\alpha e + \beta)(\tau_x/\rho)^{1/2}\right] \begin{cases} > \frac{1}{2} & \text{for } \theta > \bar{\theta} \\ < \frac{1}{2} & \text{for } \theta < \bar{\theta} \end{cases}
\end{aligned}$$

The incumbent is re-elected with a probability larger one half if she is more competent than what is expected from the challenger. The larger the difference in competences, the higher is the re-election probability.

A social planner who observes a perfect signal without noise does not make such mistakes as the median voter. Given the value of the competence θ the social planner votes for the more competent politician.

$$Pr[d^* = I|\theta] = \begin{cases} 1 & \text{for } \theta > \bar{\theta} \\ \\ 0 & \text{for } \theta < \bar{\theta} \end{cases}$$

Comparing the two re-election probabilities it is clear that for every value of θ the social planner has the higher probability to elect the more competent politician and thus gets a higher supply of public goods in the second period. Therefore, the expected utility in the second period over all possible values of θ is larger for the social planner.

C.2 Proofs for communication and differentiated weights

The proofs for Propositions 3.4 and 3.5 are similarly structured and can be jointly presented. In all these cases I assume an additive as well as multiplicative public good production function in the form $v_1 = \theta(\alpha e + \beta) + \delta e$. Voters update their signal according to

$$\begin{aligned} S_i &= \frac{\omega}{\omega + k - 1} s_i + \sum_{i \neq j} \frac{1}{\omega + k - 1} T_{ij} s_j \\ &= \theta(\alpha e + \beta) + \delta e + \frac{1}{\omega + k - 1} \left(\omega x_i + \sum_{j \neq i} T_{ij} x_j \right) \\ &= \theta(\alpha e + \beta) + \delta e + X_i, \end{aligned}$$

where $\omega \geq 1$ is the weighting parameter. An updated signal is normally distributed as it is a sum of normally distributed random variables. Its moments are

$$\begin{aligned}
E[S_i] &= \theta(\alpha e + \beta) + \delta e \\
Var[S_i] &= \frac{(\alpha e + \beta)^2}{\tau_\theta} + Var[X_i] \\
&= \frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{(\omega + k - 1)^2}\Bigg[\omega^2 Var[x_i] + \sum_{j \neq i} T_{ij} Var[x_j] \\
&\quad + 2\omega \sum_{j \neq i} T_{ij} Cov[x_i, x_j] + \sum_{j \neq i} \sum_{f \neq i,j} T_{ij} T_{if} Cov[x_j, x_f]\Bigg] \\
&= \frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{(\omega + k - 1)^2 \tau_x}\left[\omega^2 + k - 1 + (k-1)\rho(2\omega + k - 2)\right]
\end{aligned}$$

Additionally, all updated signals are multivariate normal distributed with common expected value and variance as well as common correlation coefficient.

$$\begin{aligned}
Cov[S_i, S_j] &= E\left[(\theta(\alpha e + \beta) + \delta e + X_i)(\theta(\alpha e + \beta) + \delta e + X_j)\right] - E[S_i]E[S_j] \\
&= E[\theta^2](\alpha e + \beta)^2 + 2E[\theta](\alpha e + \beta)\delta e + (\delta e)^2 - (\bar{\theta}(\alpha e + \beta) + \delta e)^2 \\
&\quad + E[X_i X_j] \\
&= \frac{(\alpha e + \beta)^2}{\tau_\theta} + E\Bigg[\frac{1}{(\omega + k - 1)^2}\left(\omega x_i + \sum_{\epsilon \neq i} T_{i\epsilon} x_\epsilon\right) \\
&\quad \cdot \left(\omega x_j + \sum_{\zeta \neq j} T_{j\zeta} x_\zeta\right)\Bigg] \\
&= \frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{(\omega + k - 1)^2}\Bigg[\omega^2 E[x_i x_j] + \omega \sum_{\zeta \neq j} E[T_{j\zeta}]E[x_i x_\zeta] \\
&\quad + \omega \sum_{\epsilon \neq i} E[T_\epsilon]E[x_j x_\epsilon] + \sum_{\epsilon \neq i} \sum_{\zeta \neq j} E[T_{i\epsilon} T_{j\zeta}]E[x_\epsilon x_\zeta]\Bigg] \\
&= \frac{(\alpha e + \beta)^2}{\tau_\theta}
\end{aligned}$$

$$+\frac{1}{(\omega+k-1)^2\tau_x}\Big[(\omega+k-1)^2\rho \\ +\frac{k-1}{n-1}(1-\rho)\left(2\omega+(n-2)\frac{k-1}{n-1}\right)\Big]$$

The winning politician only exerts the minimum effort $\tilde{e}$. Therefore, voters want to see the more competent politician in office as described in equation (3.1). The expected competence of the incumbent given the updated signal S_i and the expected effort e^* in the first period results from the bivariate normal distributed variables θ and S_i for each voter i. The covariance between competence and an updated signal is

$$\begin{aligned}
Cov[\theta, S_i] &= E\left[\theta\left(\theta(\alpha e+\beta)+\delta e+X_i\right)\right]-E[\theta]E[\theta(\alpha e+\beta)+\delta e+X_i] \\
&= \frac{(\alpha e+\beta)^2}{\tau_\theta}
\end{aligned}$$

Therefore, the expected competence of the incumbent is

$$\begin{aligned}
E[\theta|S_i, e^*] &= E[\theta]+\rho_{\theta,S_i}\left(\frac{Var[\theta]}{Var[S_i]}\right)^{1/2}(S_i-E[S_i]) \\
&= \bar{\theta}+\frac{Cov[\theta,S_i]}{Var[S_i]}\left(S_i-\left(\bar{\theta}(\alpha e^*+\beta)+\delta e^*\right)\right)
\end{aligned}$$

An individual voter i votes in favour of re-election if

$$\begin{aligned}
E[\theta|S_i, e^*] &\geq \bar{\theta} \\
S_i &\geq \bar{\theta}(\alpha e^*+\beta)+\delta e^* \equiv \hat{s}
\end{aligned}$$

For re-election at least half of the electorate has to vote for the incumbent. Accordingly, if the median voter supports the incumbent ($S_M \geq \hat{s}$), she gets re-elected.

As all signals are multivariate normal distributed with common mean, common variance and common correlation coefficient, the result by Owen and Steck (1962) can be applied. The median's signal is asymptotically normal distributed with

$$\begin{aligned}
E[S_m] &= \bar{\theta}(\alpha e + \beta) + \delta e \\
Var[S_m] &= \frac{\pi}{2n} Var[S_i] + \left(1 - \frac{\pi}{2n}\right) Cov[S_i, S_j] \\
&= \frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{(\omega + k - 1)^2 \tau_x} \\
&\quad \cdot \left\{ \frac{\pi}{2n}(\omega^2 + k - 1 + (k-1)\rho(2\omega + k - 2)) \right. \\
&\quad \left. + \left(1 - \frac{\pi}{2n}\right)\left((\omega + k - 1)^2 \rho + \frac{k-1}{n-1}(1-\rho)\left(2\omega + (n-2)\frac{k-1}{n-1}\right)\right)\right\} \\
&= \frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_M}
\end{aligned}$$

The variance of the median's signal is on the one hand determined by the variance of the incumbent's competence. On the other hand, it is determined by the variance $1/\tau_M$ that results from the noise term in the updated signals. This second element changes if one of the variables of interest changes. The derivative with respect to k is

$$\begin{aligned}
\frac{\partial}{\partial k}\left(\frac{1}{\tau_M}\right) &= \frac{1-\rho}{(\omega + k - 1)^3 \tau_x} \left[\left(1 - \frac{\pi}{2n}\right) \frac{2\omega}{(n-1)^2}((n-1)\omega - (k-1)) \right. \\
&\quad \left. - \frac{\pi}{2n}(k - 1 + \omega(2\omega - 1)) \right] < 0 \quad \forall k \geq 2
\end{aligned}$$

At $k = 1$, this derivative is positive. However, comparing this variance with the variance at $k = 2$ shows that the latter is smaller than the former.

$$\left.\frac{1}{\tau_M}\right|_{k=1} > \left.\frac{1}{\tau_M}\right|_{k=2}$$

$$\begin{aligned}\frac{1}{\tau_x}\left((1-\rho)\frac{\pi}{2n}+\rho\right) &> \frac{1}{\tau_x}\left((1-\rho)\frac{\pi}{2n}+\rho\right)\\ &\quad+\frac{1-\rho}{(\omega+1)^2\tau_x}\left\{\left(1-\frac{\pi}{2n}\right)\left(\frac{2\omega}{n-1}+\frac{n-2}{(n-1)^2}\right)-\frac{\pi}{2n}2\omega\right\}\\ \omega\frac{\pi}{n} &> \left(1-\frac{\pi}{2n}\right)\left(\frac{2\omega}{n-1}+\frac{n-2}{(n-1)^2}\right)\end{aligned}$$

This holds for all $\omega \geq 1$ and $n > 2$, where the latter condition is needed to allow for voters having two communication partners.

The second parameter of interest is the weighting parameter ω. Its effect on the variance of the median signal is

$$\begin{aligned}\frac{\partial}{\partial\omega}\left(\frac{1}{\tau_M}\right) &= \frac{1-\rho}{(\omega+k-1)^3\tau_x}\left[\frac{\pi}{n}(k-1)(\omega-1)\right.\\ &\quad\left.+2\left(1-\frac{\pi}{2n}\right)\frac{k-1}{(n-1)^2}(k-1-\omega(n-1))\right]\end{aligned}$$

This derivative with respect to ω is positive for

$$\omega > \frac{1}{(\pi-2)(n-1)}\left[\pi\left(n+\frac{k}{n}\right)-2(k+\pi-1)\right].$$

Additionally, it is positive for all $\omega > {}^{\pi}/_{(\pi-2)}$, independent from the number of voters and the number of sources.

With the normally distributed median signal, I can determine the re-election probability.

$$\begin{aligned}p(e,e^*) &= Pr[S_M \geq \hat{s}] = 1-Pr[S_M < \hat{s}] = 1-\Phi\left[\frac{\hat{s}-E[S_M]}{\sqrt{Var[S_M]}}\right]\\ &= 1-\Phi\left[(e^*-e)(\bar{\theta}\alpha+\delta)\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_M}\right)^{-1/2}\right]\end{aligned}$$

The re-election probability depends on the incumbent's effort in the first period.

$$\begin{aligned}\frac{\partial p(e,e^*)}{\partial e} &= \varphi\left[(e^*-e)(\bar{\theta}\alpha+\delta)\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_M}\right)^{-1/2}\right](\bar{\theta}\alpha+\delta)\\ &\quad\cdot\left[\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_M}\right)^{-1/2}+\alpha(e^*-e)\frac{\alpha e}{\tau_\theta}\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_M}\right)^{-3/2}\right]\end{aligned}$$

The incumbent maximises her pay-off $p(e,e^*)R-c(e)$ by choosing her effort in the first period. Accordingly, her effort is determined by

$$0=\frac{\partial p(e,e^*)}{\partial e}R-c'(e).$$

In equilibrium, voters' beliefs on first period effort are correct, such that $e^*=e$. With this condition, effort in the first period is implicitly defined by

$$\begin{aligned}0 &= \varphi[0](\bar{\theta}\alpha+\delta)\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_M}\right)^{-1/2}R-c'(e)\\ 0 &= (2\pi)^{-1/2}(\bar{\theta}\alpha+\delta)\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_M}\right)^{-1/2}R-c'(e)\equiv\lambda \qquad \text{(C.2)}\end{aligned}$$

Both k and ω affect the the equilibrium effort only via the variance of the median noise. If this variance $1/\tau_M$ increases, the minuend decreases. For the equilibrium, the subtrahend has to decrease as well which is only achieved if the effort decreases.

The expected utility in the second period can be described as well. The electoral outcome depends on the median signal

$$d^*=\begin{cases} I & \text{for } S_M\geq\bar{\theta}(\alpha e+\beta)+\delta e\\ C & \text{for } S_M<\bar{\theta}(\alpha e+\beta)+\delta e\end{cases}$$

If the incumbent is re-elected ($d^* = I$), she exerts the minimum effort $\tilde{e}$ and supply the amount $\theta(\alpha\tilde{e} + \beta) + \delta\tilde{e}$ of public goods. If the challenger gets elected, the equivalent amount of public goods is provided with his expected competence $\bar{\theta}$. The expected utility in the second period therefore is

$$
\begin{aligned}
E[v_2] &= \int_{-\infty}^{\infty} (Pr[d^* = I|\theta](\theta(\alpha\tilde{e} + \beta) + \delta\tilde{e}) \\
&\qquad + Pr[d^* = C|\theta](\bar{\theta}(\alpha\tilde{e} + \beta) + \delta\tilde{e})\Big) f(\theta)\ d\theta \\
&= \int_{-\infty}^{\infty} \left(\theta(\alpha\tilde{e} + \beta) + \delta\tilde{e} + (\bar{\theta} - \theta)\Phi\left[(\bar{\theta} - \theta)(\alpha e + \beta)\left(\frac{1}{\tau_M}\right)^{-1/2}\right]\right) f(\theta)\ d\theta
\end{aligned}
$$

With the implicit definition of effort in the first period and the expected amount of public goods in the second period, I can turn to the effects of changes in the number of sources k as well as in the weighting parameter ω

C.2.1 Proof of Proposition 3.4

The number of signals k an individual observes increases the incumbent's effort. As noted above, k influences the equilibrium effort in equation (C.2) through $1/\tau_M$, which decreases if k is increased leading to higher efforts. For $k \geq 2$, it follows

$$
\frac{\partial e}{\partial k} = -\frac{\partial\lambda/\partial k}{\partial\lambda/\partial e} > 0
$$

This results from

$$
\frac{\partial\lambda}{\partial k} = R(2\pi)^{-1/2}(\bar{\theta}\alpha + \delta)\left[-\frac{1}{2}\left(\frac{(\alpha e + \beta)^2}{\tau_\theta} + \frac{1}{\tau_M}\right)^{-\frac{3}{2}} \underbrace{\frac{\partial\left(1/\tau_M\right)}{\partial k}}_{<0}\right] > 0
$$

$$\begin{aligned}\frac{\partial\lambda}{\partial e} &= R(2\pi)^{-1/2}(\bar{\theta}\alpha+\delta)\left[-\frac{1}{2}\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_M}\right)^{-\frac{3}{2}}\frac{2\alpha(\alpha e+\beta)}{\tau_\theta}\right]\\ &\quad -c''(e)<0. \end{aligned} \tag{C.3}$$

Effort also increases for the step from $k=1$ to $k=2$, as $1/\tau_M$ decreases as shown above.

The expected utility in the second period increases as well. For all $k \geq 2$ it hold

$$\begin{aligned}\frac{\partial E[v_2]}{\partial k} &= \left(\alpha\left(\frac{1}{\tau_M}\right)^{-\frac{1}{2}}\underbrace{\frac{\partial e}{\partial k}}_{>0}-\frac{1}{2}(\alpha e+\beta)\left(\frac{1}{\tau_M}\right)^{-\frac{3}{2}}\underbrace{\frac{\partial\left(1/\tau_M\right)}{\partial k}}_{<0}\right)\\ &\quad\cdot\int_{-\infty}^{\infty}(\bar{\theta}-\theta)^2(\alpha\tilde{e}+\beta)\varphi\left[(\bar{\theta}-\theta)(\alpha e+\beta)\left(\frac{1}{\tau_M}\right)^{-\frac{1}{2}}\right]f(\theta)\ d\theta\ >0.\end{aligned}$$

For the initial increase from $k=1$ to $k=2$ it follows from

$$\begin{aligned} E[v_2]|_{k=1} &< E[v_2]|_{k=2}\\ 0 &> \int_{-\infty}^{\infty}(\bar{\theta}-\theta)(\alpha\tilde{e}+\beta)f(\theta)\\ &\quad\cdot\left\{\Phi\left[(\bar{\theta}-\theta)(\alpha e_{k=1}+\beta)\left(\frac{1}{\tau_M}\Big|_{k=1}\right)^{-1/2}\right]\right.\\ &\quad\left.-\Phi\left[(\bar{\theta}-\theta)(\alpha e_{k=2}+\beta)\left(\frac{1}{\tau_M}\Big|_{k=2}\right)^{-1/2}\right]\right\}\ d\theta \end{aligned} \tag{C.4}$$

As $1/\tau_M$ decreases and e increases with higher k it follows that

$$(\alpha e_{k=1}+\beta)\left(\frac{1}{\tau_M}\Big|_{k=1}\right)^{-1/2}<(\alpha e_{k=2}+\beta)\left(\frac{1}{\tau_M}\Big|_{k=2}\right)^{-1/2}.$$

For all $\theta \lesseqgtr \bar{\theta}$ (and $0 \lesseqgtr \bar{\theta}-\theta$) it thus follows from the properties of the cumulative normal distribution function that

$$\Phi\left[(\bar{\theta}-\theta)(\alpha e_{k=1}+\beta)\left(\frac{1}{\tau_M}\bigg|_{k=1}\right)^{-1/2}\right]$$
$$-\Phi\left[(\bar{\theta}-\theta)(\alpha e_{k=2}+\beta)\left(\frac{1}{\tau_M}\bigg|_{k=2}\right)^{-1/2}\right] \lesseqgtr 0.$$

To obtain the result in equation (C.4), the two cases if θ is larger or smaller than $\bar{\theta}$ have to be distinguished. For $\theta > \bar{\theta}$, the difference in the brackets is positive such that the condition holds, as this positive value is multiplied by the (negative) $\bar{\theta}-\theta$. For $\theta < \bar{\theta}$ the argument is analogous.

C.2.2 Proof of Proposition 3.5

The proof follows the same steps as above. Assume

$$\omega > \frac{1}{(\pi-2)(n-1)}\left[\pi\left(n+\frac{k}{n}\right)-2(k+\pi-1)\right].$$

Under this assumption $\frac{\partial}{\partial\omega}\left(\frac{1}{\tau_{\tilde{M}}}\right) > 0$. If the weighting parameter is lower, the derivative is negative and the following inequalities are reversed. Increasing the weighting parameter further under this assumption yields

$$\frac{\partial e}{\partial\omega} = -\frac{\partial\lambda/\partial\omega}{\partial\lambda/\partial e} < 0,$$

which results from equation (C.3) and

$$\frac{\partial\lambda}{\partial\omega} = R(2\pi)^{-1/2}(\bar{\theta}\alpha+\delta)\left[-\frac{1}{2}\left(\frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_{\tilde{M}}}\right)^{-\frac{3}{2}}\underbrace{\frac{\partial\left(1/\tau_{\tilde{M}}\right)}{\partial\omega}}_{>0}\right] < 0.$$

Further increases in the weighting parameter reduce the expected utility in the second period.

$$\frac{\partial E[v_2]}{\partial \omega} = \left(\alpha \left(\frac{1}{\tau_{\tilde{M}}}\right)^{-\frac{1}{2}} \underbrace{\frac{\partial e}{\partial \omega}}_{<0} - \frac{1}{2}(\alpha e + \beta) \left(\frac{1}{\tau_{\tilde{M}}}\right)^{-\frac{3}{2}} \underbrace{\frac{\partial \left({}^{1}/_{\tau_{\tilde{M}}}\right)}{\partial \omega}}_{>0} \right)$$
$$\cdot \int_{-\infty}^{\infty} (\bar{\theta} - \theta)^2 (\alpha \tilde{e} + \beta) \varphi \left[(\bar{\theta} - \theta)(\alpha e + \beta) \left(\frac{1}{\tau_{\tilde{M}}}\right)^{-\frac{1}{2}} \right] f(\theta)\ d\theta\ < 0.$$

C.3 Proof of Proposition 3.6

Again, I assume an additive as well as multiplicative public good production function in the form $v_1 = \theta(\alpha e + \beta) + \delta e$. Each voter weights all observed signals equally such that

$$\begin{aligned} S_i &= \frac{1}{k} \sum_j T_{ij} s_j \\ &= \theta(\alpha e + \beta) + \delta e + \frac{1}{k} \sum_j T_{ij} x_j \\ &= \theta(\alpha e + \beta) + \delta e + X_i \end{aligned}$$

The updated noise terms are multivariate normal distributed with

$$\begin{aligned} E[X_i] &= 0 \\ Var[X_i] &= \frac{1}{k^2} \left[\sum_j T_{ij} Var[x_j] + \sum_j \sum_{\epsilon \neq j} T_{ij} T_{j\epsilon} Cov[x_j, x_\epsilon] \right] \\ &= \frac{1}{k^2} \left[\sum_j T_{ij} \frac{1}{\tau_x} + \sum_j \sum_{\epsilon \neq j} T_{ij} T_{j\epsilon} \frac{\rho}{\tau_x} \right] \\ &= \frac{1}{k^2 \tau_x} \left[k + k(k-1)\rho \right] \end{aligned}$$

$$= \frac{1}{k\tau_x}\left[1+(k-1)\rho\right]$$

The covariance between the noise terms of two voters who are not part of the same group is zero as they cannot share common sources. Voters who are chosen randomly from the same group may share some communication partners and have a positive covariance

$$\begin{aligned}
Cov[X_i, X_j] &= E[X_i X_j] - E]X_i]E[X_j] \\
&= E\left[\frac{1}{k^2}\sum_{\epsilon} T_{i\epsilon}x_\epsilon \sum_{\zeta} T_{j\zeta}x_\zeta\right] \\
&= \frac{1}{k^2}E\left[x_i x_j + x_i\sum_{\zeta\neq j} T_{j\zeta}x_\zeta + x_j\sum_{\epsilon\neq i} T_{i\epsilon}x_\epsilon + \sum_{\epsilon\neq i}\sum_{\zeta\neq j} T_{i\epsilon}T_{j\zeta}x_\epsilon x_\zeta\right] \\
&= \frac{1}{k^2\tau_x}\left[k^2\rho + (1-\rho)\frac{k-1}{q-1}\left(k+1-\frac{k-1}{q-1}\right)\right]
\end{aligned}$$

This allows to construct the variance-covariance matrix of noise terms

$$\mathbf{\Sigma} = \begin{pmatrix} \mathbf{\Sigma}_g & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_g & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Sigma}_g \end{pmatrix}, \text{ with } \mathbf{\Sigma}_g = \frac{1}{k\tau_x}\begin{pmatrix} 1 & \rho_S & \cdots & \rho_S \\ \rho_S & 1 & \cdots & \rho_S \\ \vdots & \vdots & \ddots & \vdots \\ \rho_S & \rho_S & \cdots & 1 \end{pmatrix},$$

where $\mathbf{\Sigma}_g$ is a $q \times q$ matrix and

$$\rho_S = \left[k^2\rho + (1-\rho)\frac{k-1}{q-1}\left(k+1-\frac{k-1}{q-1}\right)\right]\left[k+k(k-1)\rho\right]^{-1}.$$

Each individual's voting decision is determined by the voting rule in (3.1) resulting in the cut-off signal $\hat{s} = \bar{\theta}(\alpha e^* + \beta) + \delta e^*$, over which a voter supports the incumbent. The (updated) median signal determines the outcome of the election. As all signals consist of the supply of public goods and the noise terms, the former affects all signals equally. An increase in public goods in the first period affects all signals equally. Hence, this element does not change the order statistics of the signals. Only the (updated) noise terms determine the order statistics as they are not fully correlated and hence may differ between voters. As a result, if the median of the noise terms is determined, this can be used to also determine the value of the median signal.

According to Rawlings (1976), the first moment of the median of unequally correlated multivariate normal random variables can be approximated for a variance-covariance structure as given above with

$$\begin{aligned} E[X_m] &= (Var[X_i])^{1/2}(1-\rho_a)^{1/2}E[Z_m] + E[X_i] \\ \text{with } \rho_a &= \frac{q-1}{n-1}\rho_S \\ \text{and } Z &\sim N(0,1). \end{aligned}$$

He shows that this approximation, where the average correlation coefficient is used, is acceptable for $\rho_S < 1/2$. In the working paper version Rawlings (1975), the author also shortly discusses the second moment of the median, but does not come to a conclusion, whether the same procedure is applicable here with

$$Var[X_m] = Var[X_i]\left[(1-\rho_a)Var[Z_m] + \rho_a\right].$$

Therefore, I verify the result with a Monte-Carlo simulation, which is reported in appendix C.4. For $\rho_S < 1/4$, this approximation to the variance is applicable.

To accommodate this constraint I introduce the condition $k < {}^{q}\!/_{4}$. Each voter may only observe up to a quarter of the signals in his group. Additionally, the intra-group correlation between the noise terms ρ has to be constrained as well. Therefore I assume in this section

$$\rho < \frac{4q - 4k^2(q-2) + k(q^2 - 2q - 7)}{4q + k(q^2 - 2q - 7) + k^2(3q^2 - 10q + 11)}.$$

The asymptotically normal distributed median noise has the two moments

$$\begin{aligned}
E[X_{\mathcal{M}}] &= 0 \\
Var[X_{\mathcal{M}}] &= Var[X_i]\left[(1-\rho_a)\frac{\pi}{2n} + \rho_a\right] \\
&= \frac{\pi}{2n}\left[Var[X_i] - \frac{q-1}{n-1}Cov[X_i, x_j]\right] + \frac{q-1}{n-1}Cov[X_i, X_j] \\
&= \frac{1}{(n-1)k\tau_x}\left[\left(1 - \frac{\pi}{2n}\right)\left((1-\rho)\frac{k-1}{k}\left(k + 1 - \frac{k-1}{q-1}\right) + k(q-1)\rho\right)\right. \\
&\qquad \left. + \frac{\pi}{2n}(n-1)(1 + (k-1)\rho)\right] \\
&\equiv \frac{1}{\tau_{\mathcal{M}}}
\end{aligned}$$

The parameters of interest are the number of voters per group q as well as the number of groups g. However, these two are in a close relationship, where $n = g \cdot q$. To determine the change in the variance if q or g are increased, this relationship has to be taken into account. I do this in two extreme ways. On the one hand, I assume that the number of voters is fixed $n = \bar{n}$ and an increase in group size has to reduce the number of groups and vice versa.

$$\frac{\partial g}{\partial q} = -\frac{n}{q^2} \quad \text{and} \quad \frac{\partial q}{\partial g} = -\frac{n}{g^2}$$

On the other hand, I assume that the number of groups $g = \bar{g}$ (the group size $q = \bar{q}$) is fixed, if q (g) is increased. In this case, the overall number of voters has to change.

$$\frac{\partial n}{\partial q} = g \quad \text{and} \quad \frac{\partial n}{\partial g} = q$$

With these results, the changes in the variance of the median noise can be determined, if q and g change.

$$\begin{aligned}
\frac{\partial}{\partial q}\left(\frac{1}{\tau_{\mathcal{M}}}\right)\Bigg|_{n=\bar{n}} &= \frac{1}{k\tau_x}\frac{1}{n-1}\left(1-\frac{\pi}{2n}\right)\left(k\rho+\frac{(k-1)^2}{k(q-1)^2}(1-\rho)\right) > 0 \\
\frac{\partial}{\partial g}\left(\frac{1}{\tau_{\mathcal{M}}}\right)\Bigg|_{n=\bar{n}} &= -\frac{1}{k\tau_x}\frac{1}{n-1}\frac{q^2}{n}\left(1-\frac{\pi}{2n}\right)\left(k\rho+\frac{(k-1)^2}{k(q-1)^2}(1-\rho)\right) < 0 \\
\frac{\partial}{\partial q}\left(\frac{1}{\tau_{\mathcal{M}}}\right)\Bigg|_{g=\bar{g}} &= \frac{1}{k\tau_x}\Bigg[(1-\rho)\frac{k-1}{k}\frac{1}{(n-1)^2(q-1)^2}\Bigg\{(n-1)(k-1)\left(1-\frac{\pi}{2n}\right) \\
&\qquad -n\frac{q-1}{1}(k(q-2)+q)\left(1-(2n-1)\frac{\pi}{2n^2}\right)\Bigg\} \\
&\qquad +k\rho\frac{1}{(n-1)^2}\Bigg\{(n-1)\left(1-\frac{\pi}{2n}\right) \\
&\qquad -n\frac{q-1}{q}\left(1-(2n-1)\frac{\pi}{2n^2}\right)\Bigg\} \\
&\qquad -\frac{\pi}{2n}\frac{1}{q}(1+(k-1)\rho)\Bigg] < 0 \\
\frac{\partial}{\partial g}\left(\frac{1}{\tau_{\mathcal{M}}}\right)\Bigg|_{q=\bar{q}} &= \frac{1}{k\tau_x}\Bigg[(1-\rho)\frac{k-1}{k}\frac{1}{(n-g)^2(n-1)^2}\Bigg\{kn(4n-2g-1)+n \\
&\qquad -\frac{n^3}{g}(k+1)-\left(1-\frac{\pi}{2n}\right)n(n-1)(k-1) \\
&\qquad +\frac{\pi}{2n}\Bigg(\frac{n^2}{g}(2n-g-1) \\
&\qquad +k\left((2n-1)\left(\frac{n^2}{g}+2g-3n\right)-n(n-1)\right)\Bigg)\Bigg\} \\
&\qquad +k\rho\frac{1}{g^2(n-1)^2}\Bigg\{n(g-1)+\frac{\pi}{2n}\left(n^2-g(2n-1)\right) \\
&\qquad -n(n-1)\left(1-\frac{\pi}{2n}\right)\Bigg\}
\end{aligned}$$

$$-\frac{\pi}{2n}\frac{1}{g}(1+(k-1)\rho)\Bigg] < 0$$

The median noise term then determines the median signal, such that

$$\begin{aligned}
S_{\mathcal{M}} &= \theta(\alpha e+\beta)+\delta e+X_{\mathcal{M}}\\
E[S_{\mathcal{M}}] &= \bar{\theta}(\alpha e+\beta)+\delta e\\
Var[S_{\mathcal{M}}] &= \frac{(\alpha e+\beta)^2}{\tau_\theta}+\frac{1}{\tau_{\mathcal{M}}}
\end{aligned}$$

These results are analogous to the calculations above. It is not necessary to repeat the derivations for the effort in the first period as well as the expected utility in the second period. The changes in the median noise's variance suffices to determine the effects. Therefore, it follows

$$\begin{aligned}
\left.\frac{\partial e}{\partial q}\right|_{n=\bar{n}} &< 0 \quad \text{and} \quad \left.\frac{\partial E[v_2]}{\partial q}\right|_{n=\bar{n}} < 0\\
\left.\frac{\partial e}{\partial g}\right|_{n=\bar{n}} &> 0 \quad \text{and} \quad \left.\frac{\partial E[v_2]}{\partial g}\right|_{n=\bar{n}} > 0\\
\left.\frac{\partial e}{\partial q}\right|_{g=\bar{g}} &> 0 \quad \text{and} \quad \left.\frac{\partial E[v_2]}{\partial q}\right|_{g=\bar{g}} > 0\\
\left.\frac{\partial e}{\partial g}\right|_{q=\bar{q}} &> 0 \quad \text{and} \quad \left.\frac{\partial E[v_2]}{\partial g}\right|_{q=\bar{q}} > 0.
\end{aligned}$$

C.4 Monte-Carlo-Simulation

In order to validate the variance of the median for the grouped electorate I employ a Monte-Carlo-simulation. By drawing the variance of the median multiple times I can determine a confidence interval for this variance which can then be compared to the approximation.[2]

For each of the parameter value combinations of g, q and ρ_S mathematica draws 1000 observations of multivariate normal distributed variables. For the distribution I assume that the (common) expected value is zero and the (common) variance is one for simplicity. The variance-covariance matrix is given by

$$\mathbf{\Sigma} = \begin{pmatrix} \mathbf{\Sigma}_g & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_g & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Sigma}_g \end{pmatrix}, \text{ with } \mathbf{\Sigma}_g = \begin{pmatrix} 1 & \rho_S & \cdots & \rho_S \\ \rho_S & 1 & \cdots & \rho_S \\ \vdots & \vdots & \ddots & \vdots \\ \rho_S & \rho_S & \cdots & 1 \end{pmatrix},$$

where $\mathbf{\Sigma}$ is a $g \cdot q \times g \cdot q$ matrix and $\mathbf{\Sigma}_g$ is a $q \times q$ matrix. Accordingly, the variance-covariance matrix is analogous to the structure above. For each of the observations I determine the median value and compute the variance of the median value from the 1000 draws. I repeat this procedure 50 times such that for each of the parameter combinations a confidence interval can be computed.

Subsequently, I determine the approximation to the median's variance using Rawling's method as well as Cramér's result. This results in the approximation

$$Var[X_m] \;=\; \left(1 - \frac{q-1}{q \cdot g - 1}\rho_S\right) \frac{\pi}{2 \cdot q \cdot g} + \frac{q-1}{q \cdot g - 1}\rho_S.$$

As Cramér's result holds asymptotically, I do not consider small electorates. In Tables 1-3 I show that the approximation lies within the 99% confidence interval for all combinations of values $g = \{2, 3, 4, 5, 10\}$, $q = \{50, 60, 70, 80, 90, 100\}$ and $\rho_S = \{1/20, 3/20, 1/4\}$.

[2]The mathematica notebook is available in supplement S3.

	$g = 2$	$g = 3$	$g = 4$	$g = 5$	$g = 10$
$q = 50$	0.040067 $[0.0390337, 0.0405182]$	0.026743 $[0.0264281, 0.0272173]$	0.020069 $[0.0195825, 0.0202881]$	0.016061 $[0.0158746, 0.0164021]$	0.0080360 $[0.00795017, 0.00821222]$
$q = 60$	0.037555 $[0.037171, 0.0384758]$	0.025063 $[0.0246188, 0.0255371]$	0.018807 $[0.018178, 0.0189495]$	0.015051 $[0.0148542, 0.0153501]$	0.0075300 $[0.00743287, 0.00767818]$
$q = 70$	0.035762 $[0.035219, 0.0363648]$	0.023864 $[0.0235604, 0.0244485]$	0.017906 $[0.0175437, 0.0180984]$	0.014329 $[0.0141367, 0.0145737]$	0.0071685 $[0.00703773, 0.00729985]$
$q = 80$	0.034416 $[0.0338386, 0.0349292]$	0.022964 $[0.0225515, 0.0234357]$	0.017230 $[0.0168951, 0.0175099]$	0.013788 $[0.0135325, 0.0139749]$	0.0068975 $[0.00674409, 0.007035]$
$q = 90$	0.033370 $[0.0325625, 0.0337628]$	0.022264 $[0.0219312, 0.022744]$	0.016705 $[0.0161694, 0.016821]$	0.013367 $[0.013269, 0.0137119]$	0.0066866 $[0.00656206, 0.0068226]$
$q = 100$	0.032533 $[0.0319648, 0.0331106]$	0.021704 $[0.0213223, 0.0220286]$	0.016284 $[0.0158847, 0.0164091]$	0.013030 $[0.0127456, 0.0131933]$	0.0065180 $[0.0064458, 0.00665896]$

Table C.1: Monte-Carlo-Simulation with $\rho_S = {}^1\!/_{20}$

Note: The first line gives the approximation and the second line the confidence interval that follows from the Monte-Carlo-Simulation.

	$g = 2$	$g = 3$	$g = 4$	$g = 5$	$g = 10$
$q = 50$	0.088784 $[0.0862169, 0.0895065]$	0.059284 $[0.0580333, 0.0598606]$	0.044499 $[0.0438751, 0.0452996]$	0.035616 $[0.034795, 0.0359742]$	0.017825 $[0.0175756, 0.0183022]$
$q = 60$	0.086486 $[0.0853524, 0.0883279]$	0.057737 $[0.0574834, 0.0591533]$	0.043332 $[0.0424366, 0.0439098]$	0.034680 $[0.0345093, 0.0357527]$	0.017354 $[0.016961, 0.0175946]$
$q = 70$	0.084845 $[0.0832771, 0.0865305]$	0.056631 $[0.0560015, 0.0579457]$	0.042499 $[0.0418487, 0.0435378]$	0.034011 $[0.0338397, 0.0347988]$	0.017018 $[0.0166516, 0.0172005]$
$q = 80$	0.083614 $[0.0824192, 0.0851957]$	0.055802 $[0.0542347, 0.0562266]$	0.041874 $[0.0411546, 0.0424923]$	0.033510 $[0.0324879, 0.0338087]$	0.016765 $[0.0163564, 0.0169042]$
$q = 90$	0.082657 $[0.0813756, 0.0840151]$	0.055157 $[0.0535025, 0.055558]$	0.041388 $[0.0409953, 0.0422844]$	0.033120 $[0.0325164, 0.0337997]$	0.016569 $[0.0164451, 0.0170329]$
$q = 100$	0.081891 $[0.0799765, 0.0828205]$	0.054641 $[0.0542371, 0.0560845]$	0.040999 $[0.0401711, 0.0415745]$	0.032808 $[0.0321465, 0.0331537]$	0.016412 $[0.0160685, 0.016638]$

Table C.2: Monte-Carlo-Simulation with $\rho_S = {}^3\!/_{20}$

Note: The first line gives the approximation and the second line the confidence interval that follows from the Monte-Carlo-Simulation.

	$g = 2$	$g = 3$	$g = 4$	$g = 5$	$g = 10$
$q = 50$	0.13750 $[0.133888, 0.137949]$	0.091826 $[0.0919549, 0.0951166]$	0.068928 $[0.0685471, 0.0707434]$	0.055171 $[0.0542092, 0.0560041]$	0.027614 $[0.0274308, 0.0282537]$
$q = 60$	0.13542 $[0.132999, 0.137659]$	0.090410 $[0.0891846, 0.0920908]$	0.067857 $[0.0675977, 0.0698782]$	0.054309 $[0.0538172, 0.0553859]$	0.027178 $[0.0271196, 0.0281299]$
$q = 70$	0.13393 $[0.13254, 0.136753]$	0.089399 $[0.0873745, 0.0908377]$	0.067091 $[0.0659794, 0.0685606]$	0.053693 $[0.0534182, 0.0549933]$	0.026867 $[0.0267655, 0.027515]$
$q = 80$	0.13281 $[0.131303, 0.135816]$	0.088640 $[0.0869438, 0.0905304]$	0.066517 $[0.0656004, 0.0679096]$	0.053231 $[0.052514, 0.0545038]$	0.026633 $[0.0263405, 0.0271211]$
$q = 90$	0.13194 $[0.131028, 0.136316]$	0.088050 $[0.0865134, 0.0891111]$	0.066071 $[0.0654248, 0.0678754]$	0.052872 $[0.0521809, 0.0540497]$	0.026452 $[0.0262617, 0.0270716]$
$q = 100$	0.13125 $[0.128422, 0.132069]$	0.087578 $[0.0875743, 0.0905623]$	0.065713 $[0.0652485, 0.0673899]$	0.052585 $[0.0520848, 0.0543005]$	0.026307 $[0.0261862, 0.0270877]$

Table C.3: Monte-Carlo-Simulation with $\rho_S = {}^1\!/_4$

Note: The first line gives the approximation and the second line the confidence interval that follows from the Monte-Carlo-Simulation.

C.5 Supplement S3

The supplement S3 (Mathematica Notebook) is available from the author upon reasonable request.

Declaration of Co-Authorship

I hereby declare that this thesis incorporates material that is the result of joint research as follows:

Chapter 1: "Asymmetric General Oligopolistic Equilibrium" is based on joint work with Jonas F. Rudsinske. The concept and the theoretical analysis were developed and conducted jointly and in equal shares, just like the writing was shared equally. Everything was mutually discussed and improved such that this chapter should be considered as joint work.

Chapter 2: "International Trade and Tax-Motivated Transfer Pricing" is based on joint work with Jonas F. Rudsinske. The concept and the theoretical analysis were developed and conducted jointly and in equal shares, just like the writing was shared equally. Everything was mutually discussed and improved such that this chapter should be considered as joint work.

Chapter 3: "Correlation, Communication and Career Concerns" is single-authored.

Chapter 4: "Accountability in Elections with Regional Structures" is single-authored.

Göttingen, 29.09.2023

Ansgar F. Quint

Declaration for Admission to the Doctoral Examination

Ph.D. program in Economics

I confirm

1. that the dissertation "Theoretic Essays in Public Economics – Profit Shifting in General Oligopolistic Equilibrium and the Role of Information in Career Concern Models" that I submitted was produced independently without assistance from external parties, and not contrary to high scientific standards and integrity,

2. that I have adhered to the examination regulations, including upholding a high degree of scientific integrity, which includes the strict and proper use of citations so that the inclusion of other ideas in the dissertation are clearly distinguished,

3. that in the process of completing this doctoral thesis, no intermediaries were compensated to assist me neither with the admissions or preparation processes, and in this process,

   - no remuneration or equivalent compensation were provided
   - no services were engaged that may contradict the purpose of producing a doctoral thesis

4. that I have not submitted this dissertation or parts of this dissertation elsewhere.

I am aware that false claims and the discovery of those false claims now, and in the future with regards to the declaration for admission to the doctoral examination can lead to the invalidation or revoking of the doctoral degree.

Göttingen, 29.09.2023

Ansgar F. Quint